



Behavior of Synovial fluid in a channel

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Abstract

The present study is devoted to investigate analytically the behaviour of synovial fluid in a channel. These studies have enabled the researcher to analyze the lubrication mechanism and structural behaviour of synovial fluid.

Keyword- Synovial fluid, load capacity, interstitial fluid, cartilage, synovial joint

Introduction - Synovial fluid an ultrafiltrate of blood with additives produced by the synovium has a number of biological roles . Articular cartilage is nourished by synovial fluid because cartilage lacks its own blood supply. Normal synovial joints exhibit salient features which are extraordinary in the mechanical sense. They are freely moving low frictional junctures characterized by minimal energy dissipation. Several types of lubrication mechanism are believed in occur in the functioning of human joints. Synovial fluid is a viscous, non-newtonian fluid found in the cavities of synovial joints. Synovial fluid is reduce friction between the articular cartilage of synovial joints during movement. The inner membrane of synovial joints is

called the synovial membrane and secretes synovial fluid into the joint cavity. The fluid contains hyaluronic acid secreted by fibroblast-like cells in the synovial membrane and interstitial fluid filtered from the blood plasma [1]. The fluid forms a thin layer at the surface of cartilage and also seeps into microcavities and irregularities in the articular cartilage surface, filling in the empty space [2].

Synovial tissue is sterile and composed of vascularized connective tissue that lacks a basement membrane. Synovial fluid is made of hyaluronic acid and lubricin proteinases and collagenases. Synovial fluid exhibit non-Newtonian flow characteristics, the viscosity coefficient is not a constant and the fluid is not linearly viscous.

Human moveable joints have coefficient of friction much smaller than any man-made machine. This efficiency is due to perfect combination of amazing materials-elastic cartilage and viscous-elastic cartilage and visco-elastic synovial fluid.

M.c cutchan [4] has suggested that in regards to hydraulic boundary condition of cartilage. The relationship between pressure within the liquid film in the gap and the time dependent deformation of the cartilage was worked out by M.c cutchan[3], Ling [1] and Yadav [6]. Mowetal[5], Mansur [2] provided result for cartilage deformation produced by compressive stress .

In this paper we have made an attempt to study the axial pressure and load bearing capacity of the cartilage between two approaching surfaces.

Formulation of the problem

The governing equation for two dimensional laminar flow in a channel given below;

$$-\frac{\partial p}{\partial x} + \mu \frac{\partial^2 u}{\partial y^2} = 0 \quad (1)$$

The equation of continuity

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0 \quad (2)$$

The solution of the equation subject to the boundary condition;

$$u=0 \quad \text{at } y= h$$

$$u=0 \quad \text{at } y=-h$$

$$\text{and } p=0 \quad \text{at } x=0 \quad (4)$$

the model equation

$$\frac{\partial^2 u}{\partial y^2} = \frac{1}{\mu} \frac{\partial p}{\partial x} \quad (5)$$

Solution of the problem

If the pressure gradient exist along Integration equation (5) , we get

$$u = \frac{1}{2\mu} (y^2 - h^2) \frac{\partial p}{\partial x} \quad (6)$$

from equation of continuity, we have

$$\int_0^h u dy = -\frac{1}{2} vx \quad (7)$$

$$\frac{1}{3\mu} h^3 \frac{\partial p}{\partial x} = \frac{1}{2} vx$$

$$\frac{1}{\mu} \frac{\partial p}{\partial x} = \frac{3}{2} \frac{vx}{h^3} \quad (8)$$

From equation (6) and (8), we get

$$u = \frac{3vx}{4h^3} (y^2 - h^2) \quad (9)$$

differentiate equation (9), we get

$$\frac{\partial u}{\partial y} = \frac{3vx}{2h^3} y \quad (10)$$

Again differentiate equation (10), Partially w.r.to y, we get

$$\frac{\partial^2 u}{\partial y^2} = \frac{3vx}{2h^3} \quad (11)$$

From equation (11)&(5), we get

$$\frac{\partial p}{\partial x} = \frac{3\mu vx}{2h^3} \quad (12)$$

Integrating equation (12), w.r.to x, we get

$$p = \frac{3\mu vx^2}{4h^3} + A \quad (13)$$

Solution of equation (13) under the boundary condition (4)

