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Alpha Logarithm Transformed Rayleigh Distribution: Properties

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Abstract: In this paper, we introduce a new two parameter Alpha Logarithm Transformed Rayleigh (ALTR)distribution. A new method has been proposed to introduce an extra parameter to a family of distributions for more flexibility .The proposed family may be named as Alpha Logarithm Transformed Rayleigh. This can be obtained via reparameterizing the exponentiated KumaraswamyG-logarithmic family and the alpha logarithmic family of distributions. The recommended distribution reveals increasing, decreasing and bath-shaped probability Density, Distribution and Hazard rate function and Survival function. Some distributional properties of new model are investigated which include the Density Function, Distribution Function (DF), Quantile Function(QF), Moments, Moment Generating Function (MGF), Cumulative Generating Function (CGF).

Keywords: **ALTR, Probability density function, Cumulative distribution function, Hazard function and Survival function, Properties**

Introduction

Lord Rayleigh (1880) introduced the Rayleigh distribution in connection with a problem in the field of acoustics. Since then, extensive work has taken place related to this distribution in different areas of science and technology. It has some nice relations with some of the well known distributions like Weibull, Chi-square or extreme value distributions. Abd-Elfattah (2006) studied the Efficiency of maximum likelihood estimators under different censored sampling schemes for Rayleigh distribution. Dey et al. (2017) introduced the alpha power exponential (APE) and alpha power transformed Weibull (APTW) distributions, respectively. Vijaya lakshmi *et al* (2018), studied the Estimation of Location (μ) and Scale (λ) for Two-Parameter Rayleigh Distribution by Median Rank Regression Method. A new three parameter α Logarithmic Transformed Family of Distributions with Application introduced sunku dey et al (2017). Vijaya Lakshmi and Anjaneyulu (2019) studied Quadratic Rank Transmuted Half Logistic Lomax Distribution: Properties and Application.A Flexible Reduced Logarithmic-X Family of Distributions with Biomedical Analysis. A Flexible Reduced Logarithmic-X Family of Distributions with Biomedical Analysis introduced YinglinLiu et al. (2020).The chapter is organized as follows. The new distribution is developed in Section 1.1 and also we define the CDF, density function, in section 1.2 survival and hazard functions of the Alpha Logarithm Transformed Rayleigh (ALTR) distribution.A comprehensive account of statistical properties of the new distribution is provided in Section 1.3. In section 1.4, 1.5 we discuss Moment Generating Function and Cumulative Generating Function for ALTR distribution.

1.1 Probability density and distribution functions of ALTR

A random variable X ~ ALTR(σ^2, α) has probability density function and is in the form

$$f(x) = \frac{\alpha(\alpha-1)\frac{x}{\sigma^2}e^{-x^2/2\sigma^2}}{\log(\alpha)\left\{\alpha-(\alpha-1)[1-e^{-x^2/2\sigma^2}]\right\}} \ldots(1),\ x > 0,\ (\sigma^2, \alpha) > 0 \ldots(1)$$

(σ^2, α) are scale and shape parameters

A random variable X ~ ALTR(σ^2, α) has cumulative distribution function and is in the form

$$F(x) = 1-\frac{\log\left\{\alpha-(\alpha-1)[1-e^{-x^2/2\sigma^2}]\right\}}{\log(\alpha)} \ldots(2),\ x > 0,\ (\sigma^2, \alpha) > 0 \ldots(2)$$

