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FORMATION OF GENERALISED ERROR EQUATION FOR GEARS

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Abstract: *The aim is to avoid the gap between the designed gear and production, which may due to human errors, material quality, etc., by adaptive manufacturing. This includes finding the generalized error equations of a critical part of the gear which is pitch circle for the designed and integrating the equation with workstation and machine with Decision support tool. Initially, gear is designed in designing tools and then manufactured by different processes. Finally, after the product is obtained there are chances of Type-1 or type-2 errors. These errors are discrete and can be formed in a library. With this library, mathematical models are created using different methods, explained in detail further. Now, these models are analyzed and formed different error equations. These equations will give us the idea of how the errors are generating for the particular material and particular dimensions of the gear. The process is much better explained in further.*

***Keywords* – Gear Manufacturing, Intelligent manufacturing, Integrated manufacturing, Machining.**

I. INTRODUCTION

A gear or cogwheel is a rotating machine part having cut teeth, or in the case of a cogwheel, inserted teeth (called cogs), which mesh with another toothed part to transmit torque. Geared devices can change the speed, torque, and direction of a power source. Gears almost always produce a change in torque, creating a mechanical advantage, through their gear ratio, and thus may be considered a simple machine. The teeth on the two meshing gears all have the same shape. Two or more meshing gears, working in a sequence, are called a gear train or a transmission. A gear can mesh with a linear toothed part, called a rack, producing translation instead of rotation. Two important concepts in gearing are pitch surface and pitch angle. The pitch surface of a gear is the imaginary toothless surface that you would have by averaging out the peaks and valleys of the individual teeth. The pitch surface of an ordinary gear is the shape of a cylinder. The pitch angle of a gear is the angle between the face of the pitch surface and the axis. The experiment is done on the bevel gears as the applications of the bevel gear are locomotives, marine applications, automobiles, printing presses, cooling towers, power plants, steel plants, railway track inspection machines, etc.

Bevel gears that have pitch angles of greater than ninety degrees have teeth that point inward and are called internal bevel gears.

In 1992, A.K. Chitale and K.K. Bharani had done experiments on set up conditions for optimum machining of bevel gears. A M Goanta and P Dumitrache had published an article on new applications in designing the bevel gears under IOP publications. Now, to get the most accurate product we can go to adaptive machining means giving knowledge to the machine through the error equations. The rest of the paper organised as Section 2 reviews the production of gear. Section 3 error library formation. Section 4 Governing equation formation and graphs. Section 5,6 are the conclusion and Future scope respectively.

Production of Gear

Manufacturing is done by taking the Catia model as input. So normal milling machines can't do the job. CAM Milling machines are used to complete the objective. There are different types of CAM Milling machines are in the market. We used the Vertical 5 axis machining centre to manufacture the gears. Specifications of the milling machine are given.

In the simplest terms, 5-axis machining involves using a CNC to move a part or cutting tool along five different axes simultaneously. This enables the machining of very complex parts, which is why 5-axis is especially popular for aerospace applications.

Manufacturing steps

- CATIA model is given as input.
- CAM software analyses the model.
- Chooses suitable machining parameters.
- Machining program is generated.
- Machining.

Manual gear hubbers or specialized CNC gear cutting machines are generally the fastest and most efficient means for producing high-precision ring gears, especially in high volumes.

Gears are one of the most common devices within the world of engineering, offering an elegant solution to the problem of effective power transmission. Modern gear drive designs must provide quiet, reliable service at high power densities, which can only be achieved by using gears which accurately embody a geometry like the involute helicoid system. Gear metrology may be divided into two subtopics, functional gaging and analytical testing. The two categories of gear, in general, provide fundamentally different types of information, each with its advantage and disadvantages.

Error library formation

The determination of pitch deviations on the tooth flanks belongs to the most important single tests on gears. The reference circle pitch, measured in the transverse section, is defined as the arc on the circumference of the reference circle of diameter, containing two consecutive right or left tooth flanks.