We get

$$p = \frac{3\mu vx^2}{4h^3} \quad (14)$$

The load capacity is given by

$$w = 2 \int_0^L p dx \quad (15)$$

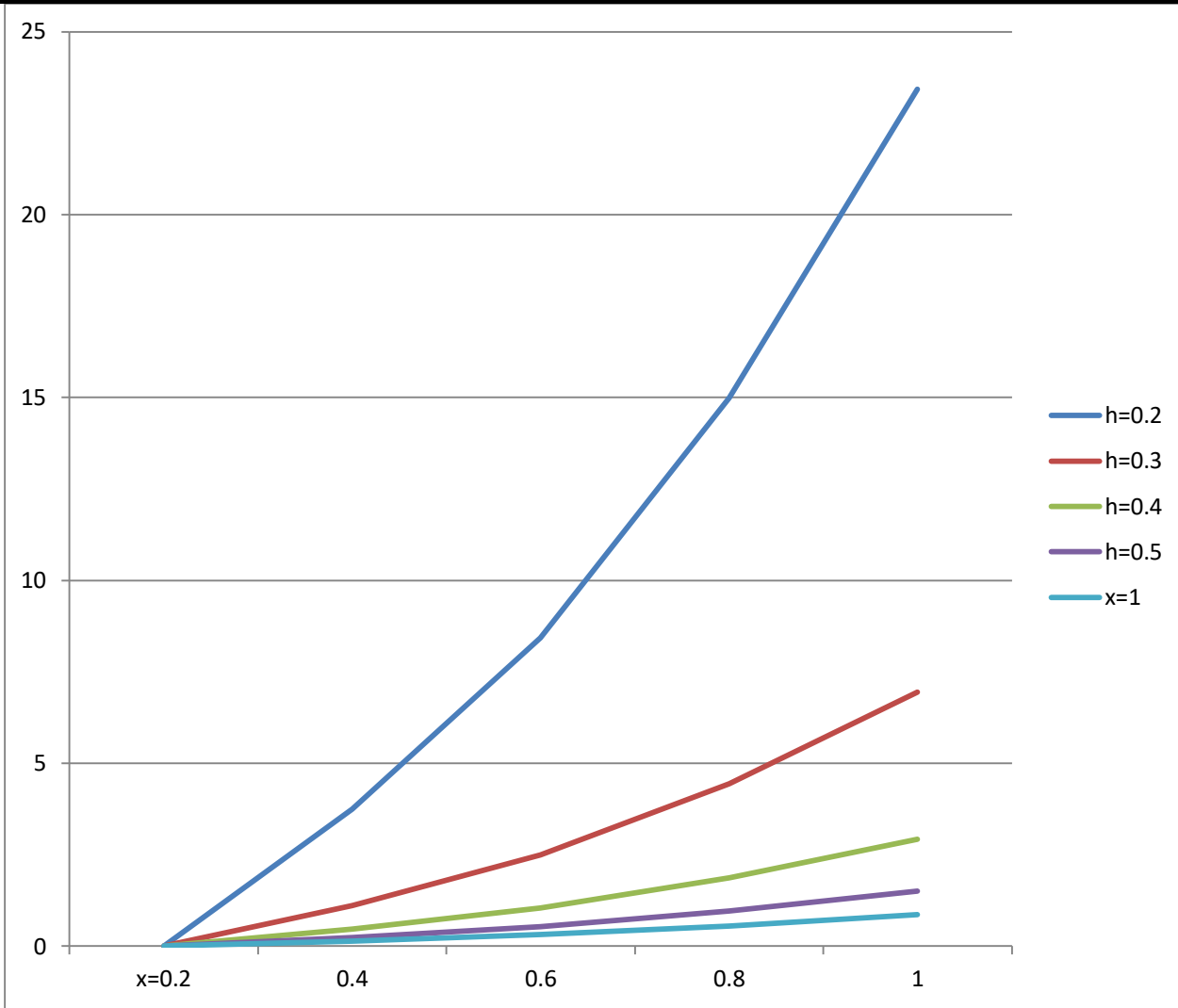
$$= \frac{3}{2} \left(\frac{\mu v L^3}{3h^3} \right)$$

