



AN OPTIMIZATION MODEL FOR MULTI-OBJECTIVE SUPPLY CHAIN WITH DISRUPTIONS AND VOLUME FLEXIBILITY

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Abstract: This paper deals with the design of supply chain models with complexities in the present industry. In this paper a mathematical model has been designed for a production – distribution supply chain network considering volume flexibility and the various risks associated with disruptions. The model is framed using mixed integer nonlinear programming and multiple periods, multiple products, suppliers, manufacturing plants, distribution center and customer market and distinct disruption scenarios have been considered. The aim is to minimize the total operational cost, minimizing the shortage cost due to disruptions and maximizing the flexibility level of the back orders.

Index Terms - Supply Chain Management Mixed integer programming, Mixed integer nonlinear programming, Supply Chain disruptions, Multi-objective optimization

I. INTRODUCTION

Supply chain management is an amalgamation of interrelated association among the suppliers, manufacturing plants, distribution center and customer markets to fulfil the customer demands through engineered flow of information, products, services, and money. These different entities form the distinct levels or echelons of the supply chain. The performance of the entire network depends on how each of these elements function and perform, and hence optimization of the performance of these entities plays a major role in the entire supply chain optimization. In the present times, the challenge is to consider various factors like disruptions, shortages, back-orders and other risks and develop an integrated model. In this work, we focus on disruptions and volume flexibility with aim to minimize total operational cost, shortage cost and maximize flexibility level of inventory. There are levels considered starting from suppliers of raw materials to manufacturing units where the products are being produced followed by the distribution center from where the products are shipped to different customer markets. The goal in any business is to maximize profit and reduce all the costs that incur in the entire process from getting the supplies to delivery of final furnished products, hence satisfying the customer demands. This requires delivery of the correct commodity or service at the desired place at the given time and most importantly for the correct prize. The initial costs are associated with supply, production, transportation, and inventory. The additional expenses occur due to certain disruptions which can lead to shortage of supply and untimely delivery of service.

In any industry, there are certain disturbances which arise due to certain factors of which some are under control and some are not. In present time, a huge interest is been taken in the supply chain disruptions which are the events which alter the regular functioning of supply chain and the consequences of these disruptions are at greater magnitude. Supply chains are getting adversely affected due to natural disasters or human activities. These include earthquakes, floods, strikes, economic crises, mishaps in storage, production, transportation and other activities.

Ali and Nakade [1] did a hypothetical case study on scenario based disruptions management through a quantitative approach proposing a model which could react to different scenarios considered. Ho [13] worked on Tabu search heuristic approach for a facility location problem with single source and capacity. Schmitt and Singh [24] worked on networks with disruptions in a multiple stage supply chain problem. Garcia et al.[27] designed supply chain network considering risks due to facility disruptions and according to their work it was observed that these networks were easily affected by the disruptions because of lean inventory management. Multiple sourcing have become extremely dependable in decision making and an enormous study of work have been listed by [3]and [6]including facility location problems.

Investigation on facility disruptions was done by Azad et al. [22] and they considered disruptions in the distribution center and modes of transportation considering stochastic network of supply chain.

Jabbarzadeh et al. [12] framed a network using mixed integer nonlinear programming model in order to add facility disruptions and they presented facility and customer allocation decisions. From the works of [1], [21] and [24] it is showcased that any disruptions caused to any of the entities in supply chain network are unavoidable and hence such risks are to be considered for research. A major study on breakdown of facilities due to disruption was done by Ghomi Avili et al.[23] and they applied simulated annealing to determine reliable and unreliable distribution center. Due to disruptions, the system fails to function normally because of which the production and distribution center fail to supply regular amount of goods. To consider this effect, distinct scenarios are considered, each of which conveys proportion of regular supply quantity in the whole system. The shortage cost is a result of these factors.

Zhou and Wang [28] did a detailed review on supply chain risk management due financial disruptions which occur due to economic uncertainty. Besides, due to these unpredictable events, there is an increasing need for the system to adapt to the situation and respond effectively to the changes which brings in the flexibility matter. Investing in flexibility of the entire supply chain system, different insurance policies related to manufacturing and storage are ways to prevent supply chain from various risks. Volume flexibility determines the output level [19] when the system faces nondeterministic demands. [17] defines volume flexibility as one such type which can alter the output level. Coping with the unpredictable customer demands is one of the crucial cases of volume flexibility. Goyal and Netessine [9]analyzed volume flexibility along with product flexibility under endogenous pricing in a double product scenario wherein they discovered that volume flexibility depended on demand correlation between various products as well as they found that volume flexibility fights total uncertainty in demands. A huge number of works has been done on supply chain optimization and various models have been designed and modified for better results.