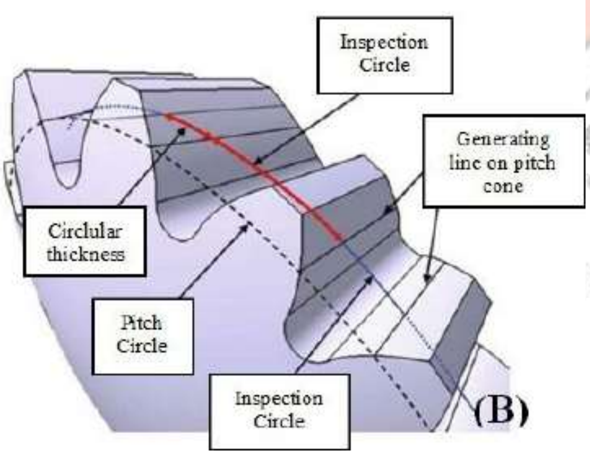


Fig:1 Anatomy of Gear

ISO-based standards define the pitch using the midpoints of two adjacent flank faces which lie on the intersection between the gears pitch reference cone and a “sampling cone”. This intersection between the two cones forms a circle whose diameter is defined in the gear’s ideal parameters. The DIN standard 3965-1 defines the cumulative pitch error, FP, as the difference between the sum of the actual pitches between any two teeth on the pitch circle and the correct value. Pitch data will be extracted from a virtual model as opposed to a real part because no real part can represent the ideal geometry to a high enough standard. All manufactured parts will contain sources of error, thereby introducing bias into any attempt to create a ‘pure’ error source.

points	x	y		x	y		
1	-33.7957	-1.85	21	33.7957	1.85		
2	-32.713	-8.6839	22	32.713	8.6839		
3	-31.5699	-12.2029	23	31.5699	12.2029		
4	-28.4287	-18.3678	24	2804287	18.3678		
5	-26.2538	-21.3613	25	26.2538	21.3613		Pitch
6	-21.361	-26.2538	26	21.361	26.2538		10.633
7	-18.3678	-28.4287	27	18.3678	28.4287		
8	-12.2029	-31.5699	28	12.2029	31.5699		
9	-8.6839	-32.7133	29	8.6839	32.7133		
10	-1.85	-33.7957	30	1.85	33.7957		
11	1.85	-33.7957	31	-1.85	33.7957		
12	8.6839	-32.7122	32	-8.6839	32.7122		
13	12.2029	-31.5699	33	-12.2029	31.5699		
14	18.3678	-28.4287	34	-18.3678	28.4287		
15	21.361	-26.2538	35	-21.361	26.2538		
16	26.2538	-21.3613	36	-26.2538	21.3613		
17	28.4287	-18.3678	37	-28.4287	18.3678		
18	31.5699	-12.2029	38	-31.5699	12.2029		
19	32.713	-8.6839	39	-32.713	8.6839		
20	33.7957	-1.85	40	-33.7957	1.85		

Fig:2 Model table

Single pitch error, FP, is the difference between the desired and the actual transverse pitch. Successive summing of the single pitch errors results in the characteristic curve of the cumulative pitch error. Location errors of the gearing axis with respect to the reference axes used during measurement and machining influence the pitch measurement results. This is because if an SBG is off-center during machining or measurement due to eccentric clamping, for example, the center of the resulting inspection circle will not correspond to the intended axis of the SBG. Such errors are represented by distinctive pitch curves. To create the single pitch error results, the X and Y coordinates are calculated using equations 1 and 2. After this, the X and Y coordinates are converted to degrees transposed about 360°, as opposed to two 180° segments. The difference in degrees between subsequent corresponding flanks is then converted into arc lengths, which is directly compared to the ideal arc length of the Gear. Any difference is recorded as the single pitch error. These values are then summed to generate the cumulative error, given in mm.

II. Governing Equation Formation and Graphs

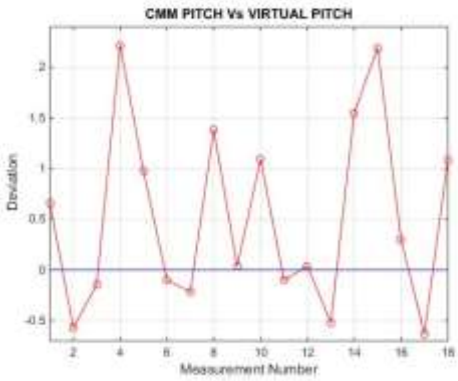
Graph plotting

Graphs are plotted by using the points that are formed by errors in pitch. Now for the gear which is having the N number of gears would have the 2N number of measurements. Because we are selecting the flanks for pitch measurement. So, we need to select the flanks which are consecutively sided. Totally we get a 2N number of measurements can be performed.

