IJCRT.ORG ISSN : 2320-2882
INTERNATIONAL JOURNAL OF CREATIVE RESEARCH THOUGHTS (IJCRT)
An International Open Access, Peer-reviewed, Refereed Journal

Super Padovan Graceful Labeling

Ashwini I

Department Of Mathematics, S.S .Government Arts and Science College,Tiruttani

Abstract: In this paper , I introduce a new concept of labeling called Super Padavon Graceful Labeling as follows: . An injective function f from V(G) into {0, p_1, p_2,..., p_q} is Padovan graceful if the induced edge labeling f *(xy) = | f(x)− f(y)| is a bijection onto the set {p_1, p_2,..., p_q}. A graph G(p,q) which admits a Super Padovan graceful labeling is called a Super Padovan graceful graph, where p_q is the qth Padovan number in the Padovan sequence. Existence of Super Padovan graceful labeling of some of the graphs are discussed here.

Keyword: **Wheel ,Comb, coconut tree,Olive tree, Super Padovan Graceful Labeling ,Super Padovan Graceful Graph.**

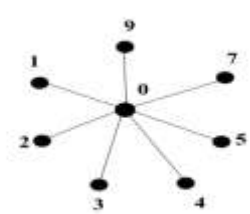
1. INTRODUCTION

Graph labeling plays a important role in Graph theory. It is used in many applications like coding theory ,x-ray crystallography and circuit design. Rosa introduced the concept of graceful labeling f of a (p,q) graph G as follows: f is a graceful labeling if f is an injection from V(G) to the set {0,1,2,...,q} such that when each edge uv is assigned the label | f(u)− f(v)|, the resulting edge labels are distinct. A Fibonacci graceful labeling and Super Fibonacci graceful labeling have been introduced by Kathiresan and Amutha [4] in 2006. As an extension to Fibonacci graceful labeling, we introduce Padovan graceful labeling here, Padovan graceful labeling of certain families of graphs are discussed.

2. Definitions

Definition 2.1: Let G(p,q) be a graph. A injective function f from V(G) into {0,1,2,..., F_q}, where F_q is the qth Fibonacci number is said to be Fibonacci graceful if the induced edge labeling f ∗ (uv) = | f(u) − f(v)| is a bijection onto the set {F_1, F_2,..., F_q}. If a graph G(p,q) admits a Fibonacci graceful labeling then G is called a Fibonacci graceful graph.

Definition 2.2: Let G(p,q) be a graph. An injective function f from V(G) into {0, p_1, p_2,..., p_q} , where p_q is the qth Padovan number in the Padovan sequence is said to be Super Padovan graceful if the induced edge labeling f ∗ (uv) = | f(u)− f(v)| is a bijection onto the set { p_1, p_2,..., p_q} .If a graph G(p,q) admits a Super Padovan graceful labeling then G is called a Super Padovan graceful graph. We fix the values to be $p_0 = 0, p_1 = 1, p_2 = 2, p_3 = 3, p_4 = 4$ and $p_n = p_{n-2} + p_{n-3}$ for all n ≥ 5 (i.e) {0,1,2,3,4,5,7,9,12,16,21,28,……..} is a Padovan sequence.



Example 2.1: In Figure 2.1 Provides an example of Super Padovan graceful labeling of a graph
Figure 2.1

3. Main results

Theorem 3.1: the cycle c_3, c_4, c_6 are a Super Padovan graceful graph.

proof: a) Cycle C_3: We assume by definition then there exists an injective function f : V(G) → {0, p_1, $p_{2,}$, p_3}since there are 3 edges we assume q=3 and hence $p_q = p_3$ such that the induced edge labels are padovan numbers are {p_1, p_2, p_3} = {1,2,3}.
let x,y,z be the vertices of the cycle C_3.then f(x) = 0, f(y) = p_1=1 and f(z) = p_3=3.
then for the vertices x and y, f *(xy) = p_1=1**,**
for the vertices y and z, f *(yz) = p_2= 2**,** which is a padovan number,
for the vertices z and x, f *(zx) = p_3= 3**,** which is a padovan number.therefore $\boldsymbol{C_3}$ is a Super Padovan graceful graph.

