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Invariant means of sequences with statistical behaviour

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Abstract: The current paper is thrusting to bring out some techniques of spaces by defining st_{σ} and $st_{\sigma_\emptyset}$ of sigma strongly convergence in statistically nature and lacunary nature of convergent sequences of strong sigma statistically in nature. Some basic topological properties will be given.

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1. Introduction and historical background: We call a sequence to be function whose domain is the set of natural numbers. Let us represent W as set of all real or complex sequences, so the sequence space is any subspace of W. With N, R and C we designate the set of non-negative integers, the set of real numbers and the set of complex numbers, respectively. Let l_∞, c and c_0, respectively, designates the set of bounded sequences, convergent sequences and those which has limit as zero [1], [8], 11-[15], [18]-[21].

Cesàro sums represents an "averaging" process. In 1890 the Italian mathematician Ernesto Cesàro used such sums while investigating products of infinite series. A expansion of the type $\sum_{r=0}^{\infty} v_r$ is said to be Cesaro summable to $L \in R$ if and only if its Cesaro sum converges i.e.,

$$\sigma_n = \sum_{r=0}^{n-1}\left[1-\frac{r}{n}\right]v_r = \frac{s_0+s_1+\cdots+s_{n-1}}{n} = \frac{1}{n}\sum_{r=0}^{n-1} s_r$$

converges to L as $i \to \infty$.

Thus, for a sequence v_r with $\sum_{r=0}^{\infty} v_r = L$ converges if and only if its sequence of partial sums s_n converges to L i.e., given $\varepsilon > 0$, we can find a natural number $n_0 \in N$ in such a way that $n \geq n_0$ implies $|s_n - L| < \varepsilon$ or equivalently, if $|\sigma_n - L| < \varepsilon$ then $\sum_{r=0}^{\infty} v_r$ is Cesàro is summable [22]. Then, we can have

$$|\sigma_n - L| = \left|\frac{s_0+s_1+\cdots+s_{n-1}}{n} - L\right| = \left|\sum_{r=0}^{n-1}\left[1-\frac{r}{n}\right]v_r - L\right|.$$

The natural density of for the set of natural numbers K is defined as

$$\delta(K) = \lim_{n\to\infty} \frac{1}{n} |K_n|$$

only if limit is finite, where $K_n = \{r \in K : r \leq n\}$ and $|.|$ represents the cardinality of the set.

Definition 1.1: We call a sequence $v = (v_j)$ to be statistically convergent to L if given $\varepsilon > 0$, we have

$$\lim_{n\to\infty} \frac{1}{n} |\{r \leq n : |v_r - L| \geq \varepsilon\}| = 0,$$

that is, $K = K(\varepsilon) = \{r \in K : |v_r - L| \geq \varepsilon\}$ has natural density zero as can be seen in [5]-[7]. With this symbolization, we represent it as $st - lim\, v = L$. With st represents the set of all statistically convergent sequences will be represented.

The idea of statistical convergence was introduced in [4] and studied by several authors as can be seen in [1]-[3], [5], [6], [22] and many more. There is a well known relationship between statistical convergence and strong Cesàro summability [2]:

$$\sigma_1 = \left\{ v = (v_n), \exists\ some\ L \ni : \lim \left(\frac{1}{n} \sum_{k=1}^{n} |v_r - L| \right) = 0 \right\}.$$

Definition 1.2: By a lacunary sequence we mean an increasing sequence $\emptyset = (k_r)$ of integers, such that $k_0 = 0$ and $h_r = k_r - k_{r-1} \to \infty$. Throughout the text the intervals determined by $\emptyset$ will be denoted by $I_r = (k_{r-1}, k_r]$ and the ratio $\frac{k_r}{k_{r-1}}$ will be abbreviated by q_r [1], [10] etc.

Definition 1.3: An Orlicz function is a function $\beth: [0,\infty) \to [0,\infty)$ which is continuous, non-decreasing and convex with $\beth(0) = 0, \beth(v) > 0$ for $v > 0$ and $\beth(v) \to \infty$ as $v \to \infty$ and can be further seen in [9],[17].

It is to be noted that if the convexity of Orlicz functions $\beth$ is replaced by

$$\beth(x + y) \leq \beth(x) + \beth(y)$$

then the function is called modulus function [16].

Let $\emptyset$ be a lacunary sequence; the number sequence v is $S_\emptyset -$ convergent to L provided that for every $\varepsilon > 0$, we have

$$\lim_{r} \frac{1}{h_r} \{k \in I_r : |v_k - L| \geq \varepsilon\} = 0.$$

In this case we write $S_\emptyset$-limit $v = L$ or $v_k \to L(S_\emptyset)$, and we define

$$S_\emptyset = \{v = (v_n), \exists\ some\ L \ni : S_\emptyset - \text{limit}\ v = L\}.$$

2. Main result

In this portion, we will now introduce new sequence spaces. Also, we will investigate inclusion relations between these new spaces and strongly σ statistically convergent and lacunary strongly σ-statistically convergent.

Definition 2.1: Let $\mathscr{M}$ be an Orlicz and $\emptyset = (k_r)$ be lacunary sequence. We define the following new spaces:

$$S_\sigma[\mathscr{M}]^{\emptyset} = \left\{ v = (v_k) : \lim_r \frac{1}{h_r} \sum_{k \in I_r} \mathscr{M}|(\zeta_{kr}(v_k - Le))| = 0, \quad uniformly\ in\ r \right\}$$

and

$$S_\sigma[\mathscr{M}] = \left\{ v = (v_k) : \lim_n \frac{1}{n} \sum_{k=0}^{m} \mathscr{M}|(\zeta_{kr}(v_k - Le))| = 0, uniformly\ in\ r \right\}.$$

Definition 2.2: We call a sequence $v = (v_k)$ to be strongly σ-statistically convergent to L if given $\varepsilon > 0$, we have

$$\lim_n \sup_r \frac{1}{n} |\{0 \le k \le n: |\zeta_{kr}(v_k - Le)| \ge \varepsilon\}| = 0.$$

We symbolize it as $S_\emptyset - \lim v = Le$.

Definition 2.3: For a lacunary sequence $\emptyset = (k_r)$, we call a sequence $v = (v_k)$ to be lacunary strongly σ-statistically convergent to L if given $\varepsilon > 0$, we have

$$\lim_n \sup_r \frac{1}{h_r} |\{k \in I_r : |\zeta_{kr}(v_k - Le)| \ge \varepsilon\}| = 0.$$

We symbolize it as $S_{\emptyset_\sigma} - \lim v = Le$.

We have following results which we state without proof.

Theorem 2.4: If $\mathscr{M}$ is an Orlicz function with $0 < \delta < 1$, then,

$$\mathscr{M}(|\zeta_{kr}|) \le 2\mathscr{M}(1)\delta^{-1}|\zeta_{kr}|.$$

Theorem 2.5: If $\mathscr{M}$ is an Orlicz function, then, $S_\sigma[\mathscr{M}] \subset S_{\emptyset_\sigma}$.

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