$$= \frac{1}{2} \left(\frac{\mu v L^3}{h^3} \right) \quad (16)$$

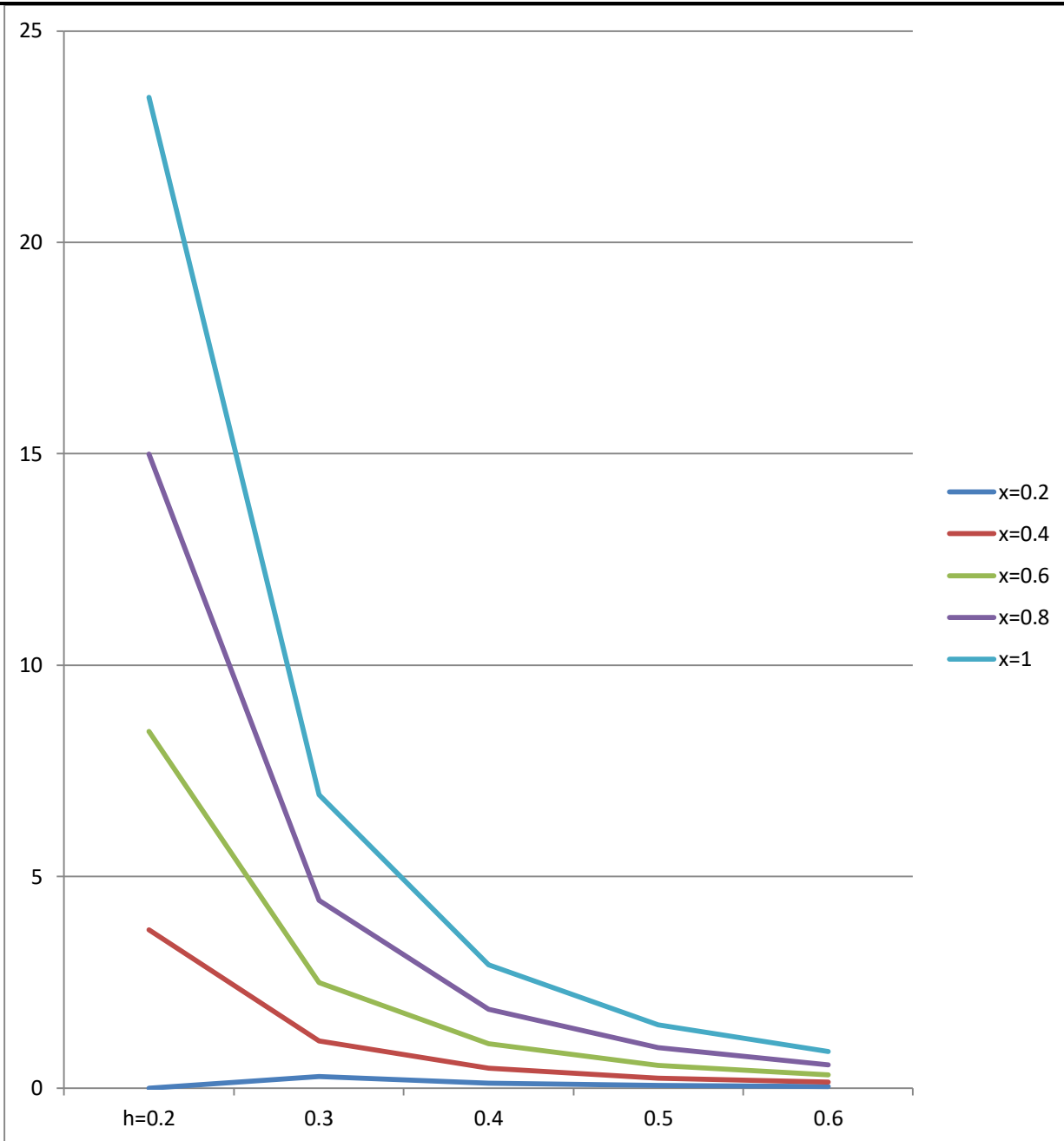
Variation of axial pressure p for different value of x , h

but $\mu = .5$ & $v = .5$

$x \backslash h$	0.2	0.3	0.4	0.5	0.6
0.2	0.9375	0.2777	0.1171	0.0600	0.03472
0.4	3.7488	1.1111	0.4672	0.2400	0.13888
0.6	8.4348	2.5000	1.0512	0.5400	0.31248
0.8	14.9952	4.4416	1.8688	0.9600	0.55552
1.0	23.4300	6.9400	2.9200	1.5000	0.8680