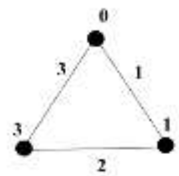


Figure 3.1.a

b) Cycle C_4: We assume by definition then there exists an injective function f : V(G) → {0, p_1, $p_{2,}$, p_3, p_4}since there are 4 edges we assume q=4 and hence $p_q = p_4$ such that the induced edge labels are padovan numbers are {p_1, p_2, p_3, p_4} = {1,2,3,4}. let a,x,y,z be the vertices of the cycle C_4.then f(a) = 0, f(x) = p_4=4 , f(y) = p_1=1and f(z) = p_2=2 .
then for the vertices a and x, f *(ax) = p_4=4**,**
for the vertices x and y, f * (xy) = p_3= 3**,** which is a padovan number,
for the vertices y and z, f * (yz) = p_1= 1**,** which is a padovan number,
for the vertices z and a, f *(za) = p_2= 2**,** which is a padovan number
therefore $\boldsymbol{C_4}$ is a Super Padovan graceful graph.

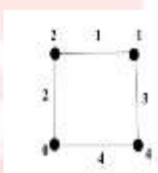


Figure 3.1.b

c) Cycle C_6: We assume by definition then there exists an injective function f : V(G) → {0, p_1, $p_{2,}$, p_3, p_4, p_5, p_6}since there are 6 edges we assume q=6 and hence $p_q = p_6 = 7$ such that the induced edge labels are padovan numbers are {p_1, p_2, p_3, p_4, p_5, p_6} = {1,2,3,4,5,7}. Let a,b,c,d,e,f be the vertices of the cycle C_6.then the labeling is as follows f(a) = 0, f(b) = p_6=7 , f(c) = p_3=3, f(d) = p_1=1, f(e) = p_2=2 and f(f) = p_5=5.
Then the resulting Super Padovan graceful labeling graph is shown below :

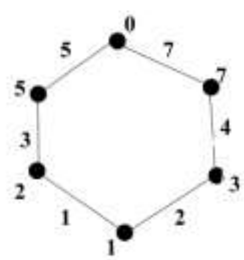
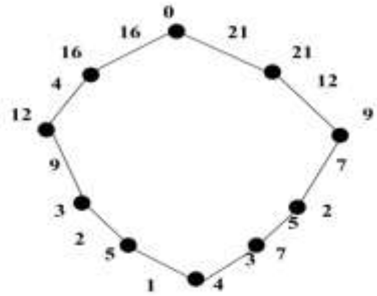


Figure 3.1.c

Observation 3.1: i) A cycle to be a Super Padovan graceful labeling has a even number of odd Padovan numbers. Cycle C_6 has 4 odd Padovan numbers and Cycle C_{10} has 6 odd Padovan numbers which is shown below:



Observation 3.1

Theorem 3.2 : The Wheel W_3 is Super Padovan graceful labeling .

Proof: By definition there exists an injective function f : V(G) → $\{0, p_1, p_2, p_3, p_4, p_5, p_6\}$since there are 6 edges we assume q=6 and hence $p_q = p_6 = 7$ such that the induced edge labels are padovan numbers are $\{p_1, p_2, p_3, p_4, p_5, p_6\}$ = {1,2,3,4,5,7}. Without loss of generality we assume u,v,x,y are the vertices of the cycle C_6.Then the labeling as follows f(u)=0,f(v)= p_6=7 ,f(x)= $p_5 = 5$,f(y)= $p_4 = 4$.

f*(uv)= $p_6 = 7$
f*(ux)= $p_5 = 5$
f*(uy)= $p_4 = 4$
f*(vx)= $p_2 = 2$
f*(vy)= $p_3 = 3$
f*(xy)= $p_1 = 1$
from which we get all padovan numbers , hence W_3 is Super Padovan graceful labeling .

