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# Harmonic Mean Labeling Of H-Super Subdivision of Y-Tree

S.Meena
Department of Mathematics,
Government Arts College, C. Mutlur,
Chidambaram – 608102

M.Sivasakthi,
Department of Mathematics,
Krishnasamy College of Science Arts and Management for Women Cuddalore.

## Abstract

A graph G with p vertices and q edges is called a harmonic mean graph if it is possible to label the vertices x∈V with distinct labels f(x) from {1,2,…..q+1} in such a way that each edge e = uv is labeled with f(uv) = $\left\lceil\frac{2f(u)f(v)}{f(u)+f(v)}\right\rceil$ (or) $\left\lfloor\frac{2f(u)f(v)}{f(u)+f(v)}\right\rfloor$ then the edge labels are distinct. In this case f is called Harmonic mean labeling of G. In this paper we prove that some families of graphs such as H- super subdivision of Y-Tree HSS($Y_{n+1}$), HSS($Y_{n+1}$) $\odot K_1$, HSS($Y_{n+1}$) $\odot\overline{K_2}$, HSS($Y_{n+1}$) $\odot K_2$ are harmonic mean graphs.

## Keywords:

Harmonic mean graph, H- super subdivision of HSS($Y_{n+1}$), HSS($Y_{n+1}$) $\odot K_1$, HSS($Y_{n+1}$) $\odot\overline{K_2}$, HSS($Y_{n+1}$) $\odot K_2$

**AMS subject classification** :- 05078

## Introduction

Let G=(V,E) be a (p,q) graph with p = |V(G)| vertices and q =|E(G)| edges, where V(G) and E(G) respectively denote the vertex set and edge set of the graph G. In this paper, we consider the graphs which are simple, finite and undirected. For graph theoretic terminology and notations we refer to Harary [4]

The concept of graph labeling was introduced by Rosa in 1967. A detailed survey of graph labeling is available in Gallian[3].The concept of Harmonic mean labeling of graph was introduced by S.Somasundaram, R.Ponraj and S.S.Sandhya and they investigated the existence of harmonic mean labeling of several family of graphs .The concept was further studied by several authors. We have proved Harmonic mean labeling of subdivision graphs such as $P_n \odot K_1$, $P_n \odot \overline{K_2}$, H-graph, crown, $C_n \odot K_1$,$C_n \odot \overline{K_2}$, quadrilateral snake, Triangular snake and also proved Harmonic mean labeling of some graphs such as Triple triangular snake T($T_n$), Alternate Triple triangular snake A[T($T_n$)],Triple quadrilateral snake T($Q_n$), Alternate Triple quadrilateral snake A[T($Q_n$)], Twig graph T(n), balloon triangular snake $T_n(C_m)$, key graph Ky(m,n),zig-zag triangle Z($T_n$), Z ($T_n$) $\odot$ $K_1$, Z( $T_n$)$\odot\overline{K_2}$ , Z( $T_n$)$\odot$ $K_2$, alternate zig-zag triangle A Z ($T_n$), spiked snake graph SS(4,n) and harmonic mean labeling of h-super subdivision of path ,cycle graphs . The following definitions are useful for the present investigation.

## Definition: 1.1 [8]

A Graph G = (V, E) with p vertices and q edges is called a Harmonic mean graph if it is possible to label the vertices $v$ ∈V with distinct labels f(v) from {1,2,…,q+1} in such a way that when each edge e = uv is labeled with f(uv) = $\left\lceil \frac{2f(u)f(v)}{f(u)+f(v)} \right\rceil$ (or) $\left\lfloor \frac{2f(u)f(v)}{f(u)+f(v)} \right\rfloor$ then the resulting edge labels are distinct. In this case f is called Harmonic mean labeling of G.

## Definition: 1.2 [2]

Let G be a (p,q) graph. A graph obtained from G by replacing each edge $e_i$ by a H-graph in such a way that the ends $e_i$ are merged with a pendent vertex in $P_2$ and a pendent vertex $P_2{}'$ is called H-Super Subdivision of G and it is denoted by HSS(G) where the H-graph is a tree on 6 vertices in which exactly two vertices of degree 3.

## Definition: 1.2 [2]

A Y-tree $Y_{n+1}$ (n≥2) is a graph obtained from the path $P_n$ by appending an edge to a vertex of a path $P_n$ adjacent to an end vertex.

## Definition: 1.2 [2]

Let $Y_{n+1}$ be a Y-tree (n≥2) with n+2 vertices and n+1 edges. Let the vertices of $Y_{n+1}$ be $v_1$,$v_2$, $v_3$,…. $v_{n+1}$,u. The HSS($Y_{n+1}$) is constructed from $Y_{n+1}$ by replacing each edge by the H-graph. The vertex and edge sets of HSS($Y_{n+1}$) are as follows

V(HSS($Y_{n+1}$)) ={{ u,$v_n u^{(1)}$, $v_n u^{(2)}$,u$v_n{}^{(1)}$, u$v_n{}^{(2)}$, $v_{n+1}$}

$\cup\{v_i \cup v_{i(i+1)}{}^{(1)} \cup v_{i(i+1)}{}^{(2)} \cup v_{(i+1)i}{}^{(1)} \cup v_{(i+1)i}{}^{(2)} \,/\, 1 \le i \le n\}\}$ and

