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SOME COMMUTATIVITY RESULTS ON CERTAIN RINGS

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Abstract:

In this we prove (1) a semi-prime ring R satisfying the condition $[xyz, [xy, yx]] = 0$, is commutative provided Char $R \neq 2$ and also (2) a non-associative 2-torsion free ring with unity 1 satisfying the condition $[x^2y^2–xy, x]=0$ or $[x^2y^2–xy, y] = 0$, then R is commutative. Initially **Gupta** [3] proved that division ring D which satisfies the polynomial identity $xy^2x = yx^2y$ for all $x, y \in D$ must be commutative. This was generalized by **Awtar** [5] as "A semi prime ring R satisfying the condition $xy^2x–yx^2y \in Z(R)$ is commutative." **Al-mojil** generalized this theorem by showing that a 2-torsion free semi-prime ring in which xy commutes with $xy^2x–yx^2y$ for all $x, y \in R$, is commutative.
We generalize this result as "In a 2-torsion free semi-prime ring, if xyz commutes with $xy^2x–yx^2y$ for all $x, y \in R$, then R is commutative. A result proved by **Ashraf and Quadri** [4] is that a semi-prime ring satisfying the condition $(xy)^2–xy \in Z(R)$ is commutative. We generalize this result by showing that a non-associative ring satisfying either of the conditions $[x^2y^2–xy, x] = 0$ and $[x^2y^2–xy, y] = 0$, is commutative provided Char $R \neq 2$.

Keywords: Periodic Ring, Center, Torsion Free Ring, Semi-Prime Ring.

I. INTRODUCTION:

The study of associative and non- associative rings has evoked great interest and assumed importance. The results on associative and non- associative rings in which one does assume some identities in the center have been scattered throughout the literature.Many sufficient conditions are well known under which a given ring becomes commutative. Notable among them are some given by **Jacobson, Kaplansky** and Herstein. Many Mathematicians of recent years studied commutativity of certain rings with keen interest. Among these mathematicians **Herstein, Bell, Johnsen, Outcalt, Yaqub, Quadri and Abu-khuzam** are the ones whose contributions to this field are outstanding.

II. PRELIMINARIES:

Commutator

For every x, y in a ring R satisfying [x,y] = xy-yx then [x,y] is called a commutator

Commutative Ring

For every x, y in a ring R if xy=yx then R is called a commutative ring.

Non–commutative ring is split from the commutative ring, i.e., R is not commutative with respect to multiplication. i.e., we cannot take xy=yx for every x, y in R as an axiom.

Periodic Ring

For positive integers m, n with $m(x), n(x)$ such that $x^m = x^n$ for all x in R then R is called a periodic ring i.e., m=m(x) and n=n(x). Due to Chacron R is periodic if and only if for each $x \in R$, there exists a positive integers k=k(x) and a polynomial $f(\lambda) = f_x(\lambda)$ with integer co–efficient such that $x^k = x^{k+1} f(x)$.

Prime Ring

A ring R is called a prime ring if whenever A and B are ideals of R such that AB = 0 then either A = 0 or B = 0.

Semi Prime Ring

A ring R is semi prime if for any ideal A of R, $A^2 = 0$ implies A = 0. These rings are also referred to as rings free from trivial ideals.

Primitive Ring

A ring R is defined as primitive in case it possesses a regular maximal right ideal, which contains no two–sided ideal of the ring other than the zero ideal.

Division Ring

A ring R is said to be a division ring if its non–zero elements form a group with respect to multiplication.

Center

In a ring R, the center denoted by Z(R) is the set of all elements $x \in R$ such that xy=yx for all $x \in R$. It is important to note that this definition does not depend on the associative of multiplication and in fact, we shall have occasion to deal with derivation of non-associative algebras.

III. MAIN RESULTS:

Theorem 1. Let *R* be a 2-torsion-free semi-prime ring such that $[xyz,[xy, yx]] = 0$ for all *x*, *y* in *R*. Then *R* is commutative.

Proof : By Hypothesis.

$(xy^2x - yx^2y)\, xyz = xyz\, (xy^2x - yx^2y)$ …1.1

Replacing *x* by *x* + *y* in 1.1, we obtain

$(xy^2x - yx^2y)(xyz + y^2z) + [y^2(yx - xy) + (xy - yx)y^2](xyz + y^2z)$
$= (xyz + y^2z)(xy^2y) + (xyz + y^2z)$ …1.2
$[y^2(yx - xy) + (xy - yx)y^2]$

Again replace *x* by *x* + *y* in 1.2 to yield.