Consider the measurement number on X-axis and error on Y-axis. In each measurement, we can find some error value in pitch. This error is the Y coordinate value. All the points are plotted by using the MATLAB program. We can form a graph for each cross section. So, as we performed on three gears each having three cross sections we can observe 9 graphs.

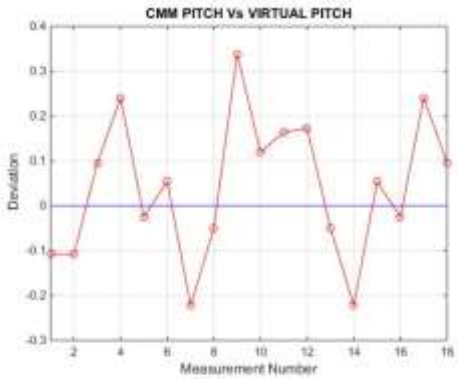
Gears with 9Teeth

The 1st cross section at a depth of 10 mm



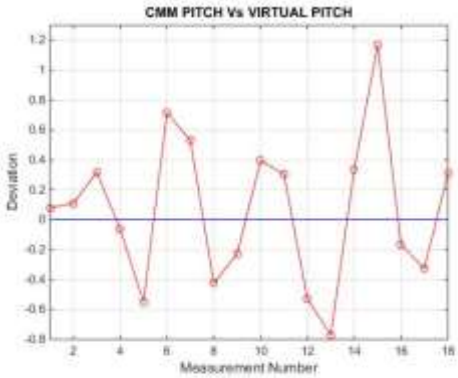
Graph 1

The 2nd cross section at a depth of 5mm



Graph 2

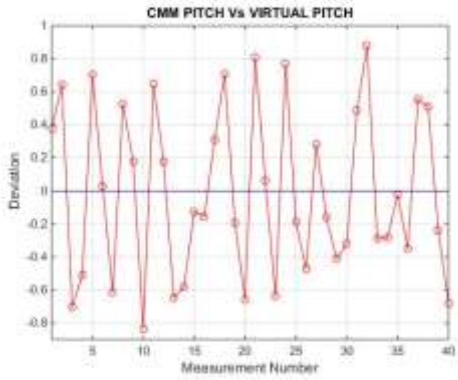
The 3rd cross section at a depth of 7.5 mm