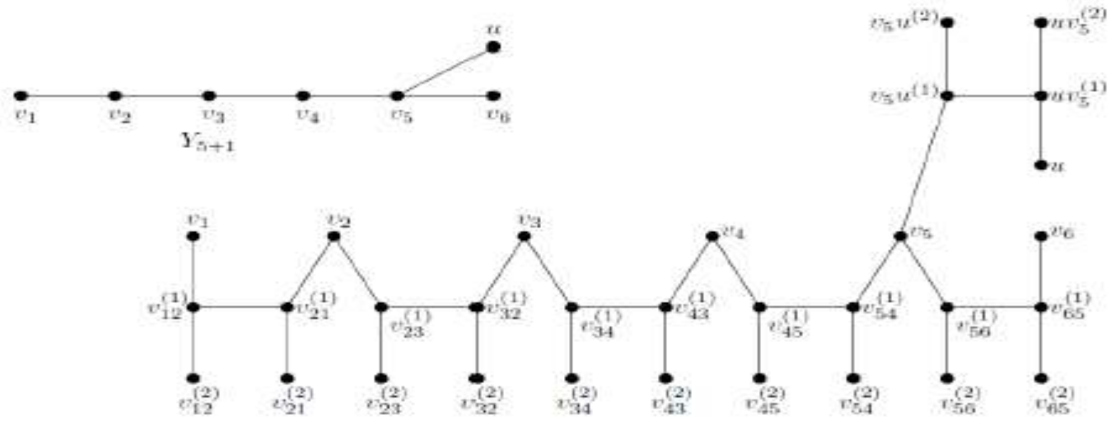
E (HSS($Y_{n+1}$)) = $E_1 \cup E_2$ where

$E_1$ = { $v_n$ $v_n u^{(1)}$,$v_n u^{(1)}$ $v_n u^{(2)}$ ,$v_n u^{(1)}$ u$v_n{}^{(1)}$, u$v_n{}^{(1)}$ u$v_n{}^{(2)}$, u$v_n{}^{(1)}$u },

$E_2$ = { $v_i$ $v_{i(i+1)}{}^{(1)}$, $v_{i(i+1)}{}^{(1)}$ $v_{i(i+1)}{}^{(2)}$, $v_{i(i+1)}{}^{(1)}$ $v_{(i+1)i}{}^{(1)}$, $v_{(i+1)i}{}^{(1)}$ $v_{(i+1)i}{}^{(2)}$,

$v_{(i+1)i}{}^{(1)}$ $v_{i+1}$ / $1 \le i \le n$}.

Then HSS($Y_{n+1}$) has 5n+6 vertices and 5n+5 edges.



$Y_{5+1}$

$HSS(Y_{5+1})$

In this paper we prove that H- super subdivision of HSS($Y_{n+1}$), HSS($Y_{n+1}$) $\odot$ $K_1$, HSS($Y_{n+1}$) $\odot \overline{K_2}$, HSS($Y_{n+1}$) $\odot$ $K_2$ are harmonic mean graph

## II. Harmonic mean labeling of graphs

**Theorem:2.1**

The structure of H- super subdivision of Y-tree HSS($Y_{n+1}$) is a harmonic mean graphs

**Proof:**

Let HSS($Y_{n+1}$) be the H- super subdivision of a Y-tree $Y_{n+1}$ which has 5n+6 vertices and 5n+5 edges. The vertex set

V(G) = {{ u, u$v_n{}^{(1)}$, u$v_n{}^{(2)}$, $v_n u^{(1)}$, $v_n u^{(2)}$, $v_{n+1}$}∪{$v_i \cup x_i \cup r_i \cup s_i \cup y_i$} / 1≤ i≤n }} and the edge set.

E(G) = {{ $v_n$ $v_n u^{(2)}$, $v_n u^{(2)}$ $v_n u^{(1)}$, $v_n u^{(2)}$ u$v_n{}^{(1)}$, u u$v_n{}^{(1)}$, u$v_n{}^{(1)}$ u$v_n{}^{(2)}$} ∪ { $v_i x_i$, $x_i r_i$, $x_i y_i$, $s_i y_i$, $y_i v_{i+1}$ / 1≤ i≤n }}.

Then the resultant graph is harmonic mean labeling of structure of H- super subdivision of Y-tree graph.

Define a function f : V(G) $\rightarrow \{1,2,\ldots,q+1\}$ by

$f(v_i)$ = 5i- 4 for $1\leq i \leq n$

$f(x_i)$ = 5i- 3 for $1\leq i \leq n$

$f(y_i)$ = 5i for $1\leq i \leq n$

$f(r_i)$ = 5i- 2 for $1\leq i \leq n$

$f(s_i)$ = 5i-1 for $1\leq i \leq n$

$f(u)$ = 5n+2

$f(u\, v_n{}^{(1)})$ = 5n+3

$f(u\, v_n{}^{(2)})$ = 5n+4

$f(v_n u^{(1)}) = 5n+5$

$f(v_n u^{(2)}) = 5n+6$

Then the resulting edge labels are distinct.

$f(v_i x_i) = 5i - 4$ for $1 \leq i \leq n$

$f(x_i r_i) = 5i - 3$ for $1 \leq i \leq n$

$f(x_i y_i) = 5i - 2$ for $1 \leq i \leq n$

$f(s_i y_i) = 5i - 1$ for $1 \leq i \leq n$

$f(y_i v_{i+1}) = 5i$

$f(v_n \; v_n u^{(2)}) = 5n$

$f(u \; uv_n{}^{(1)}) = 5n+2$

$f(uv_n{}^{(1)} \; uv_n{}^{(2)}) = 5n+3$

$f(uv_n{}^{(1)} \; v_n u^{(2)}) = 5n+4$

$f(v_n u^{(1)} \; v_n u^{(2)}) = 5n+5$

Thus f provides a harmonic mean labeling of graph G.

Hence G is a harmonic mean graph.