Limits of the ALTR function

The limit of the Probability Distribution Function is given by

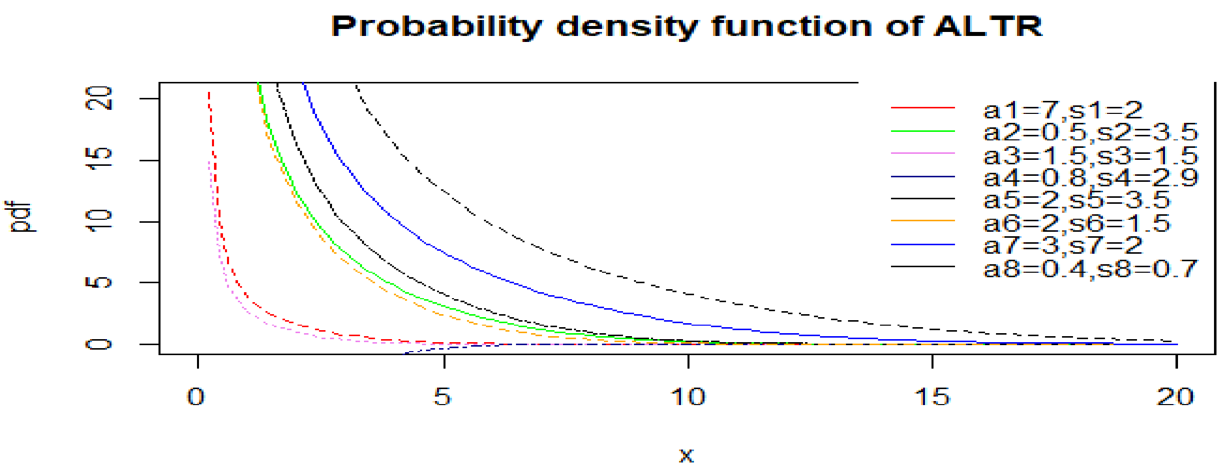
$$\lim_{x\to 0} F(x;\sigma^2,\alpha) = \lim_{x\to 0} 1 - \frac{log\left\{\alpha-(\alpha-1)[1-e^{-x^2/2\sigma^2}]\right\}}{\log(\alpha)} = 0 \quad \ldots (3)$$

$$\lim_{x\to \infty} F(x;\sigma^2,\alpha) = \lim_{x\to \infty} 1 - \frac{log\left\{\alpha-(\alpha-1)[1-e^{-x^2/2\sigma^2}]\right\}}{\log(\alpha)} = 1 \ldots (4)$$

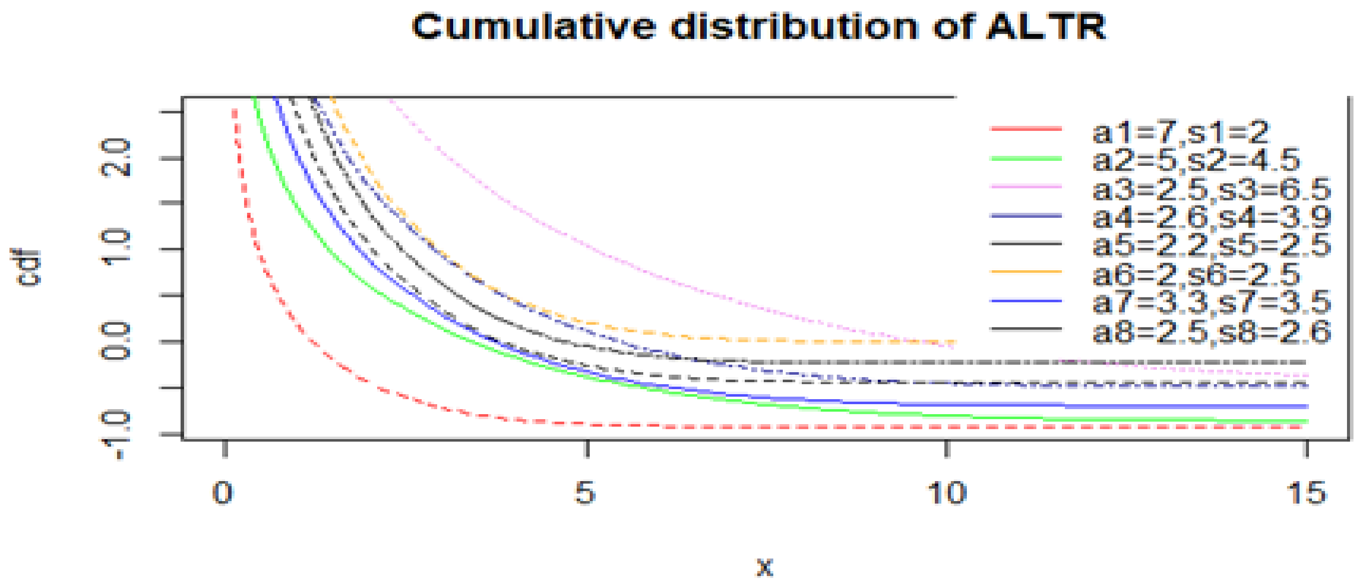
$$\lim_{x\to 0} F(x;\sigma^2,\alpha) = \lim_{x\to 0} 1 - \frac{log\left\{\alpha-(\alpha-1)[1-e^{-x^2/2\sigma^2}]\right\}}{\log(\alpha)} = 0 \qquad \ldots (5)$$