Graph 3

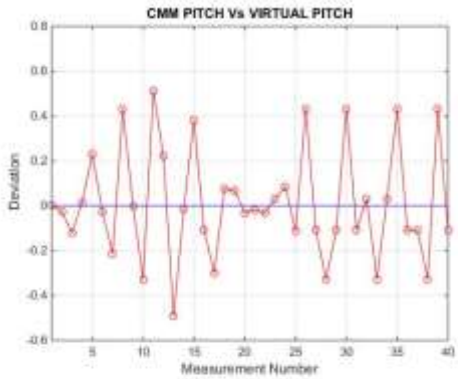
Gears with 20 Teeth

The 1st cross section at a depth of 5 mm



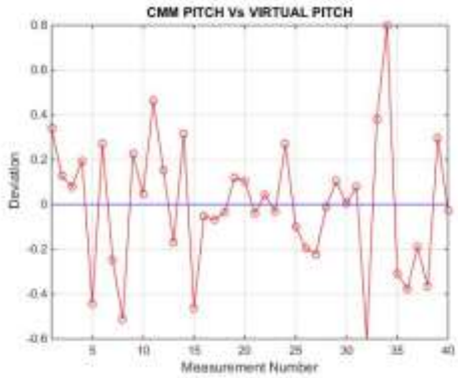
Graph 4

The 2nd cross section at a depth of 8 mm



Graph 5

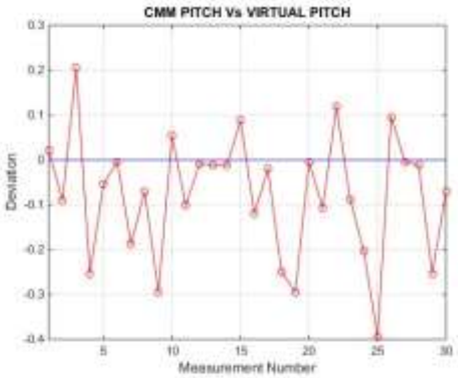
The 3rd cross section at a depth of 7.5 mm



Graph 6

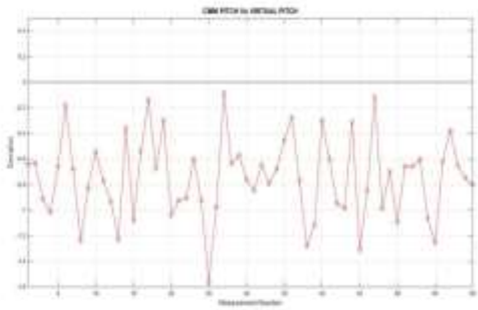
Gears with 30 Teeth

The 1st cross section at a depth of 5 mm



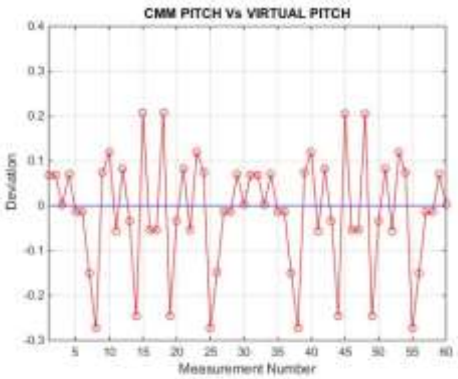
Graph 7

The 2nd cross section at a depth of 10 mm



Graph 8

The 3rd cross section at a depth of 15 mm



Graph 9

METHOD OF LEAST SQUARES

For a set of points which are represented on a graph and are subjected to Method of least squares then we will obtain the line equation. Here we are including a model graph in which the points are plotted and the line is drawn.

By using the method of least squares we will form the lines. For this, we will consider the points from each cross-section of the gear. Then the least square method would be used and straight-line equations are formed. For this, we will follow the process in two steps.

In the first step, we will consider all the positive errors and we will find the equation for those points. In the second step, we will consider all the negative points and we will form another line. In this way, we can see two lines for each graph. In this, we will get a total of 6 equations for each gear. But in the case of 30Teeth gear, we will see only 5 equations. This due to a cross section we will get all points on the negative side.

The equations we got for each gear are listed below. By using these equations lines are drawn and are superimposed on the graphs.

