IJCRT.ORG ISSN : 2320-2882

INTERNATIONAL JOURNAL OF CREATIVE RESEARCH THOUGHTS (IJCRT)

An International Open Access, Peer-reviewed, Refereed Journal

A choice function which is not dictatorship

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Abstract

Within Arrovian framework, purpose of this work is to find a definite social preference (at least mathematically) corresponding to a profile of preferences. By relaxing the condition 'independence of irrelevant alternatives' of Arrow's Impossibility Theorem and redefining the concept 'unanimity' (to get the image of a profile of preferences, it is necessary to redefine the concept of unanimity and that has been shown in the discussion section), we can arrive at a choice function that respects transitivity and newly defined unanimity but not genuinely dictatorship.

Keywords : Choice function, Unanimity, Transitivity.

JEL Classification Code : **C02, H19, I38, O21**

Introduction

In the Arrovian framework comparability is demanded for individual preferences. But in this work, comparability is also demanded for social preference. With this demand, one can show the existence of a collective choice rule which is non-dictatorship and respects transitivity and unanimity.

1. Transitive preference and choice

Let A = $\{\propto, \beta, \ldots.., \gamma\}$ be a finite set of at least three alternatives, no two alternatives are non comparable. A *transitive preference* over A, is a ranking of the alternatives from top to bottom with ties in every possible way allowed (i.e. a chain w.r.t. binary relation of 'weak preference'). By 'tie', we mean 'as good as', not 'indifference'. In this sense 'weak preference' is anti-symmetric.

Consider a society of N individuals, each of whom has a (potentially different) transitive preference. Let S be the set of all possible transitive preferences over A.

Consider $S^N = \{(x_{1,} x_{2,} \ldots.., x_N) \,/\, x_i \text{ } is\ the\ transitive\ preference\ of\ the\ i-th\ individual \}$ and each element in S^N is called a *profile*.

A *choice* is a function $f : S^N \rightarrow S$, the image of a profile under f is called a *social preference*.

A choice respects *unanimity* if for society to put alternative α strictly above β, it is necessary that every individual must puts α strictly above β (or equivalently, if every individual does not put α> β then society cannot put α> β).

Lemma 1 : Suppose the choice respects unanimity. Let the alternatives α and β be chosen arbitrarily. If at a profile in which some voter puts strictly α over β and some other voter does not the same, then correspondingly society must put α and β in tie.

Proof : We first observe that three following cases may arise –

Case 1 : Let all other voters put strictly β over α. By unanimity, in this case, society cannot put strictly α over β or strictly β over α.

Case 2 : Let all other voters put α and β in tie. In this case also, by unanimity, society cannot put strictly α over β.

Case 3 : Let there exists some other voters who put strictly β over α and there exists some other voters who put α and β in tie. In this case also, by unanimity, society cannot put strictly α over β or strictly β over α.

Since no two alternatives are non comparable, by law of trichotomy we have only feasible conclusion that society must put α and β in tie. This completes the lemma.

2. Existence of choice that respects transitivity and unanimity

Without loss of generality consider $A = \{\alpha, \beta, \gamma\}$. Now we can have three following cases –

Case 1 : Let every individual puts α at very top (or very bottom) and there exists differences of opinion about the ranking of β and γ at a profile. Then by Lemma 1, society must put β and γ in tie. Now using transitivity, social preference will be

$\left\{\begin{matrix}\alpha \\ \beta=\gamma\end{matrix}\right\}$ or $\left\{\begin{matrix}\beta=\gamma \\ \alpha\end{matrix}\right\}$,

both of them are transitive preferences (i.e the member of S).

Case 2 : Suppose we have three following transitive preferences at a profile :

$$\begin{matrix}\alpha & \gamma & \alpha \\ \gamma & \alpha & \beta \\ \beta & \beta & \gamma\end{matrix}$$

In this case by Lemma 1, α = γ, β = γ and hence by transitivity, social preference will be α = β = γ, which is a transitive preference (i. e. a member of S).

Case 3 : Let there exist differences of opinion about the ranking of all α, β and γ at a profile. In this case using Lemma 1 and transitivity social preference will be α = β = γ, which is a member of S.