Many optimization problems of supply chain management use mixed integer programming and in the seventies an attempt was made by Geoffrion and Graves [8]. They worked on determining the locations of distribution center and customer markets along with flow of products and transportation. Bravo and Vidal [4] worked on bring together production and inventory for controlled decision making at a multiple entity level. In a similar context Piewthongngam et al. [26]did a study on multiple entity case of feed production and distribution planning determining required number of vehicles for transportation and production batch size so as to reduce the cost is minimized. Over the years mixed integer nonlinear programming has been applied in framing models for supply chain optimization with major focus on distribution, production and inventory. Nasiri et al.[11, 18] planned an integrated model which included all these entities with a mixed integer nonlinear model and solved using Lagrangian relaxation approach in the former and in the later they incorporated stochastic demand constraint. In [7], the authors have done a detailed study on logistics chain framing a linear mixed integer model. The problem was solved using a technique named LINDO. An integer programming problem with two criteria was modelled by the authors in [10] and they used an e-constraint technique to obtain a pareto solution.

Monteiro et al.[15] applied a mixed integer nonlinear model which involved three randomly created cases of a system plan in the chain incorporating inventory in storage building on specific non deterministic consumer demands. A nonlinear model with the use of mixed integer programming was developed by Liao et al.[16] for a firm with 15 different distribution units and 50 buyers to attain different objectives with management of location, inventory and vendor system. Guerrero et al.[5] designed a logistics nonlinear complex system with multiple levels and uncertain demands. They transformed the nonlinear problem to a mixed integer linear programming model and inferred that through linearization decision making under different scenarios took shorter time. Kaya and Urek [14]worked on a supply chain(closed loop) using mixed integer nonlinear model wherein inventory was affected by price demand function. This model was solved using heuristic approach. Pourjadav and Mayorga [20] optimized both the forward and reverse flow in a closed loop supply chain for a glass manufacturing industry using mixed integer linear model.



II. MATHEMATICAL MODEL FORMULATION

This work is inspired by the works of [25]and [2].In this paper we consider four levels in the supply chain with multiple suppliers, multiple plants, multiple distribution center and multiple customer markets. In this formulation of our problem we consider multiple products and time periods along with multiple scenarios due to disruptions. In the setup, the suppliers provide the plants with raw materials to manufacture different products, from the production plants the products are shipped to various distribution center from where the products are delivered to different customer markets for sales. The problem considers determining the apt number of distribution center and productions plants to be functioning at different locations considering different number of scenarios as a result of disruptions. We consider that all the suppliers are operating but not all plants and distribution center necessarily operate. The customer markets receive products from the distribution center which are open. It is also assumed that the products incur certain damage due to disruptions, and hence the products thus received by the different distribution center would be inferior to the usual flow quantity. This results in unfulfilled demands by the distribution center leading to shortage costs. The losses could be poor sales or lower product prize etc. The objective is to reduce the total transportation, setup, production, inventory and shortage costs and maximize flexibility level to the changes which takes place.

2.1 Sets and Indices

X	Products set
p	Index for products
S	Suppliers set
s	Index for suppliers
M	Manufacturing plants set
m	Index for manufacturing plants
N	Distribution center set
i	Index for distribution center
R	Customer markets set
j	Index for customer markets
T	Time periods set
t	Index for time periods
F	Scenarios set
f	Index for scenarios