30 T Gear equations are

For 1st cross section at 5 mm depth

1. Y = 0.0934 + 0.0002697 X ....(1) positive error
2. Y = - 0.1032 - 0.00111 X ....(2) negative error

For 1st cross section at 10 mm depth

Y = - 0.73584 - 0.0003246 X ....(3) negative error

For 1st cross section at 15 mm depth

1. Y = 0.08539 +0.0001326X ....(4) positive error
2. Y = -0.10435 -0.000009003X ....(5) negative error

20 T Gear equations are

For 1st cross section at 5 mm depth

1. Y = 0.3939 + 0.00483 X ...(6) positive error
2. Y = - 0.6192 + 0.009099 X ...(7) negative error

For 2nd cross section at 8 mm depth

1. Y = 0.17064 + 0.002815 X ...(8) positive error
2. Y = - 0.11879 - 0.001519 X ...(9) negative error

For 3rd cross section at 7.5 mm depth

1. Y = 0.17448 + 0.00194 X ...(10) positive error
2. Y = -0.36313 + 0.005239 X ...(11) negative error

9 T Gear equations are

For 1st cross section at 5 mm depth

1. Y = 1.09546 - 0.0045456 X ...(12) positive error
2. Y = - 0.18489 - 0.016797 X ...(13) negative error

For 1st cross section at 10 mm depth

1. Y = 0.072017 + 0.0063659 X ...(14) positive error
2. Y = -0.10408 + 0.00040707 X ...(15) negative error

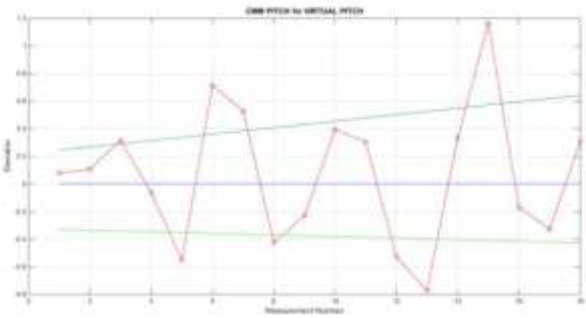
For 1st cross section at 1.5 mm depth

1. Y = 0.22223 + 0.023385 X ...(16) positive error
2. Y = -0.32259 - 0.0057006 X ...(17) negative error

Super impose the lines in the respective graphs. This will give us the idea how the errors are following a function. So, these lines represent the pitch errors. So, these graphs allow us to get the functional change in the pitch. If we form these pitch lengths and draw the arcs between the flanks, we can get the deviation of error pitch circle and original pitch circle.

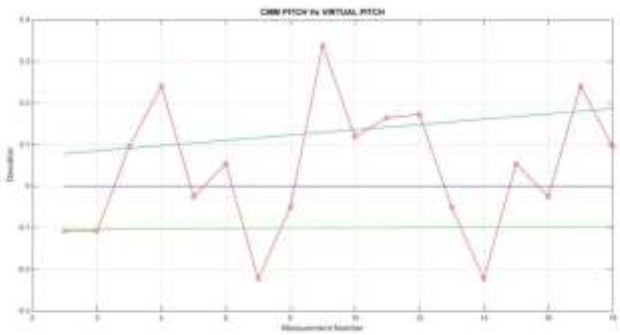
Gear with 9Teeth (Equations combined)

The 1st cross section at a depth of 10 mm



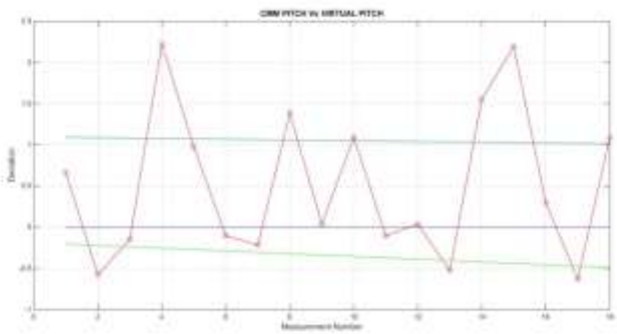
Graph 10

The 2nd cross section at a depth of 5mm



Graph 11

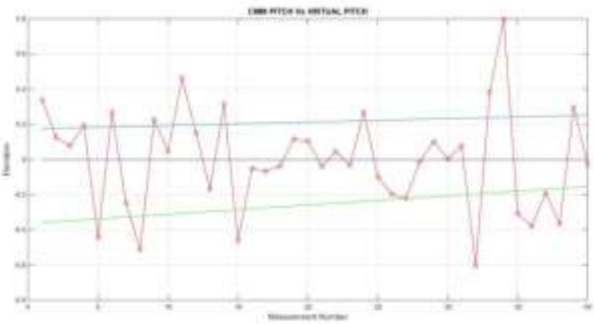
The 3rd cross section at a depth of 7.5 mm



Graph 12

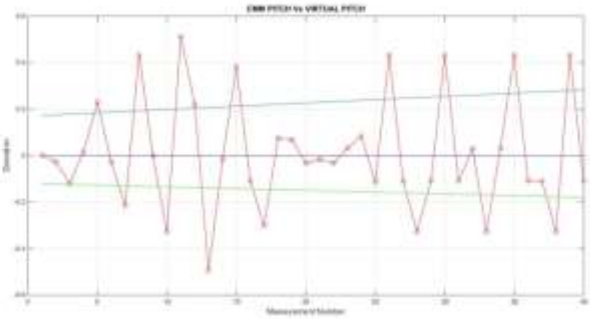
Gear with 20 Teeth (Equations combined)