## Example:2.1.1

A harmonic mean labeling of graph G obtained by structure of H- super subdivision of Y-tree HSS $(Y_{4+1})$ are shown in fig 2.1.1

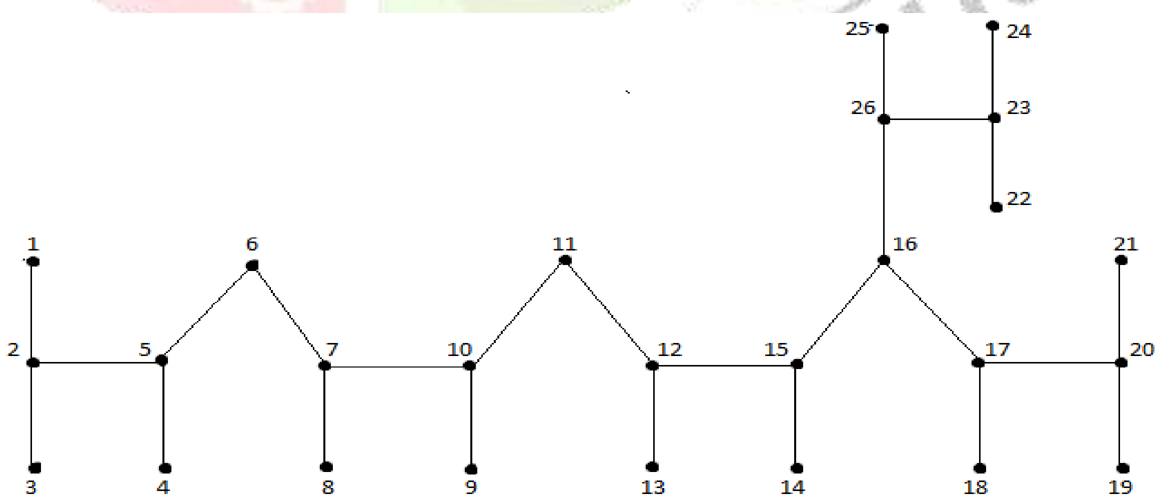



fig 2.1.1

## Theorem:2.2

The structure of H- super subdivision of Y-tree HSS$(Y_{n+1}) \odot K_1$ is a harmonic mean graph.

**Proof:**

Let HSS($Y_{n+1}$) be the H- super subdivision of a Y-tree $Y_{n+1}$ which has 5n+6 vertices and 5n+5 edges and every vertex attached by $K_1$ graph. Then the resultant graph is HSS($Y_{n+1}$) ⊙ $K_1$ graph whose vertex set

V(G) = {{ z, $zw_n{}^{(1)}$, $zw_n{}^{(2)}$, $w_nz^{(1)}, w_nz^{(2)}$, $k_n{}^{(1)}, k_n{}^{(2)}$, $k_n{}^{(3)}, l_n{}^{(1)}$, $l_n{}^{(2)}$, $u_{n+1}$, $v_{n+1}, w_{n+1}, s_{n+1}$ } ∪ { $u_i, v_i, w_i, s_i, r_i, t_i, x_i, y_i$ / 1≤ i≤n } ∪ {$p_i, q_i$ / 1≤ i≤n -1}}.

And the edge set

E(G) = {{ z $zw_n{}^{(1)}$, z $k_n{}^{(1)}$, $zw_n{}^{(1)}\, k_n{}^{(2)}$, $zw_n{}^{(1)}\, zw_n{}^{(2)}$, $zw_n{}^{(2)}$ $k_n{}^{(3)}$, $zw_n{}^{(1)}w_nz^{(2)}$, $w_nz^{(1)}$ $l_n{}^{(1)}$, $w_nz^{(1)}w_nz^{(2)}$, $w_nz^{(2)}l_n{}^{(2)}$, $w_nz^{(2)}w_n$ } ∪ { $v_iu_{i+1}, v_iw_i, u_{i+1}w_{i+1}, w_is_i,$ $v_ir_i,$ $t_iu_{i+1},\ r_ix_i,\ t_iy_i,$ / 1≤ i≤n } ∪ {$p_iu_{i+1}, q_iv_{i+1}$ / 1 ≤ i ≤ n -1}∪{ $u_1v_1,$ $u_{n+1}v_{n+1},\ u_{n+1}w_{n+1},$ $w_{n+1}s_{n+1}$ }}.

Then the resultant graph is harmonic mean labeling of structure of H- super subdivision of HSS($Y_{n+1}$) ⊙ $K_1$ Y-tree graph.

Define a function f : V(G) → $\{1,2,\ldots,q+1\}$ by

| | | | |
|---|---|---|---|
| $f(u_1)$ | = | 1 | |
| $f(u_i)$ | = | 10i- 11 | for 2≤ $i \le n$ +1 |
| $f(v_i)$ | = | 10i- 6 | for 1≤ $i \le n$ |
| $f(v_{n+1})$ | = | 10n+12 | |
| $f(w_1)$ | = | 3 | |
| $f(w_i)$ | = | 10i- 8 | for 2≤ $i \le n$ |
| $f(w_{n+1})$ | = | 10n | |
| $f(p_i)$ | = | 10i | for 1≤ $i \le n$-1 |
| $f(q_i)$ | = | 10i+3 | for 1≤ $i \le n$ -1 |
| $f(s_1)$ | = | 2 | |
| $f(s_i)$ | = | 10i- 9 | for 2≤ $i \le n$ +1 |
| $f(r_i)$ | = | 10i- 5 | for 1≤ $i \le n$ |
| $f(t_i)$ | = | 10i- 2 | for 1≤ $i \le n$ |
| $f(x_i)$ | = | 10i- 4 | for 1≤ $i \le n$ |
| $f(y_i)$ | = | 10i- 3 | for 1≤ $i \le n$ |
| $f(z)$ | = | 10n+ 3 | |
| $f(zw_n{}^{(1)})$ | = | 10n+ 5 | |

$f(zw_n{}^{(2)}) = 10n+7$

$f(k_n{}^{(1)}) = 10n+2$

$f(k_n{}^{(2)}) = 10n+4$

$f(k_n{}^{(3)}) = 10n+6$

$f(w_n z^{(1)}) = 10n+9$

$f(w_n z^{(2)}) = 10n+11$

$f(l_n{}^{(1)}) = 10n+8$

$f(l_n{}^{(2)}) = 10n+10$

Then the resulting edge labels are distinct.