$$\lim_{x\to \infty} F(x;\sigma^2,\alpha) = \lim_{x\to \infty} 1 - \frac{log\left\{\alpha-(\alpha-1)[1-e^{-x^2/2\sigma^2}]\right\}}{\log(\alpha)} = 1 \qquad \ldots (6)$$

Graph 1 Probability density function ofAlpha Logarithm Transformed Rayleigh (ALTR) distribution for different values of the parameters.



Graph 2 Distribution function ofAlpha Logarithm Transformed Rayleigh (ALTR) distribution for different values of the parameters.



1.2. Survival and Hazard Functions of ALTR

Survival Function

A random variable X ~ ALTR (σ^2, α) has Survival function and is in the form

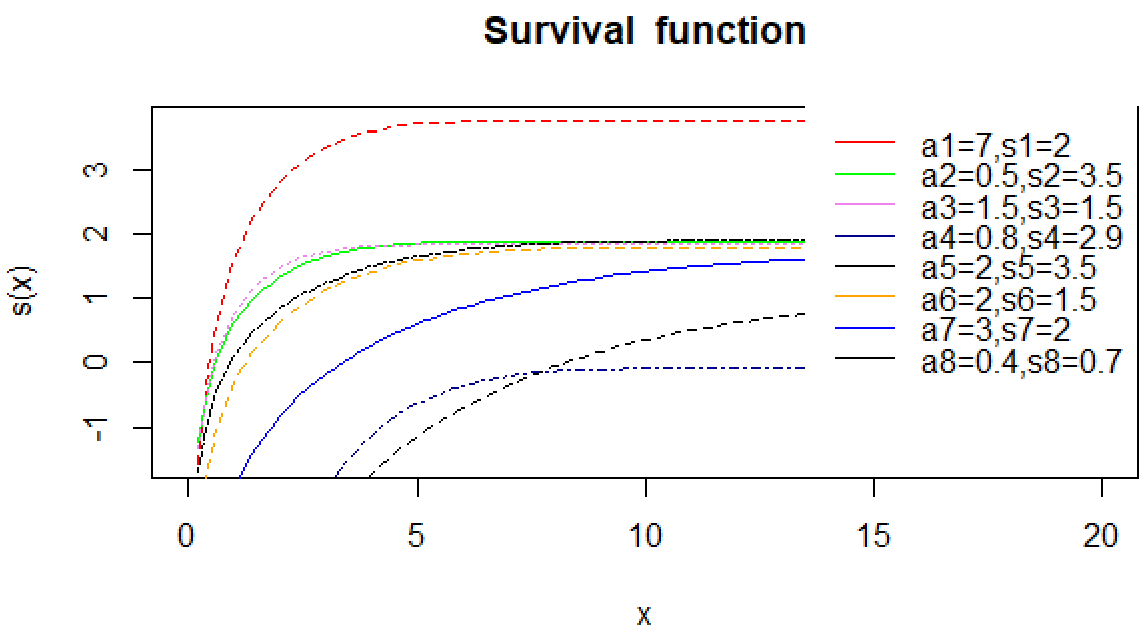
$$S(x) = \frac{log\left\{\alpha-(\alpha-1)[1-e^{-x^2/2\sigma^2}]\right\}}{\log(\alpha)} \ldots (7)$$