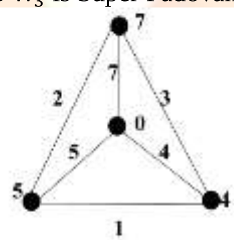


Figure 3.2

Theorem 3.3: The Path $p_2, p_3, p_4, p_5, p_6, p_7, p_8$ are all Super Padovan graceful graphs.

Proof: a) Path p_4: According to the definition of Padovan graceful graphs the range of labels of the vertex varies from 0 to p_q where q is the size of the graph and p_q is the qth Padovan number (V = $\{0, p_1, p_2, ..., p_q\}$).So the order of p_4 is 4 and size of the graph is 3 .There exists a function f : V(G) → $\{0, p_1, p_2, p_3\}$ such that the induced edge labels are padovan numbers are $\{p_1, p_2, p_3\}$ = {1,2,3}. Super Padovan labeling is illustrated in the figure .

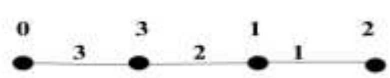


Figure 3.3.a: Super Padovan Graphs , $\boldsymbol{p_4}$

b) Path p_5 : There exists a function f : V(G) → $\{0, p_1, p_2, p_3, p_4\}$ such that the induced edge labels are padovan numbers are $\{p_1, p_2, p_3, p_4\}$ = {1,2,3,4}. Super Padovan labeling is illustrated in the figure .

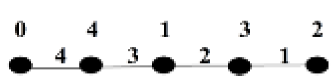


Figure 3.3.b

c)Path p_6 : There exists a function f : V(G) → $\{0, p_1, p_2, p_3, p_4, p_5\}$ such that the induced edge labels are padovan numbers are $\{p_1, p_2, p_3, p_4, p_5\}$ = {1,2,3,4,5}. Super Padovan labeling is illustrated in the figure .

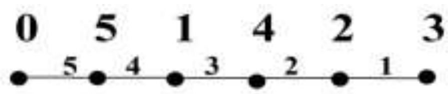


Figure 3.3.c

d) Path p_7 : There exists a function f : V(G) → $\{0, p_1, p_2, p_3, p_4, p_5, p_6\}$ such that the induced edge labels are padovan numbers are $\{p_1, p_2, p_3, p_4, p_5, p_6\}$ = {1,2,3,4,5,7}. Super Padovan labeling is illustrated in the figure .

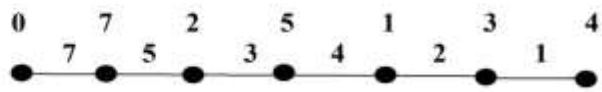


Figure 3.3.d

e)Path p_8 : There exists a function f : V(G) → $\{0, p_1, p_2, p_3, p_4, p_5, p_6, p_7\}$ such that the induced edge labels are padovan numbers are $\{p_1, p_2, p_3, p_4, p_5, p_6, p_7\}$ = {1,2,3,4,5,7,9}. Super Padovan labeling is illustrated in the figure

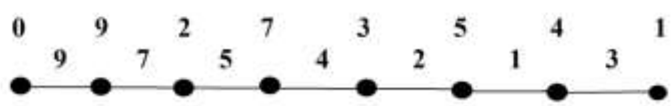


Figure 3.3.e

Theorem 3.4: Friendship graph f_4 is Super Padovan graceful labeling.

Proof: Let f_4 be the friendship graph. The order of f_4 is p=9 and the size of f_4 is q=12. By definition of labeling ,There exists a function f : V(G) → $\{0, p_1, p_2, p_3, p_4, p_5, p_6, p_7, p_8, p_9, p_{10}, p_{11}, p_{12}\}$ such that the induced edge labels are padovan numbers are $\{p_1, p_2, p_3, p_4, p_5, p_6, p_7, p_8, p_9, p_{10}, p_{11}, p_{12}\}$ = {1,2,3,4,5,7,9,12,16,21,28,37}. Then the labeling as follows: let $f(v_0)$=0 , x_1, x_2, x_3, x_4 are the first vertices of the triangle and y_1, y_2, y_3, y_4 are the second vertices of the triangle ,if $f(x_1)$= 28 ,$f(x_2)$ =21,$f(x_3)$=12,$f(x_4)$=3,$f(y_1)$=37,$f(y_2)$=16,$f(y_3)$=5,$f(y_4)$=1 .