2.2 Parameters

f_m	Fixed coast of each plant
d_i	Fixed cost of each distribution center
K	Large positive number
cs_{ps}	Supplier cost for making of one unit of a product p by supplier s
y_{pmt}	Per unit production cost of product p by plant m
ICN_{pit}	Inventory holding cost of one unit of product p at distribution center i
TMN_{pmi}	Shipping cost of delivering one unit of product p from plant m to distribution center i
TSM_{psm}	Shipping cost of supplies for unit product p from supplier s to plant m
TNR_{pij}	Shipping cost of delivering unit product p from distribution center i to customer market j
D_{pjt}	Demand of product p at customer market j
HCN_i	Holding capacity at distribution center i
T'	Time unit available for manufacturing in any given period
B_{pm}	Processing time for producing one unit of product p by plant m
VF_t	Volume flexibility in time period t
V_p	Volume of product p
λ	Volume flexibility performance index
g_1, g_2	Weight factor for capacity utilization
A_{pj}	Penalty cost for unsatisfied demand of product p by customer market j
pr_f	Chance of scenario f
α_{pmf}	Small Percentage of supply from plant m of product p in scenario f
α_{pif}	Small Percentage of supply from distribution center i of the product p in scenario f

2.3 Decision Variables

x'_p	Raw materials for unit product p
x_{psmt}	Capacity of supply for product p from supplier s to plant m in period t
q_{pmt}	Capacity of product p produced by plant m in period t
q'_{pmit}	Capacity of product p delivered from plant m to distribution center i in period t
q_{pijt}	Capacity of product p delivered from distribution center i to customer market k in period t
IN_{pit}	Inventory level of product p at distribution center i in time period t
q''_{pijft}	Shortage amount of product p from customer j at distribution center i at time period t in scenario f
q_{pijft}	Amount of product p supplied from distribution center i to customer j at period t in scenario f
M_m	$\begin{cases} 1, & \text{if plant operates} \\ 0, & \text{else} \end{cases}$
N_i	$\begin{cases} 1, & \text{if Distribution Center operates} \\ 0, & \text{else} \end{cases}$

2.4 Objective Function

$$\text{Min } Z_1 = \sum_p \sum_s x'_p cs_{ps} + \sum_m f_m M_m + \sum_i d_i N_i + \sum_p \sum_m \sum_t q_{pmt} y_{pmt} M_m + \sum_p \sum_i \sum_j \sum_t q_{pijt} TNR_{pijt} + \sum_p \sum_i \sum_t ICN_{pit} IN_{pit} + \sum_p \sum_s \sum_m \sum_t x_{psmt} TSM_{psm} + \sum_p \sum_m \sum_i \sum_t q'_{pmit} TMN_{pmi} + \sum_f pr_f (\sum_p \sum_i \sum_j \sum_t A_{pj} q''_{pijft}) \quad (1)$$

$$\text{Min } Z_2 = \sum_t VF_t - \lambda T' \quad (2)$$

2.5 Constraints

$$\sum_i q_{pijt} = D_{pjt} \quad \forall p, j, t \quad (3)$$

$$\sum_i q'_{pijt} = q_{pmt} \quad \forall \quad p, m, t \quad (4)$$

$$q_{pijt} \leq KN_i \quad \forall \quad p, i, j, t \quad (5)$$

$$q'_{pmit} \leq KN_i \quad \forall \quad p, m, i, t \quad (6)$$

$$x_{psmt} \leq KM_m \quad \forall \quad p, s, m, t \quad (7)$$

$$IN_{pit} = IN_{pi(t-1)} + \sum_m q'_{pmit} - \sum_j q_{pijt} \quad \forall \quad p, i, t \quad (8)$$

$$\sum_p q'_{pmit} V_p + \sum_p IN_{pi(t-1)} V_p \leq HCN_i \quad \forall \quad t, m, i \quad (9)$$

$$\sum_p B_{pm} q_{pmt} M_m \leq T' \quad \forall \quad t, m \quad (10)$$

$$\sum_i q''_{pijft} \geq D_{pjt} - \sum_i \alpha_{pif} q_{pijt} \quad \forall \quad p, j, f, t \quad (11)$$

$$\sum_i q''_{pijft(t+1)} \geq D_{pjt} - \sum_i q_{pijft(t+1)} \quad \forall \quad p, j, f, t \quad (12)$$

$$\sum_j q_{pijft(t+1)} = \sum_m \alpha_{pmf} q'_{pmit} \quad \forall \quad p, i, t \quad (13)$$

$$VF_t = \left[\frac{T'}{\sum_p B_{pm}} - M_m q_{pmt} \right] g_1 + \left[\sum_i N_i HCN_i - \sum_p \sum_i q'_{pmit} M_m \right] g_2 \quad \forall \quad t, m \quad (14)$$