Hazard Function

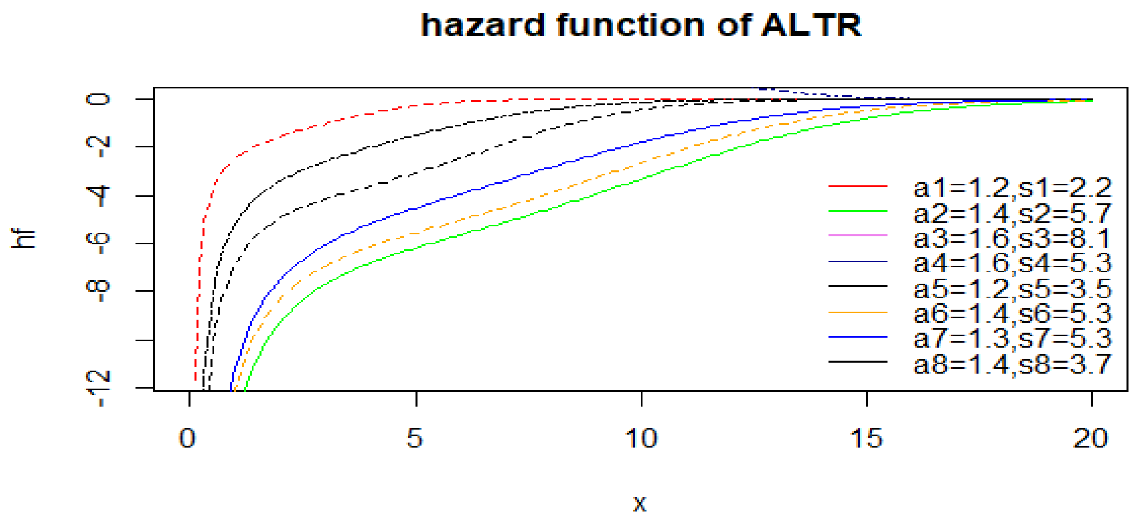
A random variable X ~ ALTR (σ^2, α) has Hazard function and is in the form

$$H(x) = \frac{\alpha(\alpha-1)\frac{x}{\sigma^2}e^{-x^2/2\sigma^2}}{\left\{\alpha-(\alpha-1)[1-e^{-x^2/2\sigma^2}]\right\}log\left\{\alpha-(\alpha-1)[1-e^{-x^2/2\sigma^2}]\right\}} \ldots (8)$$

Graph 3 Survival function of Alpha Logarithm Transformed Rayleigh (ALTR) distribution for different values of the parameters.



Graph 4 Hazard function of Alpha Logarithm Transformed Rayleigh (ALTR) distribution for different values of the parameters.



1.3 Statistical Properties of ALTR

1.3.1 Quantiles and Random number generation of ALTR

Quantile function

A random variable X ~ ALTR (σ^2, α) has Quantile function and is in the form
The p^{th} quantile x_p of ALTR distribution is the root of the equation

$$x_p = \sigma\sqrt{2log\left[\frac{\alpha-\alpha^{(1-p)}}{(\alpha-1)} - 1\right]} \quad \ldots (9)$$

Random generating function

Let U ~ U(0,1), then equation (9) can be used to simulate a random sample of size n from the ALTR distribution as follows

$$x_i = \sigma\sqrt{2log\left[\frac{\alpha-\alpha^{(1-u_i)}}{(\alpha-1)} - 1\right]}, \text{ i=1, 2, …, n. … (10)}$$

1.3.2 Moments of ALTR

The follows theorem gives the moments of HLTR

Theorem 1: The r[th] moment about the origin of X ~ ALTR (σ^2, α) is given by

$$\mu_r^1 = \sum_{j,k,l=0}^{\infty}(-1)^{l+m}\binom{k}{l}\left(\frac{\alpha-1}{\alpha}\right)^{k+1}\frac{l^m}{m!}(2\sigma^2)^{\frac{r+2m}{2}}\left(\gamma\left(\frac{(r+2m)}{2}\right)+1\right)$$