Thus we conclude that the choice which respects transitivity and unanimity exists.

3. Question of Dictatorship

A choice is a *dictatorship* by individual *n* if for every pair of alternatives α and β, society strictly prefers α to β whenever *n* strictly prefers α to β.

Let each voter puts the alternative β (arbitrarily chosen) at very bottom of his / her ranking of alternatives. By unanimity and transitivity society must are as well. Now let the individuals from 1 to N successively move β from the bottom of their ranking to the very top, leaving other relative ranking in place. Let n^* be the first voter whose change causes the social ranking of β to the very top. By unanimity and transitivity, this change occurs at the latest when $n^* = N$. Denote by profile I the list of all voter rankings just after n^* moves β to the very top.

Now if possible let n^* is a dictator over any pair α and γ not involving β. Construct profile II from profile I by letting n^* moves α above β, so that $\alpha > \beta > \gamma$ and by letting all other voters arbitrarily rearrange their ranking of α and γ keeping β in its extreme position.

Since n^* is a dictator, corresponding to profile II, social preference should put $\alpha > \gamma$. But by lemma 1, social preference corresponding to profile II, must put $\alpha = \beta$.

Now if some voter other than n^* puts $\gamma > \alpha$ or $\alpha = \gamma$ in profile II, corresponding social preference would put $\gamma = \alpha = \beta$ (by lemma 1 and transitivity), a contradiction.

Again if all the voters put $\alpha > \gamma$ in profile II, we can have following two sub cases –

Sub case 1 : Let all the voters put γ and rest of the alternatives strictly bellow α and β. In this case social preference would put $\beta = \alpha > \gamma$ and in this case each individual is a dictator over the pair α and γ.

Sub case 2 : Let there exists two voters, other than n^*, whose preferences are following

β	β
α	δ
γ	α
δ	γ

for any other alternative δ. In this case by lemma 1 and transitivity social preference would put $\beta = \alpha = \gamma\ (= \delta)$. This is also a contradiction.

Thus we can say that above described choice function is not genuinely dictatorship.

4. Discussion

By lemma 1, if some voter puts $\alpha > \beta$, correspondingly society cannot put $\beta > \alpha$. Thus the condition 'independence of irrelevant alternatives' of Arrow's Impossibility Theorem is relaxed but not absolutely removed. Actually 'independence of irrelevant alternatives' is satisfied w.r.t. partial order relation '$\geq$'. In fact, 'weak preference' is a partial order relation.

Now look back at case 2 of section 2. Here every individual puts $\alpha > \beta$ but society cannot put $\alpha > \beta$ otherwise it will violate the transitivity and no image can be found. Thus for society to put $\alpha > \beta$, it is not sufficient that every individual must put $\alpha > \beta$, transitivity relation should also be satisfied. This is the reason for redefining the concept of 'unanimity'.

Since the choice function f exists, it is always decisive. Clearly the choice function is symmetric of its arguments. Also if some voter puts $\alpha > \beta$ correspondingly society would put $\alpha > \beta$ or $\alpha = \beta$ and if some voter puts $\beta > \alpha$ correspondingly society would put $\beta > \alpha$ or $\alpha = \beta$. Thus the choice function does not favour either alternative. This is true for any pair of alternatives.

Again if some voter changes his / her vote from $\alpha > \beta$ or $\alpha = \beta$ to $\beta > \alpha$, correspondingly social preference would changes from $\alpha > \beta$ to $\alpha = \beta$ or from $\alpha = \beta$ to $\alpha = \beta$ or $\beta > \alpha$. Now if we assign values -1, 0, 1 respectively to $\alpha > \beta$, $\alpha = \beta$, $\beta > \alpha$, then social preference changes from -1 to 0 or from 0 to 0 or 1, for the pair α and β. Thus the choice function f is non – negative responsive for any pair of alternatives.

Conclusion

Above described choice function can be considered as social welfare function, since corresponding to a profile of preferences we can find a social preference. Hence this function shows that under certain changes of definitions (mentioned above), the Arrovian impossibility can be avoided. This function may be applicable for holding referendums on public policies to facilitating people's economic and social opportunities and safeguarding the rights and liberties of individuals and minorities.

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