$f^*(v_0x_1) = 28$

$f^*(v_0y_1) = 37$

$f^*(v_0x_2) = 21$

$f^*(v_0y_2) = 16$

$f^*(v_0x_3) = 12$

$f^*(v_0y_3) = 5$

$f^*(v_0x_4) = 3$

$f^*(v_0y_4) = 1$

$f^*(x_1y_1) = 9$

$f^*(x_2y_2) = 4$

$f^*(x_3y_3) = 7$

$f^*(x_4y_4) = 2$

from which we get the induced edge labels are padovan numbers are $\{p_1, p_2, p_3, p_4, p_5, p_6, p_7, p_8, p_9\}$ = {1,2,3,4,5,7,9,12,16,21,28,37}, Hence f_4 is a Super Padovan graceful labeling.

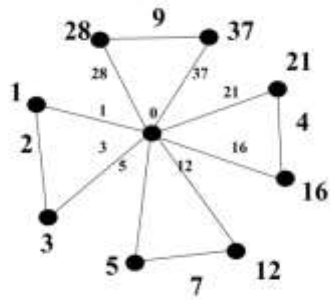


Figure 3.4

Theorem 3.5: Complete graphs k_n is Super Padovan graceful labeling iff n<=4.

Proof: i) case n=3: We assume by definition then there exists an injective function f : V(G) → $\{0, p_1, p_{2,}, p_3\}$since there are 3 edges we assume q=3 and hence $p_q = p_3$ such that the induced edge labels are padovan numbers are $\{p_1, p_2, p_3\}$ = {1,2,3}.
let x,y,z be the vertices of the complete graph k_3.then f(x) = 0, f(y) = p_1=1 and f(z) = p_3=3.
then for the vertices x and y, f *(xy) = p_1=1**,**
for the vertices y and z, f * (yz) = p_2= 2**,** which is a padovan number,
for the vertices z and x, f *(zx) = p_3= 3**,** which is a padovan number.therefore $\boldsymbol{k_3}$ is a Super Padovan graceful graph.

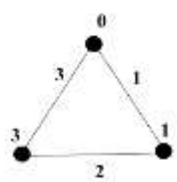


Figure 3.5.i

ii) case n=4: We assume by definition then there exists an injective function f : V(G) → $\{0, p_1, p_{2,}, p_3, p_4\}$since there are 4 edges we assume q=6 and hence $p_q = p_6$ such that the induced edge labels are padovan numbers are $\{p_1, p_2, p_3, p_4, p_5, p_6\}$ = {1,2,3,4,5,7}. let a,x,y,z be the vertices of the complete graph k_4.then f(a) = 0, f(x) = p_6=7 , f(y) = p_5=5 and f(z) = p_4=4 .
then for the vertices a and x, f *(ax) = p_6=7**,**
for the vertices x and y, f *(xy) = p_2= 2**,** which is a padovan number,
for the vertices y and z, f *(yz) = p_1= 1**,** which is a padovan number,
for the vertices z and a, f *(za) = p_4= 4**,** which is a padovan number,
for the vertices z and x, f*(zx) = p_3 =3,which is a padovan number,
for the vertices y and a,f*(ya)= $p_5 = 5$,which is a padovan number
therefore $\boldsymbol{k_4}$ is a Super Padovan graceful graph.

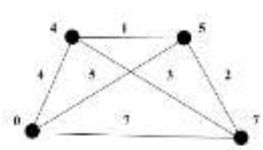


Figure 3.5.ii

Theorem 3.6: Combs are Super Padovan graceful labeling for n=3,4,5,6.