$f(u_1 v_1) = 1$

$f(v_i u_{i+1}) = 10i - 4$ for $1 \leq i \leq n$

$f(u_{n+1} v_{n+1}) = 10n+5$

$f(v_1 w_1) = 3$

$f(v_i w_i) = 10i - 8$ for $2 \leq i \leq n$

$f(u_i w_i) = 10i - 10$ for $2 \leq i \leq n$

$f(u_{n+1} w_{n+1}) = 10n-1$

$f(p_i u_{i+1}) = 10i - 1$ for $1 \leq i \leq n-1$

$f(q_i v_{i+1}) = 10i + 3$ for $1 \leq i \leq n-1$

$f(s_1 w_1) = 2$

$f(s_i w_i) = 10i - 9$ for $2 \leq i \leq n+1$

$f(r_i v_i) = 10i - 6$ for $1 \leq i \leq n$

$f(t_i u_{i+1}) = 10i - 2$ for $1 \leq i \leq n$

$f(r_i x_i) = 10i - 5$ for $1 \leq i \leq n$

$f(t_i y_i) = 10i - 3$ for $1 \leq i \leq n$

$f(z\, k_n{}^{(1)}) = 10n+2$

$f(z\ zw_n{}^{(1)}) = 10n+3$

$f(zw_n{}^{(1)}\, k_n{}^{(2)}) = 10n+4$

$f(zw_n{}^{(1)}\, zw_n{}^{(2)}) = 10n+6$

$f(zw_n{}^{(2)}\, k_n{}^{(3)}) = 10n+7$

$f(zw_n{}^{(1)}\, w_n z^{(2)}) = 10n+8$

$f(w_n z^{(1)}\ l_n{}^{(1)}) = 10n+9$

$$f(w_n z^{(1)}\, w_n z^{(2)}) \;=\; 10n+10$$

$$f(w_n z^{(2)}\, l_n{}^{(2)}) \;=\; 10n+11$$

Thus f provides a harmonic mean labeling of graph G.

Hence G is a harmonic mean graph.

## Example:2.2.1

A harmonic mean labeling of graph G obtained by structure of H- super subdivision of Y-tree HSS($Y_{5+1}$) $\odot$ $K_1$ are shown in fig 2.2.1

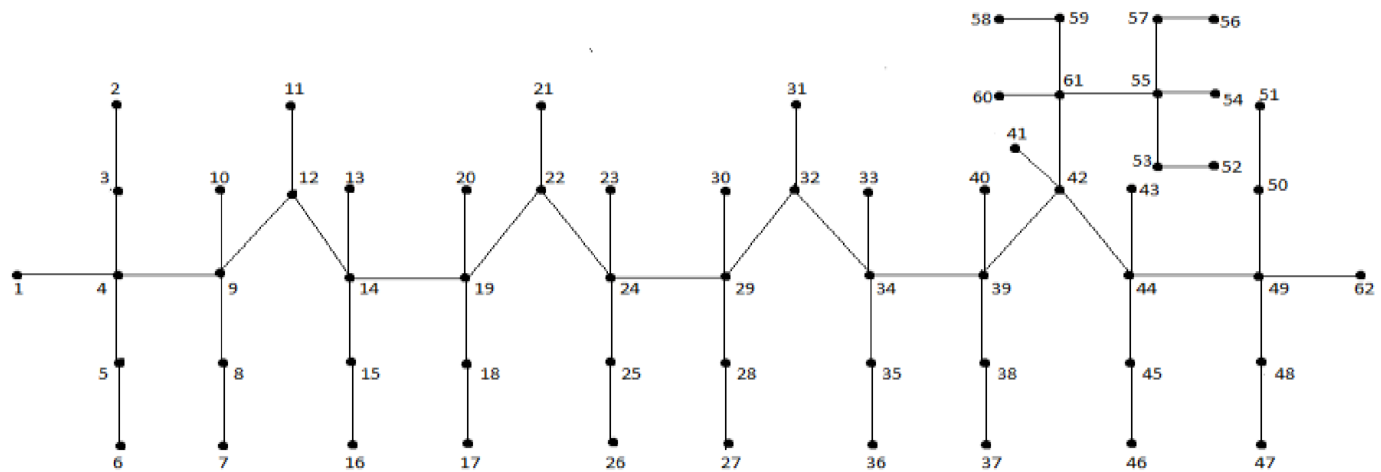



fig 2.2.1

## Theorem:2.3

The structure of H- super subdivision of Y-tree HSS($Y_{n+1}$) $\odot\overline{K_2}$ is a harmonic mean graph.