Proof: We have

$$\mu_r^1 = E(x^r)$$

$$= \int_{-\infty}^{\infty} x^r f(x)dx$$

$$= \int_0^{\infty} x^r \frac{\alpha(\alpha-1)\frac{x}{\sigma^2}e^{-x^2/2\sigma^2}}{\log(\alpha)\left\{\alpha-(\alpha-1)[1-e^{-x^2/2\sigma^2}]\right\}} dx \ldots (11)$$

By using Maclaurin series and binomial expansion the equation (11) can be written as

$$= \sum_{j=0}^{\infty}\sum_{k=0}^{\infty}\sum_{l=0}^{\infty}(-1)^{l+m}\binom{k}{l}\left(\frac{\alpha-1}{\alpha}\right)^{k+1}\frac{l^m}{m!}\int_0^{\infty} x^{r+2m+1}e^{\frac{-x^2}{\sigma^2}}dx \ldots (12)$$

$$= \sum_{j,k,l=0}^{\infty}(-1)^{l+m}\binom{k}{l}\left(\frac{\alpha-1}{\alpha}\right)^{k+1}\frac{l^m}{m!}(2\sigma^2)^{\frac{r+2m}{2}}\left(\gamma\left(\frac{(r+2m)}{2}\right)+1\right) \ldots (13)$$

Hence proved.

1.4 Moment Generating Function of ALTR

The follows theorem gives the moment generating function of HLTR

Theorem 2: The moment generating function of X ~ ALTR (σ^2, α) is given by

$$M_{(X)}^t = \frac{e^{t^2\sigma^2}}{\sigma^2}\sum_{i=0}^{\infty}\binom{2m+1}{i}(\sigma^2)^{2m+1-i}\left(\sqrt{2\sigma}\right)^{i+1}\left(\frac{\gamma\left(i+2,\frac{\sigma^4t^2}{2}\right)(-\sigma^2t)^i}{2(\sigma^2t)^i}+\frac{\gamma(\frac{i}{2}+1)}{2}\right)$$

Proof: We have

$M_X^t = E(e^{tx})$

$= \int_{-\infty}^{\infty} e^{tx} f(x)dx$

$= \int_0^{\infty} e^{tx} \frac{\alpha(\alpha-1)\frac{x}{\sigma^2}e^{-x^2/2\sigma^2}}{\log(\alpha)\left\{\alpha-(\alpha-1)[1-e^{-x^2/2\sigma^2}]\right\}} dx \ldots (14)$

By using Maclaurin series and binomial expansion the equation (20) can be written as

$= \sum_{j=0}^{\infty}\sum_{k=0}^{\infty}\sum_{l=0}^{\infty}(-1)^{l+m}\binom{k}{l}\left(\frac{\alpha-1}{\alpha}\right)^{k+1}\frac{l^m}{m!}\int_0^{\infty}\frac{x^{2m+1}}{\sigma^2}e^{tx+\frac{-x^2}{\sigma^2}}dx \ldots (15)$

$= \sum_{j=0}^{\infty}\sum_{k=0}^{\infty}\sum_{l=0}^{\infty}(-1)^{l+m}\binom{k}{l}\left(\frac{\alpha-1}{\alpha}\right)^{k+1}\frac{l^m}{\sigma^2 m!}\int_0^{\infty} x^{2m+1}e^{\frac{-(x-\sigma^2 t)^2}{2\sigma^2}}dx \quad \ldots (16)$

Let consider,

w = $\frac{-(x-\sigma^2 t)}{\sqrt{2\sigma^2}}$ => x = $w\sigma\sqrt{2}+\sigma^2 t$ and dx = $\sigma\sqrt{2}dw \ldots (17)$

Limits $x = 0 \Rightarrow w = \frac{-\sigma^2 t}{\sqrt{n}}, x = \infty \Rightarrow w = \infty \ldots (18)$