$[xy^2x - yx^2y + y^2(yx - xy) + (xy - yx)y^2[xyz + y^2z + y^2z]$
$+[y^2(yx - xy) + (xy - yx)y^2][xyz + y^2z]$
$= (xyz + y^2z + y^2z)\, [xy^2x - yx^2y + y^2\, (yx - xy) + (xy - yx)\, y^2]$
$+ (xyz + y^2z + y^2z)\, (y^2\, (yx - xy) + (xy - yx)\, y^2\,]$ …1.3

Using (1.2) in (1.3), we obtain

$[xy^2x - yx^2y + y^2(yx - xy) + (xy - yx)y^2]y^2x$
$+[y^2(yx - xy) + (xy - yx)y^2][xyz + y^2z + y^2z]$
$= y^2z[xy^2x - yx^2y + y^2(yx - xy) + (xy - yx)y^2]$
$+[xyz + y^2z + y^2z][y^2(yx - xy) + (xy - yx)y^2]$ ...1.4

Again replacement of *x* by *x* + *y* in (1.4) yields

$[xy^2x - yx^2y + y^2(yx - xy) + (xy - yx)y^2]y^2z + [y^2(yx - xy) + (xy - yx)y^2]y^2z$
$+[y^2(yx - xy) + (xy - yx)y^2][xyz + y^2z + y^2z] =$
$y^2z[xy^2x - yx^2y + y^2(yx - xy) + (xy - yx)y^2]$
$+[xyz + y^2z + y^2z][y^2(yx - xy) + (xy - yx)y^2]$

Simplifying and using (1.4), we obtain

$2[y^2\, (yx - xy) + (xy - yx)\, y^2\,]y^2z = 2y^2z\, [y^2\, (yx - xy) + (xy - yx)\, y^2]$

But since *R* is 2-torsion free, so we obtain

$[y^2(yx - xy) + (xy - yx)y^2z = y^2z[y^2(yx - xy) + (xy - yx)y^2]$
$[(xy - yx)y^2 - y^2(xy - yx)]y^2z = y^2z[(xy - yx)y^2 - y^2(xy - yx)]$

i.e., ...1.5

Replacing z by $z + y$ in (1.5) and using (1.5) we obtain

$[(xy-yx)y^2 - y^2(xy - yx)]y^2 z = y^2[(xy - yx)y^2 - y^2(xy - yx)]$

i.e., $[[[x, y], y^2], y^3] = 0$ ...1.6

Let I_r denote the inner derivation with respect to r i.e., $I_r : X \to [r, x]$, then (1.6) becomes $I_{y3}I_{y2}I_y(x) = 0$. Using lemma which is applicable in prime rings wehave either $I_{y3}I_{y2}= 0$ or $I_y = 0$. If $I_{y3}I_{y2}= 0$ then for x, $y \in R$. $I_{y3}I_{y2}$ (x)=0 Then again by lemma. Either $I_{y3} = 0$ or $I_{y2}=0$ i.e., $y^3 \in Z(R)$ or $y^2 \in Z(R)$.

Then in both the cases we have either $[y^3, x] = 0$ or $[y^2, x] = 0$ which by lemma yields that R is commutative. Now consider the case $I_y = 0$ which implies

$I_y(x) = 0$ or $xy–yx = 0$, i.e., $xy–yx$.

Thus in all the cases R is commutative. Since R is isomorphic to subdirect sum of prime ring R_o each of which as homomorphic image of R satisfies the hypothesis imposed on R so theorem holds for semi-prime rings also.□

Theorem 2.: A non–associative ring with unity 1 satisfying either of the conditions :

(a) $[x^2y^2–xy, x] = 0$ (b) $[x^2y^2–xy, y] = 0$

Is commutative provided it is 2-torsion free.

Proof . By hyhpothesis (a) we have

$(x^2y^2–xy)x = x(x^2y^2–xy)$ ...2.1

Replacing y by $y + 1$in 2.1and using it, we obtain

$2x(x^2y) = 2(x^2y)x$

Since R is 2-torsion free hence

$x(x^2y) = (x^2y)x$ ...2.2

Now replacing x by $x + 1$ in 2.2 and using it we yield

$2x(xy) + xy = 2(xy)x + yx$ …2.3

Again replacing x by $x + 1$ and using 2.3 we obtain

$2xy = 2yx$

i.e., $2(xy–yx) = 0$. But R is 2-torsion free.

So we have $xy = yx$. Thus R is commutative. Hypothesis (b) gives us

$(x^2y^2–xy)y = y(x^2y^2–xy)$ …2.4

Replacing x by $x + 1$in 2.4 and using 2.4 we obtain

$2(xy^2)y = 2y(xy^2)$

But R is 2-torsion free, hence

$(xy^2)y = y(xy^2)$ …2.5

Now replace y by $y + 1$ in 2.5 and use 2.5 to obtain

$2(xy)y + xy = 2y(xy) + yx$ ...2.6

Again replacing $y$by $y+1$ in 2.6 and using 2.6 we obtain

$2(xy–yx) = 0$,

since$R$ is 2-torsion free, this yields $xy = yx$. Thus R is commutative.□

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