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Medical Application Of Fuzzy Relation By Coupling With Transition Probability Matrix

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Abstract

In modern times Fuzzy mathematics is applied to various fields and a lot of research is going on in this area of Fuzzy. Some authors have applied Fuzzy mathematics in the field of medicine . In this paper Fuzzy relation is applied to the medical field for the first time to diagonise and predict results in the long run. The diseases are in general taken as d_1, d_2 etc. In this paper finally d_1 and d_2 are replaced with particular diseases like COVID 19.

Keywords: Fuzzy relation; Fuzzy Matrix; Markov Chain; Transition Probability Matrix.

1. Introduction

In recent times much research has been done in fuzzy mathematics and it is used in various fields. In this paper how fuzzy relation can be used in the field of medicine is researched and explained. The highlight of the paper is how this concept can be applied in the testing of COVID 19 is included in the last section of this paper. This paper is constructed as follows.

In section 2 the basic concept of fuzzy matrices is given. In section 3 the prerequisite concepts of fuzzy relations are discussed.

In section 4 the basics of Markov chains are given. In section 5, the method to obtain medical results are given. In section 6, a numerical example is provided. In section 7 an application to the treatment of COVID 19 is provided.

2. Basic concepts of Fuzzy matrices

The characteristics function of a crisp set assigns a value of either 1 or 0 to each individual in the universal set theory discriminating between members and non- members of the crisp set. This function can be generalised such that the values assigned to the elements of the universal set fall within a specific range and indicate the membership grade of these elements in the set in question. Larger values denote higher degrees of set membership. Such a function is called a membership function and the set defined by it as a fuzzy set. Thus the membership function μ_A by which a fuzzy set A is defined has the form of $\mu_A : X \to [0,1]$.

A Fuzzy matrix is a matrix with elements having values in the closed interval $[0,1]$.

Let F_{mn} denotes the set of all $m \times n$ Fuzzy matrices over $[0,1]$. If $m = n$ we write F_n.

3. Fuzzy relation

If a crisp relation R represents that from set A to B, for $x \in A$ and $y \in B$ its membership function $\mu_R(x,y)$ is

$$\mu_R(x,y) = \begin{cases} 1 & (x,y) \in R \\ 0 & (x,y) \notin R \end{cases}$$

The membership function maps $A \times B$ to set $\{0,1\}$.

$$\mu_R : A \times B \to \{0,1\}$$

A Fuzzy relation has degree of membership whose values lies in $[0,1]$

$$\mu_R(x,y) : A \times B \to \{0,1\}$$

$$R = \{(x,y), \mu_R(x,y) / \mu_R(x,y) \geq 0, x \in A, y \in B\}$$

Here $\mu_R(x,y)$ is interpreted as strength of relation between x and y.

When $\mu_R(x,y) \geq \mu_R(x',y')$, (x,y) is more strongly related than (x',y').

4. Markov chain

If $P[X_n = a_n / X_{n-1} = a_{n-1}, X_{n-2} = a_{n-2} \ldots X_0 = a_0] = P[X_n = a_n / X_{n-1} = a_{n-1}]$ for all n, then the process $\{X_n, n = 0,1,2,\ldots\}$ is called a Markov chain.

The conditional probability $P[X_n = a_j / X_{n-1} = a_i]$ is called the one step transition probability from state a_i to a_j at the nth step and is denoted by $P_{ij}(n-1,n)$.

If the one step transition probability does not depend on the step $P_{ij}(n-1,n) = P_{ij}(m-1,m)$, the Markov chain is called a homogeneous Markov chain. When the Markov chain is homogeneous the one step transition probability is denoted by P_{ij}. The matrix $P = [P_{ij}]$ is called transition probability matrix, shortly TPM.

5. Working Rule to obtain the medical results

When crisp relation R represents the relation from crisp set A to B its domain and range are given by

$$dom(R) = \{x / x \in A, y \in A, \mu_R(x,y) = 1\}$$

$$ran(R) = \{y / x \in A, y \in A, \mu_R(x,y) = 1\}$$

When fuzzy relation R is defined in crisp sets A and B, the domain and range of this relation are

$$\mu_{dom(R)}(x) = \max_{y \in B} \mu_R(x, y)$$

$$\mu_{ran(R)}(x) = \max_{x \in A} \mu_R(x, y)$$

It is clear that $dom(R) \subseteq A$ and $ran(R) \subseteq B$.

From the fuzzy relation we can form the TPM of the Markov chain. If the sum of the each row is 1 we can make use of the known result $\pi P = \pi$ where $\pi = [\pi_1 \quad \pi_2]$

Where π_1 and π_2 are the probabilities in the long run and hence $\pi_1 + \pi_2 = 1$.

6. A numerical examples with imaginary values

Let d_1 and d_2 be two diseases. Let us consider the following Fuzzy relation

$$d_1 \xrightarrow[1]{} d_2$$

$$d_1 \xleftarrow[\frac{1}{2}]{} d_2$$

From the Fuzzy relation it is clear that if disease d_1 occurs surely it leads to d_2 in the next stage. If d_2 occurs it leads to the disease d_1 with 50% surety.

From this we can obtain the TPM $\begin{array}{c} \\ d_1 \\ d_2 \end{array} \begin{array}{c} d_1 \quad d_2 \\ \begin{bmatrix} 0 & 1 \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix} \end{array}$ assuming a 0.5 grade for missing data.

Here the sum of each row is 1.

Making use of the result $\pi P = \pi$ we get

$$[\pi_1 \quad \pi_2] \begin{bmatrix} 0 & 1 \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix} = [\pi_1 \quad \pi_2]$$

$$\Rightarrow \frac{\pi_2}{2} = \pi_1 \text{ and } \pi_1 + \frac{\pi_2}{2} = \pi_2$$

Also $\pi_1 + \pi_2 = 1$ always.

Solving we get

$$\pi_1 = \frac{1}{3}, \quad \pi_2 = \frac{2}{3}$$

Therefore in the long run the likelihood of getting the disease d_2 is more.

7. An application in testing for COVID 19

In section 6, if we take d_1 to be the disease due to COVID 19 virus and d_2 to be the common cold. Here d_1 mostly leads to d_2 but d_2 does not result in d_1. That is those who are having COVID 19 may have the cold symptoms but the converse that those who are having cold symptoms will have COVID 19 is not necessarily true. So it is clear that the common cold may not result in COVID 19. As mentioned earlier section 6 is worked out with imaginary values. If genuine data is collected we can ascertain more results like which type of cold is an indication for COVID 19 etc.

8. Conclusion and Scope for further research

In this paper Fuzzy relation is coupled with transition probability matrix to obtain results in the long run. If physically proper data is collected we can make use of Fuzzy relation to predict results in the long run with the help of the Markov chain tool.

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