## Proof:

Let HSS($Y_{n+1}$) be the H- super subdivision of a Y-tree $Y_{n+1}$ which has 5n+6 vertices and 5n+5 edges and every vertex attached by $\overline{K_2}$ graph. Then the resultant graph is HSS($Y_{n+1}$) $\odot\overline{K_2}$ graph whose vertex set

$$V(G) = \{\{ u_i, v_i, x_i, y_i, z_i, p_i, q_i, r_i, s_i, t_i, e_i, d_i, f_i, k_i, l_i, / \, 1 \le i \le n\}\} \cup$$
$$\{z_{n+1}, s_{n+1}, r_{n+1}, w, w\, z_n{}^{(1)}, w\, z_n{}^{(2)}, z_n w^{(1)}, z_n w^{(2)},$$
$$g_n{}^{(1)}, g_n{}^{(2)}, g_n{}^{(3)}, g_n{}^{(4)}, g_n{}^{(5)}, g_n{}^{(6)}, h_n{}^{(1)}, h_n{}^{(2)}, h_n{}^{(3)}, h_n{}^{(4)}\}.$$

And the edge set

$$E(G) = \{\{ w\, g_n{}^{(1)}, w g_n{}^{(2)}, w\, z_n{}^{(1)}\, g_n{}^{(3)}, w\, z_n{}^{(1)}\, g_n{}^{(4)}, w\, z_n{}^{(2)}\, g_n{}^{(5)}, w\, z_n{}^{(2)}\, g_n{}^{(6)},$$
$$w\, w\, z_n{}^{(1)}, w\, z_n{}^{(1)}\, w\, z_n{}^{(2)}, w\, z_n{}^{(1)}\, z_n w^2, z_n w^{(1)} h_n{}^{(1)}, z_n w^{(1)} h_n{}^{(2)}, z_n w^{(2)} h_n{}^{(3)},$$
$$z_n w^{(2)} h_n{}^{(4)}, z_n w^{(1)}\, z_n w^{(2)}, z_n w^{(2)}\, z_n \} \cup \{ u_i x_i, v_i x_i, x_i y_i, z_i x_i,$$
$$y_i z_{i+1}, z_i r_i, z_i s_i, x_i t_i, y_i e_i, t_i d_i, t_i f_i, e_i k_i, e_i l_i / \, 1 \le i \le n \} \cup \{ y_n z_{n+1},$$
$$z_{n+1} s_{n+1}, z_{n+1} r_{n+1}\}.$$

Then the resultant graph is harmonic mean labeling of structure of H- super subdivision of $HSS(Y_{n+1}) \odot \overline{K_2}$ Y-tree graph.

Define a function f : V(G) $\rightarrow \{1,2,\ldots,q+1\}$ by

$f(x_i)$ = 15i- 9 for $1\leq i \leq n$

$f(y_i)$ = 15i- 1 for $1\leq i \leq n$

$f(u_1)$ = 1

$f(u_i)$ = 15i-11 for $2\leq i \leq n$

$f(v_i)$ = 15i- 10 for $1\leq i \leq n$

$f(p_i)$ = 15i- 2 for $1\leq i \leq n$

$f(q_i)$ = 15i for $1\leq i \leq n$

$f(z_1)$ = 4

$f(z_i)$ = 15i- 14 for $2\leq i \leq n$ -1

$f(z_{n+1})$ = 15n+3

$f(r_i)$ = 15i- 13 for $1\leq i \leq n$ +1

$f(s_i)$ = 15i- 12 for $1\leq i \leq n$ -1

$f(s_{n+1})$ = 15n+1

$f(t_i)$ = 15i -8 for $1\leq i \leq n$

$f(e_i)$ = 15i- 4 for $1\leq i \leq n$

$f(d_i)$ = 15i- 7 for $1\leq i \leq n$

$f(f_i)$ = 15i- 6 for $1\leq i \leq n$

$f(k_i)$ = 15i- 5 for $1\leq i \leq n$

$f(l_i)$ = 15i- 3 for $1\leq i \leq n$

$f(w)$ = 15n+5

$f({g_n}^{(1)})$ = 15n+ 6

$f({g_n}^{(2)})$ = 15n+ 4

$f({g_n}^{(3)})$ = 15n+ 9

$f({g_n}^{(4)})$ = 15n+ 7

$f({g_n}^{(5)})$ = 15n+ 12

$f({g_n}^{(6)})$ = 15n+ 10

$f(w\,{z_n}^{(1)})$ = 15n+8

$f(w\,{z_n}^{(2)})$ = 15n+11

$f(z_n w^{(1)})$ = 15n+ 14

$f(z_n w^{(2)})$ = 15n+ 17

$f(h_n{}^{(1)})$ = 15n+ 13

$f(h_n{}^{(2)})$ = 15n+ 15

$f(h_n{}^{(3)})$ = 15n+ 16

$f(h_n{}^{(4)})$ = 15n+ 18

Then the resulting edge labels are distinct.