The 1st coss section at a depth of 5 mm



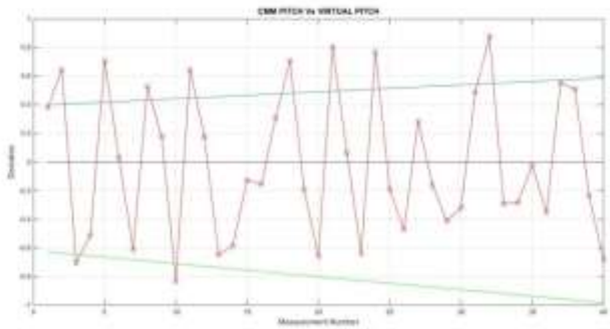
Graph 13

The 2nd cross section at a depth of 8 mm



Graph 14

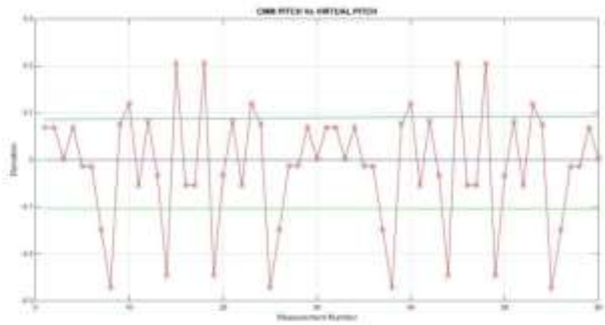
The 3rd cross section at a depth of 7.5 mm



Graph 15

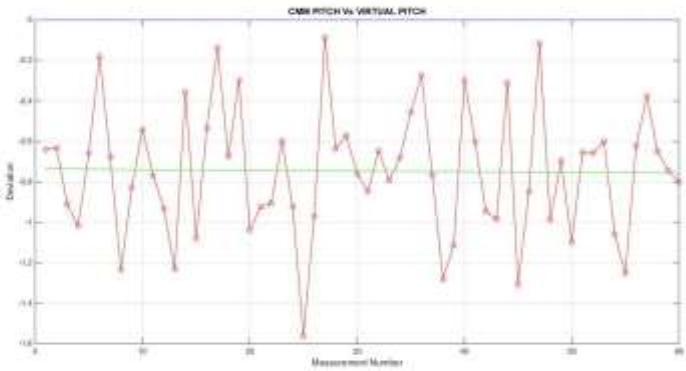
Gear with 30 Teeth (Equations combined)

The 1st cross section at a depth of 5 mm



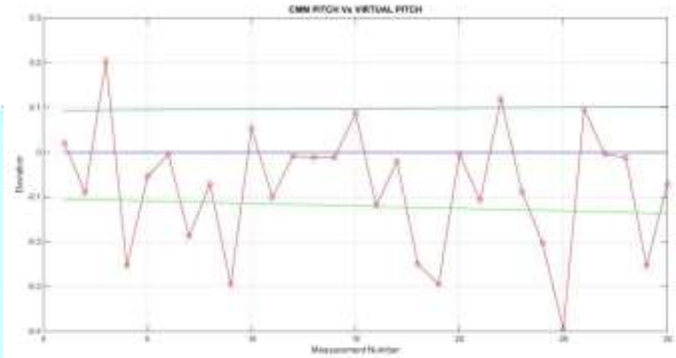
Graph 16

The 2nd cross section at a depth of 10 mm



Graph 17

The 3rd cross section at a depth of 15 mm



Graph 18

Governing equation formation

To form the governing equation we need to use all the equations. So we will combine all the equations we got in three cross sections of gear. To form this we need to consider the equations. We will consider the positive and negative equations individually. Then we will frame a combined function which will give us the resultant governing equation.

As we observe the function should be of the variable of the Z axis. But as there is no relation, in-depth there is no functional relation between the functional equations at cross-sections. So we will consider the errors are independent of gear width. Then we can superimpose all the lines in one graph.