$$VF_t \geq \lambda \quad \forall \quad t \quad (15)$$

2.6 Non-Negativity Constraints

$$q_{pijt} \geq 0 \quad \forall \quad p, i, j, t \quad (16)$$

$$q'_{pmit} \geq 0 \quad \forall \quad p, m, i, t \quad (17)$$

$$x_{psmt} \geq 0 \quad \forall \quad p, s, m, t \quad (18)$$

$$IN_{pit} \geq 0 \quad \forall \quad p, i, t \quad (19)$$

$$q''_{pijft} \geq 0 \quad \forall \quad p, i, j, f, t \quad (20)$$

$$q_{pijft} \geq 0 \quad \forall \quad p, i, j, f, t \quad (21)$$

$$M_m = 0, 1 \quad \forall \quad m \quad (22)$$

$$N_i = 0, 1 \quad \forall \quad i \quad (23)$$

Equation 1 represents the first objective function which is a minimization function that decreases the entire cost of operation. This includes costs from setup to distribution to shortage costs. Equation 2 represents volume flexibility which is sum of all performance measures per period. The aim is to maximize inventory flexibility level by minimizing 2. Among the constraints, equation 3 implies that the demands of the consumers are met, constraint 4 assures that the total amount of commodities supplied from plant to the distribution center equals quantity produced in that period, equations 5, 6, 7 ensures that there is flow of materials among suppliers to plants to distribution center to customer markets provided the plants and distribution center have been established, equation 8 indicates that the inventory levels are well balanced, equation 9 represents capacity constraints of the distribution center, constraint 10 indicates that capacities of plant is respected, equation 11 represents shortage amount in initial time period and 12 indicated the same in the succeeding period, the 13th constraint refers to quantity of products provided to the distribution center in period (t+1) as a results of disruptions in initial period of time (t), equation 14 represents the sum of plant volume and distribution volume flexibility and equation 15 represent minimum flexibility level. The equation 16 to 23 represents the non-negativity conditions and the decision variable type.

III. CONCLUSION

In this work a mixed integer non-linear model has been designed which includes flexibility improvement of inventory as well as considers the disruption scenarios. The model provides optimum number of manufacturing plants and distribution center to be functioning to save on the excess cost considering multiple scenarios and periods. It optimizes the complete supply chain network with a major intention towards inventory optimization. The future scope is to apply certain metaheuristics approach to solve this nonlinear model with real time data and infer on identifying an optimum heuristic for this model.