$f(x_i y_i)$ = 15i - 6 for $1 \le i \le n$

$f(x_1 z_1)$ = 4

$f(x_i z_i)$ = 15i - 12 for $2 \le i \le n$

$f(y_i z_{i+1})$ = 15i for $1 \le i \le n$

$f(y_n z_{n+1})$ = 15n+1

$f(x_1 u_1)$ = 1

$f(x_i u_i)$ = 15i - 11 for $2 \le i \le n$

$f(x_i v_i)$ = 15i - 10 for $1 \le i \le n$

$f(y_i p_i)$ = 15i - 2 for $1 \le i \le n$-1

$f(y_n p_n)$ = 15n - 1

$f(y_i q_i)$ = 15i-1 for $1 \le i \le n$-1

$f(y_n p_n)$ = 15n

$f(z_1 r_1)$ = 2

$f(z_i r_i)$ = 15i - 14 for $2 \le i \le n$

$f(z_{n+1} r_{n+1})$ = 15n +3

$f(z_1 s_1)$ = 3

$f(z_i s_i)$ = 15i - 13 for $2 \le i \le n$

$f(x_i t_i)$ = 15i - 9 for $1 \le i \le n$

$f(y_i e_i)$ = 15i - 3 for $1 \le i \le n$

$f(t_i d_i)$ = 15i - 8 for $1 \le i \le n$

$f(t_i f_i)$ = 15i - 7 for $1 \le i \le n$

$f(e_i k_i)$ = 15i - 5 for $1 \le i \le n$

$f(e_i l_i)$ = 15i - 4 for $1 \le i \le n$

$f(z_n\, z_n w^{(2)})$ = 15n-2

$f(w\, g_n{}^{(1)})$ = 15n+5

$f(w\, g_n{}^{(2)}) = 15n+4$

$f(w\, wz_n{}^{(1)}) = 15n+6$

$f(\mathrm{w}\, z_n{}^{(1)} g_n{}^{(3)}) = 15n+8$

$f(\mathrm{w}\, z_n{}^{(1)} g_n{}^{(4)}) = 15n+7$

$f(\mathrm{w}\, z_n{}^{(1)} \mathrm{w}\, z_n{}^{(2)}) = 15n+9$

$f(\mathrm{w}\, z_n{}^{(2)} g_n{}^{(5)}) = 15n+11$

$f(\mathrm{w}\, z_n{}^{(2)} g_n{}^{(6)}) = 15n+10$

$f(\mathrm{w}\, z_n{}^{(1)} z_n w^{(2)}) = 15n+12$

$f\left(z_n w^{(1)} h_n{}^{(1)}\right) = 15n+13$

$f\left(z_n w^{(1)} h_n{}^{(2)}\right) = 15n+14$

$f\left(z_n w^{(2)} h_n{}^{(3)}\right) = 15n+16$

$f\left(z_n w^{(2)} h_n{}^{(4)}\right) = 15n+17$

$f(z_n w^{(1)} z_n w^{(2)}) = 15n+15$

Thus f provides a harmonic mean labeling of graph G.

Hence G is a harmonic mean graph

**Example:2.3.1**

A harmonic mean labeling of graph G obtained by structure of H- super subdivision of Y-tree HSS($Y_{n+1}$) $\odot \overline{K_2}$ are shown in fig 2.3.1

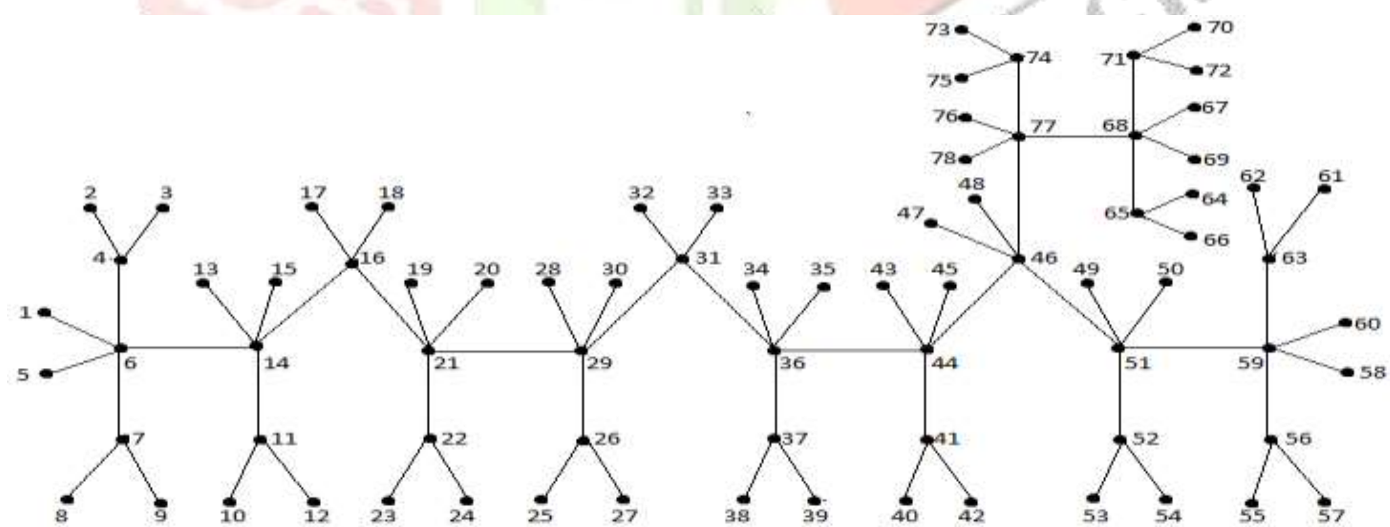



fig 2.3.1

**Theorem:2.4**

The structure of H- super subdivision of Y-tree HSS($Y_{n+1}$) $\odot K_2$ is a harmonic mean graph.

**Proof:**

Let HSS($Y_{n+1}$) be the H- super subdivision of a Y-tree $Y_{n+1}$ which has 5n+6 vertices and 5n+5 edges and every vertex attached by $K_2$ graph. Then the resultant graph is HSS($Y_{n+1}$) ⊙ $K_2$ graph whose vertex set

V(G) = {{ $u_i, v_i, x_i, y_i, z_i, p_i, q_i, r_i, s_i, t_i, e_i, d_i, f_i, k_i, l_i$ / 1 ≤ i ≤ n }}∪{$p_i, q_i$ / 1≤ $i \le n$ -1}

∪ {$z_{n+1}$,$s_{n+1}$,$r_{n+1}$,$w$,$w\, z_n{}^{(1)}, w\, z_n{}^{(2)}, z_n w^{(1)}, z_n w^{(2)}$, $g_n{}^{(1)}, g_n{}^{(2)}, g_n{}^{(3)}, g_n{}^{(4)}, g_n{}^{(5)}, g_n{}^{(6)}, h_n{}^{(1)}, h_n{}^{(2)}, h_n{}^{(3)}, h_n{}^{(4)}$}.