As we observe the are not parallel. So they will have a point of intersection with each other. So we are going to adopt a new technique to solve the common equation. In this, we are going to find the angular bisector of two lines and we will find the angular bisector of the new line and 3rd line.

$L_1 + k\, L_2$ represents the family of lines passing through the intersection of L_1 and L_2. For different values of k, we get different line equations.

Now let us write all the equations of 30 T negative error side

$Y = - 0.1032 - 0.00111\ X$

$Y = - 0.73584 - 0.0003246\ X$

$Y = - 0.10435 - 0.000009003\ X$

Consider first two equation in case of 30 t lets proceed in this way

Now the family of lines are in the form of this equation $L1 + KL2 = 0$

$(Y + 0.1032 + 0.00111\ X) + K(Y + 0.73584 + 0.0003246\ X) = 0$

slope of this family of lines are

$$m = \frac{-(0.00111 + K0.0003246)}{(1+k)}$$

We also know the slope of two lines which is 0.0001326 and 0.0002697.

Now we know the angle between the two lines when the slope is known is

$$\theta = \tan^{-1}\frac{m1 - m2}{1 + (m1 * m2)}$$

As we know the line which is intended is an angular bisector it will make an equal angle with both the lines.

So $\theta 1 = \theta 2$

$$\tan^{-1}\frac{-0.0003246 - \frac{0.00111+0.0003246k}{1+k}}{1 - 0.0003246\frac{0.00111+0.0003246k}{1+k}} = \tan^{-1}\frac{-0.00111 - \frac{0.00111+0.0003246k}{1+k}}{1 - 0.00111\frac{0.00111+0.0003246k}{1+k}}$$

$$\frac{-0.0003246 - \frac{0.00111+0.0003246k}{1+k}}{1 - 0.0003246\frac{0.00111+0.0003246k}{1+k}} = \frac{-0.00111 - \frac{0.00111+0.0003246k}{1+k}}{1 - 0.00111\frac{0.00111+0.0003246k}{1+k}}$$

Taking an approximation

$0.0003246 * \frac{0.00111+0.0003246k}{1+k} \cong 0$ and the same on the other side.

After solving we will get k = -1 and +1

Both lines are angular bisectors. But we need acute angle bisector.

That's why choose $k = 1$

Then we get this equation

(Y + 0.1032 + 0.00111 X) + (Y + 0.73584 + 0.0003246 X) = 0

Y = -0.41952 - 0.0007173 X

This is the angular bisector we obtained from 1st and 2nd lines. Now we need to find the angular bisector of this line and the 3 rd line. To get the angular bisector to follow the above procedure and get the results.

As we are dealing with the lines with the almost same slope, we will always get k = 1;

We performed the check for each and every calculation and results are sorted out.

So, the final governing equation of 30 T gear negative side would be

Y = - 0.261935 - 0.0003631X

In this manner obtain all the solutions and formulated bellow.