Proof: Comb graph are of the form p_nAK_1. The order of the comb is p=2n and the size of the comb is q=2n-1 .

let $x_1, x_2, x_3, x_4 \dots . x_n$ are the vertices of the path p_n and $y_1, y_2, y_3, y_4 \dots . y_n$ are the pendent vertices attached to the path .

a)n=3, p_3AK_1: the order of the comb is 6 and the size of the comb is 5. There exists a function f : V(G) → $\{0, p_1, p_{2,}, p_3, p_4, p_5\}$ such that the induced edge labels are padovan numbers are $\{p_1, p_2, p_3, p_4, p_5\}$ = {1,2,3,4,5}.then the labeling as follows , we assume $f(x_1) = 0$, $f(x_2) = 4$,$f(x_3) = 2$, $f(y_1) = 5$,$f(y_2) = 1$, $f(y_3) = 3$

$f^*(x_1y_1)$=5

$f^*(x_1x_2)$=4

$f^*(x_2y_2)$=3

$f^*(x_2x_3)$=2

$f^*(x_3y_3)$=1,

from which we get the induced edge labels are padovan numbers are {1,2,3,4,5} .Hence comb p_3AK_1 is a Super Padovan graceful labeling .

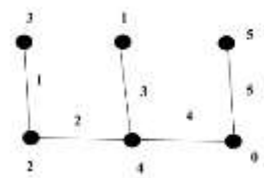


Figure 3.6.a

b) n=4, p_4AK_1: The order of the comb is 8 and the size of the graph is 7. There exists a function f : V(G) → $\{0, p_1, p_{2,}, p_3, p_4, p_5, p_6, p_7\}$ such that the induced edge labels are padovan numbers are $\{p_1, p_2, p_3, p_4, p_5, p_6, p_7\}$ = {1,2,3,4,5,7,9} then the labeling as follows , we assume $f(x_1) = 0$, $f(x_2) = 7$,$f(x_3) = 3$,$f(x_4) = 4$, $f(y_1) = 9$,$f(y_2) = 2$, $f(y_3) = 5$,$f(y_4) = 1$

$f^*(x_1y_1)$=9

$f^*(x_1x_2)$=7

$f^*(x_2y_2)$=5

$f^*(x_2x_3)$=4

$f^*(x_3y_3)$=2

$f^*(x_4y_4)$ =3

$f^*(x_3x_4) = 1$, from which we get the induced edge labels are padovan numbers are {1,2,3,4,5,7,9} .Hence comb p_4AK_1 is a Super Padovan graceful labeling.

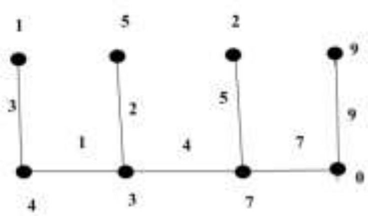


Figure 3.6.b: comb p_4AK_1 is a Super Padovan graceful labeling.

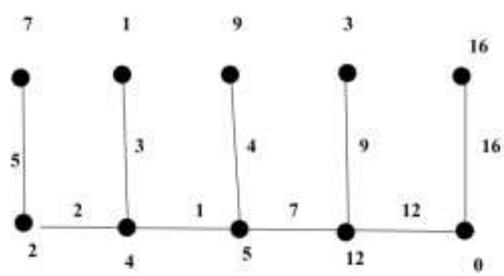


Figure 3.6:comb p_5AK_1 is a Super Padovan graceful labeling.

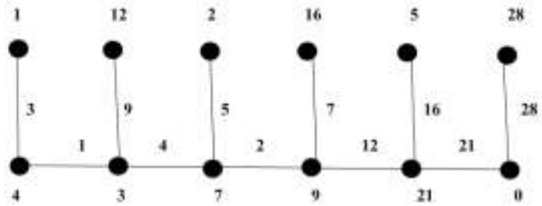


Figure 3.6:comb p_6AK_1 is a Super Padovan graceful labeling.

Theorem 3.7: Coconut tree CT(6,n) is Super Padovan graceful labeling for all n.