Substitute (17) and (18) in (16), then it obtained

$=\frac{e^{t^2\sigma^2}}{\sigma^2}\sum_{i=0}^{\infty}\binom{2m+1}{i}(\sigma^2 t)^{2m+1-i}(\sigma\sqrt{2})^{i+1}\int_{\frac{-\sigma^2 t}{\sqrt{n}}}^{\infty} e^{-w^2}w^i\,dw \ldots (19)$

$=\frac{e^{t^2\sigma^2}}{\sigma^2}\sum_{i=0}^{\infty}\binom{2m+1}{i}(\sigma^2)^{2m+1-i}(\sqrt{2\sigma})^{i+1}\left(\frac{\gamma\left(i+2,\frac{\sigma^4 t^2}{2}\right)(-\sigma^2 t)^i}{2(\sigma^2 t)^i}+\frac{\gamma(\frac{i}{2}+1)}{2}\right) \quad \ldots (20)$

1.5 Cumulative Generating Function of ALTR

A random variable X ~ ALTR (σ^2, α) has function and is in the form

Kx(t)=log(Mx(t)=Log($\frac{e^{t^2\sigma^2}}{\sigma^2}\sum_{i=0}^{\infty}\binom{2m+1}{i}(\sigma^2)^{2m+1-i}(\sqrt{2\sigma})^{i+1}\left(\frac{\gamma\left(i+2,\frac{\sigma^4 t^2}{2}\right)(-\sigma^2 t)^i}{2(\sigma^2 t)^i}+\frac{\gamma(\frac{i}{2}+1)}{2}\right)$ $=t^2\sigma^2 - \log(\sigma^2) +$

$\prod_{i=1}^{n}\log\left(\binom{2m+1}{i}(\sigma^2)^{2m+1-i}(\sqrt{2\sigma})^{i+1}\left(\frac{\gamma\left(i+2,\frac{\sigma^4 t^2}{2}\right)(-\sigma^2 t)^i}{2(\sigma^2 t)^i}+\frac{\gamma\left(\frac{i}{2}+1\right)}{2}\right)\right) \ldots (21)$

Conclusion:

1. The LTF failure rate function can have the following forms depending on its shape parameters: (i) decreasing (ii) upside down bathtub and (iii) reversed J-shaped shaped. Therefore, it can be used quite electively in analyzing lifetime data. Additionally, the new ALTR.
2. This ALTR new distribution gives Incomplete moments.
3. When $\alpha = 1$, distribution does not exist. This is the important drawback of this new distribution.

References

1. Abd-Elfattah, A. M., Hassan, A. S. & Ziedan, D. M. (2006), Efficiency of maximum likelihood estimators under different censored sampling schemes for Rayleigh distribution. Institute of Statistical Studies & Research, vol. 1, pp. 1-16.
2. Dey S, Sharma V K, and Mesfiou Mi (2017), "A new extension of Weibull distribution with application to lifetime data" Annals of Data Science, 4(1), pp. 31–61.
3. Rayleigh, J.W.S. (1880). On the resultant of a large number of vibrations of the some pitch and of arbitrary phase. Philosophical Magazine, 5-th Series, 10, 73 - 78.
4. Sanku Dey, Mazen Nassar & Devendra Kumar (2017) "α Logarithmic Transformed Family of Distributions with Application","Annals of Data Science" , vol. 4, pp: 457–482.
5. Vijaya Lakshmi, M. and Anjaneyulu, G. V. S. R. (2019), "Quadratic Rank Transmuted Half Logistic Lomax Distribution: Properties and Application", International Journal in IT & Engineering (IJITE), 7(6), pp. 1-12.
6. Vijaya Lakshmi M, Rajasekhram O. V. and Anjaneyulu G. V. S. R. (2018). " Estimation of Scale (λ) and Location (μ) of Two-Parameter Rayleigh Distribution Using Median Ranks method", International Journal of Science and Research (IJSR),1(1), pp. 1948-1952.
7. YinglinLiu, MuhammadIlyas, SaimaK.Khosa, EisaMuhmoudi , ZubairAhmad , Dost Muhammad Khan and G. G Hamedani (2020), "A Flexible Reduced Logarithmic-X Family of Distributions with Biomedical Analysis",Vol.2020, pp:1-15