Governing equations

30 T **Gear governing equations:**

Y = 0.088495 + 0.00020115 X ...(18) positive side

Y = -0.261935 - 0.0003631 X ...(19) negative side

20 T **Gear governing equations:**

Y = 0.22837 + 0.004452 X ...(20) positive side

Y = - 0.366062 + 0.0045145 X ...(21) negative side

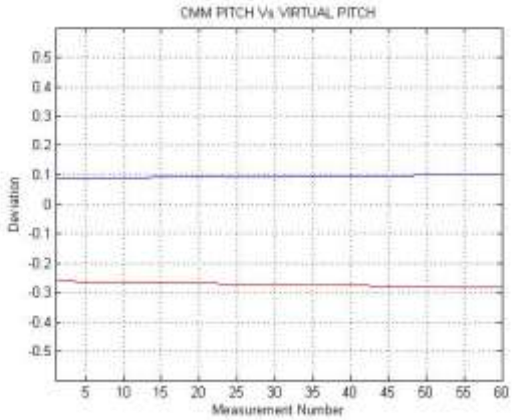
9 T **Gear governing equations:**

Y = 0.40298 + 0.01214 X ...(22) positive side

Y = - 0.233535 - 0.0069473 X ...(23) negative side

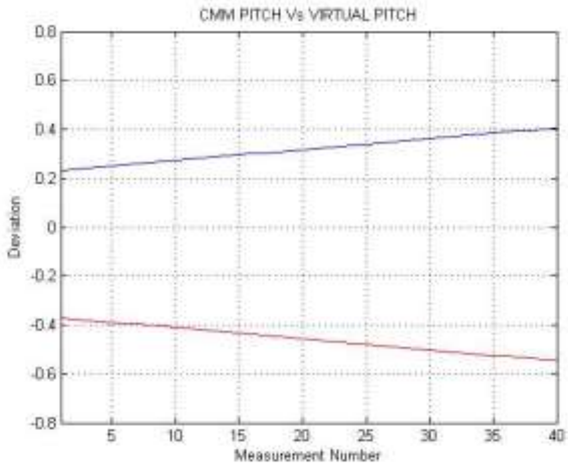
Now let us see the graphs of all gears with their governing equations.

30 T **Gear governing equations graph**



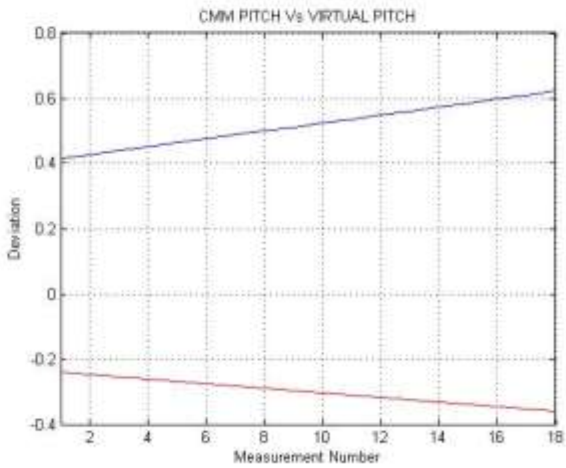
Graph 19

20 T **Gear governing equations graph**



Graph 20

9 T **Gear governing equations graphs**



Graph 21

III. Conclusions

The errors that are found in the pitch measurement of the following gears are discrete values. Even though by using statistical methods we c0an form a function by using all deviations from the original value. So, these errors are following a function. At any stage for particular gear, we can find the errors by using the function. This would result in the selection of the gear among all the gears.

We can also observe that the error lines slope is more in the 9Teeth gear than others and 20Teeth gear has more slope than 30Teeth gear. This would result in a conclusion that

$m_{9t} > m_{20t} > m_{30t}$

So, we can say that the errors get stabilized and minimized as we go for a high number of teethed gears.

We can use differential methods to minimize the slope. We can find the optimal teeth number of the gear at which we can find optimum errors and get good gears.

IV. Future Scope

The Decision Support Tool (DST) constructed using Visual Basic for Applications in Microsoft Excel. The reason for this is due both to the ease of usability of the two applications, and the high level of product integration into many work environments. The structure of the application is split across several functions. The two main functions are those that calculate the pitch errors based on coordinate data, as described in Chapter 6, and the function that determines which of the error library entries matches closest to the CMM results. The right and left flanks are treated as separate entities, rather than the whole gear considered as one.

This is because of the complicated machine movements involved during gear machining. As not all errors are derived from clamping issues for example, it cannot be assumed that the right and left flanks will always show the same source of error. If for example, a machine spindle lengthens during the course of machining, this may result in the Z and Y axes being affected in such a way as to influence two flanks differently. The user of the application is required to simply input pitch results taken directly from a measured part, and select the 'Analyze' button on the app user interface. The app is then coded so that the pitch results that were input get compared one by one to the errors in the error library, using Pearson's Correlation method.

The application then performs two main tasks. First, for every flank, it returns the name of errors which have the highest correlation coefficients. This represents the three errors with the closest match to the input data. Secondly, the app returns the one error which has the highest combined correlation coefficient to the input data, derived from the correlation coefficients of the right and left flanks. This displays to the user the 'best match' scenario, meaning that error with the highest combination of both right and left flank correlation coefficients. The CMM data is then plotted separately against the top three results for every flank and the 'best match' error or error combination. This enables the user to also visually determine the error most probable to be influencing the part.

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