Proof: Let CT(6,n) be the Coconut tree of order p= 6+n and size q=(6-1)+n . By Definition , we have $u_1, u_2, u_3 \ldots\ldots u_m$ be the vertices of the path and $v_1, v_2, v_3, v_4, \ldots\ldots v_n$ be the pendent vertices attached to the end vertex of the path. Define the labeling as follows : f: V(G) → {0, p_1, p_2,..., p_q}

$f(u_1) = 0$

$f(v_i) = p_{6+(i-1)}$, i=1,2,....n

$f(u_i) = p_5 - p_4 + p_3 - p_2 + p_1$, i = 2,3,4,5,6

then the above labeling admits Super Padovan graceful labeling, Hence Coconut tree CT(6,n) is Super Padovan graceful labeling for all n. The generalized graph for Coconut tree CT(6,n) for all n is shown in the figure .

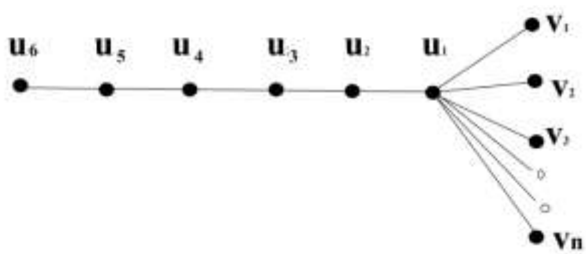


Figure 3.7.a: Coconut tree CT(6,n) is shown in the figure .

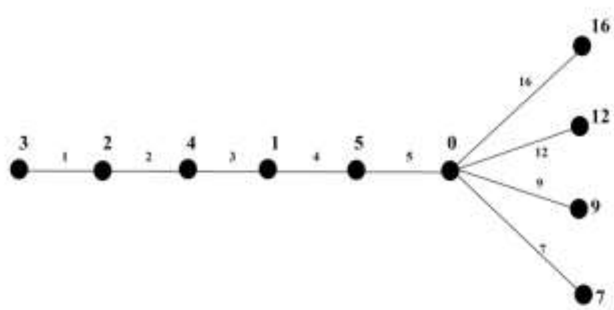


Figure 3.7.b: Coconut tree CT(6,4) is shown in the figure .

Theorem 3.8: Olive tree O_4 is Super Padovan graceful labeling .

Proof: Olive Trees O_n has the order $(n^2+n+2)/2$ and the size is $n(n+1)/2$. So in O_4 we have 11 vertices and 10 edges. By definition of labeling ,There exists a function f : V(G) → $\{0, p_1, p_2, p_3, p_4, p_5, p_6, p_7, p_8, p_9, p_{10}\}$ such that the induced edge labels are padovan numbers are $\{p_1, p_2, p_3, p_4, p_5, p_6, p_7, p_8, p_9, p_{10}\}$ = {1,2,3,4,5,7,9,12,16,21}. Its proved to be O_4 is Super Padovan graceful graph from the figure.

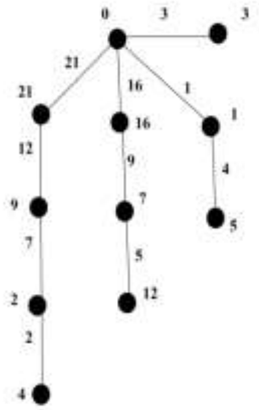


Figure 3.8

4.Conclusion

In this paper we have shown that the cycle (c_3, c_4, c_6), The Path $(p_2, p_3, p_4, p_5, p_6, p_7, p_8)$, The Wheel (W_3), Friendship graph (f_4) ,Combs, Coconut tree CT(6,n),Olive tree O_4 are Super Padovan graceful labeling .

5. References

[1] Rosa .A, “On certain valuation of vertices of graph” (1967).

[2] Kathiresan K. and Amutha S., Fibonacci graceful graphs, Ph.D. Thesis, Madurai Kamaraj University, (October 2006)

[2] D. Muthuramakrishnan and S. Sutha ,Pell graceful labeling of graphs(2019)