References

- [1] S M Ali and K Nakade. Scenario based supply chain disruptions management framework-a quantitative approach. *Innovation and Supply Chain Management*, 8(3):81–91, 2014.
- [2] Syed Mithun Ali and Nakade Koichi. A mathematical optimization approach to supply chain disruptions management considering disruptions to suppliers and distribution centers. *Operations and Supply Chain Management*, 8(2):57–66, 2015.
- [3] M Bieniek. A note on the facility location problem with stochastic demands. *Omega*, 55:53–60, 2015.
- [4] J.J. Bravo and C.J. Vidal. Freight transportation function in supply chain optimization models: A critical review of recent trends. *Expert Systems with Applications*, 40(17):6742–6757, 2013.
- [5] Aaron Guerrero Campanur, Elias Olivares-Benitez, Pablo A. Miranda, Rodolfo Eleazar Perez-Loaiza, and Jose Humberto Ablanedo-Rosas. Design of a logistics nonlinear system for a complex, multiechelon, supply chain network with uncertain demands. *Complexity*, 2018:1–16, nov 2018.
- [6] S.Dantrakul, C.Likasiri and R.Pongvuthithum. Applied p-median and p-center algorithms for facility location problems. *Expert Systems with Applications*, 41(8):3596–3604, 2014.
- [7] A. Creazza, F. Dallari, and T. Rossi. Applying an integrated logistics network design and optimisation model: the pirelli tyre case. *International Journal of Production*, pages 37–41, 2012.
- [8] A M Geoffrion and G W Graves 1974. Multicommodity distribution system design by benders decomposition. *Management science*, 20(5):822–44, 1974.
- [9] Manu Goyal and Serguei Netessine. Volume flexibility, product flexibility, or both: The role of demand correlation and product substitution. *Manufacturing & Service Operations Management*, 13(2):180–193, apr 2011.
- [10] G. Guillen-Gosalbez, F. Mele, and I. Grossmann. A bi-criterion optimization approach for the design and planning of hydrogen supply chains for vehicle use. *AIChE Journal*, (56(3)), 2010.
- [11] Nasiri G. R., Davoudpour H., and Karimi B. The impact of integrated analysis on supply chain management: A coordinated approach for inventory control policy. *Supply Chain Management: An International Journal*, (15(4)):277–289., 2010.
- [12] A.Jabbarzadeh, S.G.Jalali Naini, H.Davoudpour and N.Azad. Designing a supply chain network under the risk of disruptions. *Mathematical Problems in Engineering*, pages 1–23, 2012.
- [13] H C Ho. An iterated tabu search heuristic for the single source capacitated facility location problem. *Applied Soft Computing*, 27:169–178, 2015.
- [14] O. Kaya and B. Urek. A mixed integer nonlinear programming model and heuristic solutions for location, inventory and pricing decisions in a closed loop supply chain. *Computer and Operations Research*, 65:93–103, 2016.
- [15] M. M. Monteiro, J. E. Leal and F. M. P. Raupp. A four-type decision-variable MINLP model for a supply chain network design. *Mathematical Problems in Engineering*, 2010:1–16, 2010.
- [16] Shu-Hsien Liao, Chia-Lin Hsieh, and Yu-Siang Lin. A multi-objective evolutionary optimization approach for an integrated location-inventory distribution network problem under vendor-managed inventory systems. *Annals of Operations Research*, 186(1):213–229, nov 2010.
- [17] M.T. Melo, S. Nickel, and F. Saldanha da Gama. Facility location and supply chain management a review. *European Journal of Operational Research*, 196(2):401–412, jul 2009.
- [18] G. Reza Nasiri, Rohollah Zolfaghari, and Hamid Davoudpour. An integrated supply chain production distribution planning with stochastic demands. *Computers & Industrial Engineering*, 77:35–45, 2014.
- [19] N.Slack. The flexibility of manufacturing systems. *International Journal of Operations & Production Management*, 7(4):35–45, 1987.
- [20] Ehsan Pourjavad and Rene V. Mayorga. An optimization model for network design of a closed-loop supply chain: a study for a glass manufacturing industry. *International Journal of Management Science and Engineering Management*, 14(3):169–179, sep 2018.
- [21] H. Hishamuddin, R.A.Sarker and D. Essam. A recovery mechanism for a two-echelon supply chain system under supply disruption. *Economic Modelling*, 38:555–563, 2014.
- [22] N Azad, H Davoudpour, G K D Saharidis and M Shiripour. A new model to mitigating random disruption risks of facility and transportation in supply chain network design. *The International Journal of Advanced Manufacturing Technology*, 70(9-12):1757–1774, 2013.
- [23] M. Ghomi Avili, A.Makui, S.B.Naghbi and E.Yadegari. Facility location with the disruption risk in the supply chain network design. *Journal of Basic and Applied Scientific Research*, 3(7):898–910, 2013.
- [24] A J Schmitt and M Singh. A quantitative analysis of disruption risk in a multi-echelon supply chain. *International Journal of Production Economics*, 139:22–32, 2012.
- [25] Payam Chiniforooshan Shahrooz Shahparvari and Ahmad Abareshi. Designing an integrated multi-objective supply chain network considering volume flexibility. In *Proceedings of the World Congress on Engineering and Computer Science*, volume 2. IAENG, October 2013.
- [26] K. Piewthongngam, S.Pathumnakul and S.Homkhampad. An interactive approach to optimize production distribution planning for an integrated feed swine company. *International Journal of Production Economics*,

142(2):290–301, 2013.

[27] P Garcia Herrerros , J M Wassick and I E Grossmann. Design of supply chains under the risk of facility disruptions. *Computer Aided Chemical Engineering*, 32(6):577–582, 2013.

[28] Bin Zhou and Zhongxian Wang. Revisit of supply chain risk management and disruption under the recent financial crisis. *International Journal of Operations Research and Information Systems*, 4(1):51–63, jan 2013.