And the edge set

E(G) = {{ w $g_n{}^{(1)}$,w$g_n{}^{(2)}$, $g_n{}^{(1)} g_n{}^{(2)}$,w $w\, z_n{}^{(1)}$, $w\, z_n{}^{(1)}\, g_n{}^{(3)}$, $w\, z_n{}^{(2)}\, g_n{}^{(4)}$, $g_n{}^{(3)} g_n{}^{(4)}$, $w\, z_n{}^{(2)}\, w\, z_n{}^{(1)}$, $w\, z_n{}^{(2)}\, g_n{}^{(5)}$, $w\, z_n{}^{(2)}\, g_n{}^{(6)}$, $g_n{}^{(5)}\, g_n{}^{(6)}$, $w\, z_n{}^{(1)}\, z_n w^{(2)}$, $z_n w^{(1)} h_n{}^{(1)}$, $z_n w^{(1)}\, h_n{}^{(2)}$, , $h_n{}^{(1)}\, h_n{}^{(2)}$, $z_n w^{(1)}\, z_n w^{(2)}$, $z_n\, z_n w^{(2)}$, $z_n w^{(2)}\, h_n{}^{(3)}$, $z_n w^{(2)}\, h_n{}^{(4)}$, $h_n{}^{(3)}, h_n{}^{(4)}$} ∪ { $u_i x_i$, $u_i v_i$, $v_i x_i$, $x_i y_i$, $z_i x_i$, $y_i z_{i+1}$, $z_i r_i$, $z_i s_i$, $r_i s_i$, $x_i t_i$, $y_i e_i$, $t_i d_{i,}$, $d_{i,} f_i$, $e_i k_i$, $e_i l_i$, $k_i l_i$ / 1 ≤ i ≤ n } ∪ { $y_n z_{n+1}$, $z_{n+1} s_{n+1}$, $z_{n+1} r_{n+1}$ }.

Then the resultant graph is harmonic mean labeling of structure of H- super subdivision of HSS($Y_{n+1}$) ⊙ $K_2$ Y-tree graph.

Define a function f : V(G) → $\{1,2,\ldots,q+1\}$ by

$f(x_1)$ = 8

$f(x_i)$ = 20i- 13 for 2≤ $i \le n$

$f(y_i)$ = 20i- 1 for 1≤ $i \le n$

$f(z_1)$ = 3

$f(z_i)$ = 20i- 19 for 2≤ $i \le n$

$f(z_{n+1})$ = 20n+3

$f(u_i)$ = 20i- 15 for 1≤ $i \le n$

$f(v_1)$ = 7

$f(v_i)$ = 20i- 14 for 2≤ $i \le n$

$f(p_i)$ = 20i- 3 for 1≤ $i \le n$ -1

$f(p_n)$ = 20n-4

$f(q_i)$ = 20i- 2 for 1≤ $i \le n$ -1

$f(q_n)$ = 20n+1

$f(r_1)$ = 1

$f(r_i)$ = 20i- 18 for 2≤ $i \le n$

$f(s_1)$ = 2

$f(s_i)$ = 20i- 17 for $2 \leq i \leq n$

$f(s_{n+1})$ = 20n+4

$f(t_i)$ = 20i- 11 for $1 \leq i \leq n$

$f(e_i)$ = 20i- 5 for $1 \leq i \leq n$

$f(d_i)$ = 20i- 10 for $1 \leq i \leq n$

$f(f_i)$ = 20i- 9 for $1 \leq i \leq n$

$f(k_i)$ = 20i- 8 for $1 \leq i \leq n$

$f(l_i)$ = 20i- 6 for $1 \leq i \leq n$

$f(w)$ = 20n+6

$f(g_n{}^{(1)})$ = 20n+7

$f(g_n{}^{(2)})$ = 20n+5

$f(g_n{}^{(3)})$ = 20n+11

$f(g_n{}^{(4)})$ = 20n+9

$f(g_n{}^{(5)})$ = 20n+15

$f(g_n{}^{(6)})$ = 20n+13

$f(w\, z_n{}^{(1)})$ = 20n+10

$f(w\, z_n{}^{(2)})$ = 20n+14

$f(z_n w^{(1)})$ = 20n+18

$f(z_n w^{(2)})$ = 20n+22

$f(h_n{}^{(1)})$ = 20n+17

$f(h_n{}^{(2)})$ = 20n+19

$f(h_n{}^{(3)})$ = 20n+21

$f(h_n{}^{(4)})$ = 20n+23

Then the resulting edge labels are distinct.

$f(x_i y_i)$ = 20i - 8 for $1 \leq i \leq n$

$f(x_i z_i)$ = 20i - 16 for $1 \leq i \leq n$

$f(y_i z_{i+1})$ = 20i for $1 \leq i \leq n$

$f(y_n z_{n+1})$ = 20n+1

$f(x_i u_i)$ = 20i - 14 for $1 \leq i \leq n$

$f(x_i v_i)$ = 20i - 13 for $1 \leq i \leq n$

$f(u_i v_i) = 20i - 15$ for $1 \leq i \leq n$

$f(y_i p_i) = 20i - 2$ for $1 \leq i \leq n-1$

$f(y_n p_n) = 20n - 3$

$f(y_i q_i) = 20i - 1$ for $1 \leq i \leq n-1$

$f(y_n q_n) = 20n$

$f(p_i q_i) = 20i - 3$ for $1 \leq i \leq n-1$

$f(p_n q_n) = 20n - 1$

$f(z_i r_i) = 20i - 19$ for $1 \leq i \leq n$

$f(z_{n+1} r_{n+1}) = 20n + 2$

$f(z_1 s_1) = 3$

$f(z_i s_i) = 20i - 18$ for $2 \leq i \leq n$

$f(z_{n+1} s_{n+1}) = 20n + 4$

$f(r_1 s_1) = 2$

$f(r_i s_i) = 20i - 17$ for $2 \leq i \leq n$

$f(x_i t_i) = 20i - 12$ for $1 \leq i \leq n$

$f(y_i e_i) = 20i - 4$ for $1 \leq i \leq n$

$f(t_i d_i) = 20i - 11$ for $1 \leq i \leq n$

$f(t_i f_i) = 20i - 10$ for $1 \leq i \leq n$

$f(d_i f_i) = 20i - 9$ for $1 \leq i \leq n$

$f(e_i k_i) = 20i - 6$ for $1 \leq i \leq n$

$f(e_i l_i) = 20i - 5$ for $1 \leq i \leq n$

$f(k_i l_i) = 20i - 7$ for $1 \leq i \leq n$

$f(w\, g_n^{(1)}) = 20n+7$

$f(w\, g_n^{(2)}) = 20n+5$

$f(g_n^{(1)}\, g_n^{(2)}) = 20n+6$

$f(w\, wz_n^{(1)}) = 20n+8$

$f(\mathrm{w}\, z_n^{(1)} g_n^{(4)}) = 20n+9$

$f(g_n^{(4)} g_n^{(3)}) = 20n+10$

$f(\mathrm{w}\, z_n^{(2)} g_n^{(6)}) = 20n+13$

$f(\mathrm{w}\, z_n^{(1)} \mathrm{w}\, z_n^{(2)}) = 20n+12$

$f(g_n^{(6)} g_n^{(5)}) = 20n+14$

$f\left(\mathrm{w}\, z_n{}^{(2)} g_n{}^{(5)}\right)$ = 20n+15 $f\left(z_n w^{(1)} h_n{}^{(1)}\right)$ = 20n+17

$f\left(h_n{}^{(1)} h_n{}^{(2)}\right)$ = 20n+18

$f\left(z_n w^{(2)} h_n{}^{(3)}\right)$ = 20n+21

$f\left(h_n{}^{(3)} h_n{}^{(4)}\right)$ = 20n+22

$f\left(z_n w^{(2)} z_n w^{(1)}\right)$ = 20n+20

$f\left(z_n w^{(2)} h_n{}^{(3)}\right)$ = 20n+21

$f\left(h_n{}^{(4)} z_n w^{(2)}\right)$ = 20n+23

$f\left(z_n w^{(2)} z_n\right)$ = 20n

Thus f provides a harmonic mean labeling of graph G.

Hence G is a harmonic mean graph

## Example:2.4.1

A harmonic mean labeling of graph G obtained by structure of H- super subdivision of Y-tree HSS($Y_{n+1}$) $\odot$ $K_2$ are shown in fig 2.4.1

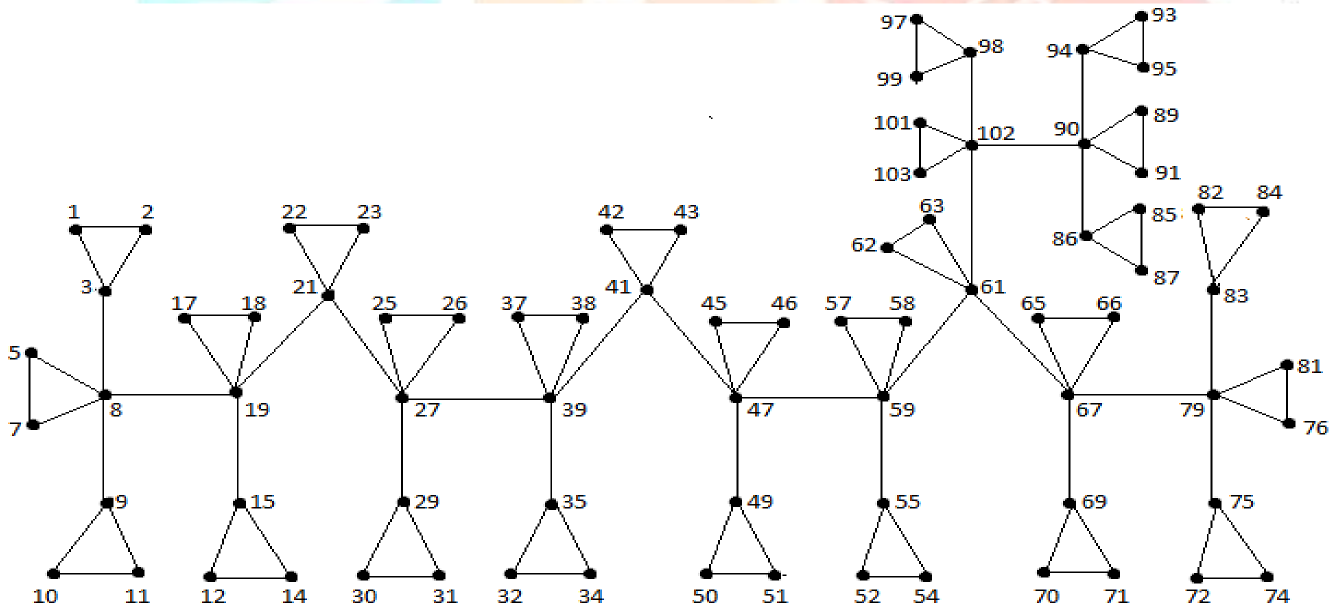



fig 2.4.1

## Conclusion:

We have presented a few new results on Harmonic mean labeling of certain classes of graphs like the that H-super subdivision of HSS($Y_{n+1}$), HSS($Y_{n+1}$) $\odot$ $K_1$, HSS($Y_{n+1}$) $\odot\overline{K_2}$, HSS($Y_{n+1}$) $\odot$ $K_2$. Analogous work can be carried out for other families and in the context of different types of graph labeling techniques.

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