Variation of axial pressure with flow behavior for different value of h & x

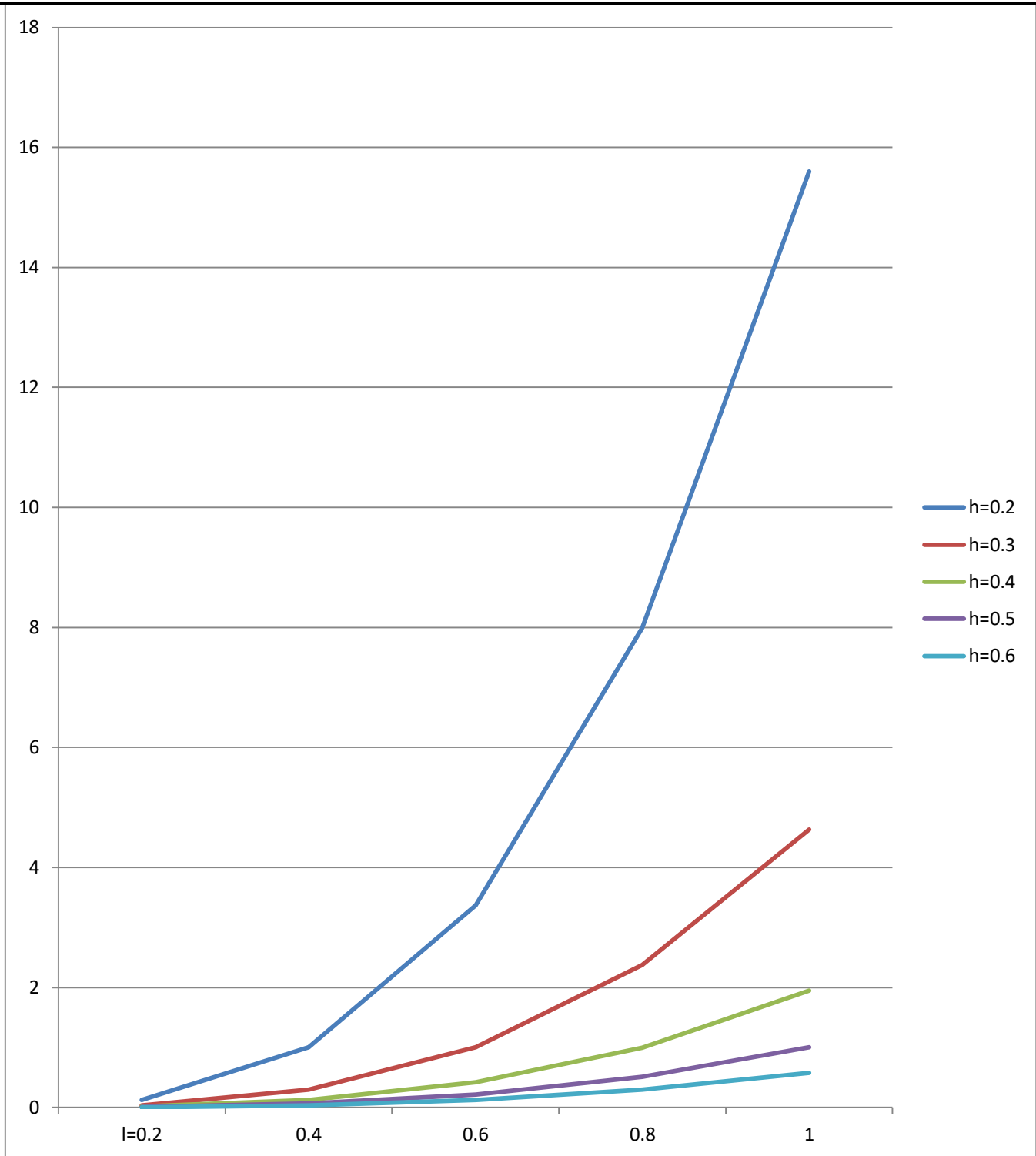


Variation of axial pressure with flow behavior for different value of h & x

Variation of Load carrying capacity w for different value of h, L

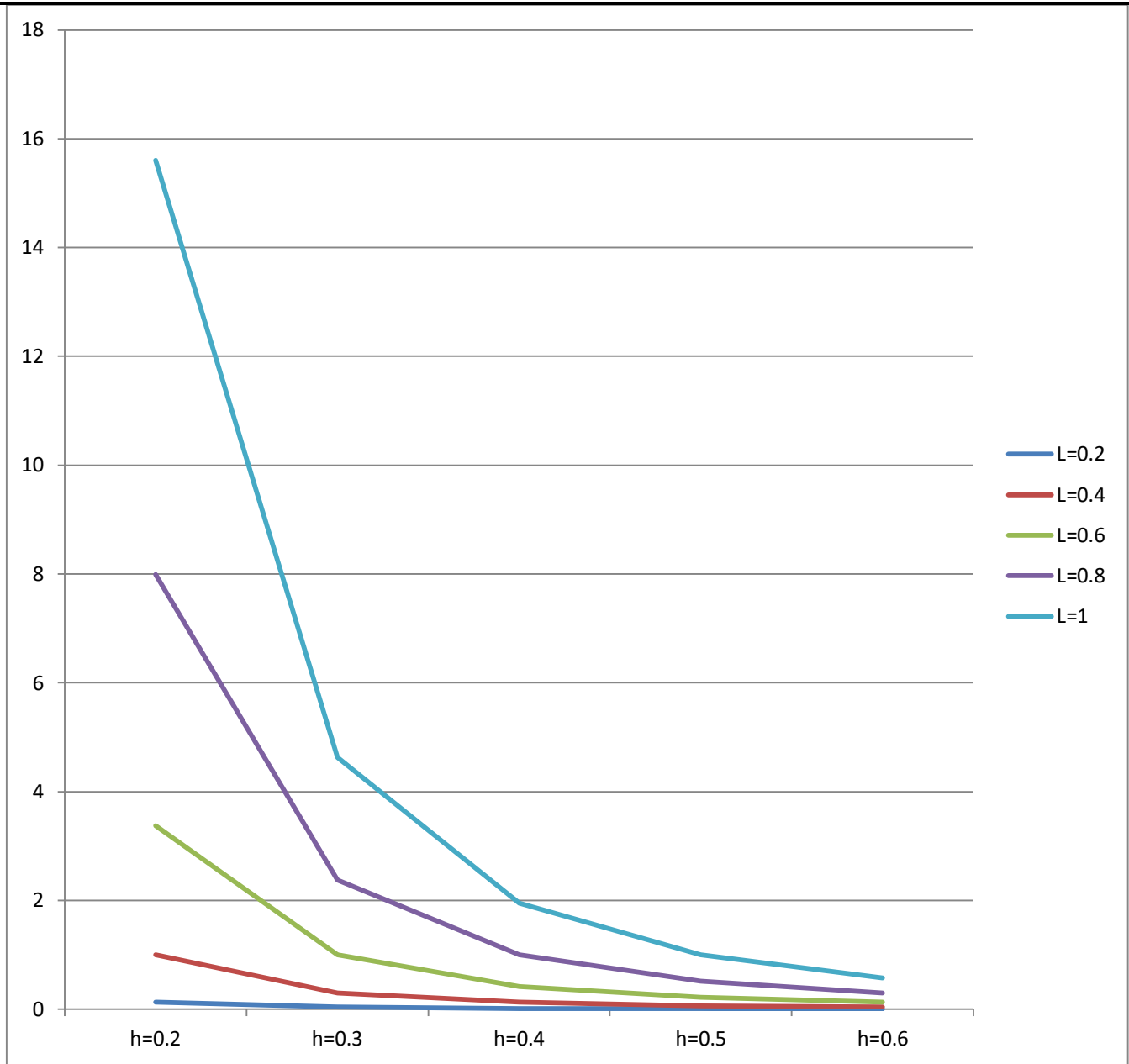
But $\mu = .5$ & $v = .5$

$h \backslash L$	0.2	0.3	0.4	0.5	0.6
0.2	0.125000	0.037032	0.015600	0.00800	0.0046290
0.4	1.000000	0.296256	0.124800	0.06400	0.0370368
0.6	3.369600	0.999864	0.421200	0.21600	0.1249900
0.8	7.987200	2.370048	0.998400	0.51200	0.2962944
1.0	15.600000	4.629000	1.950000	1.00000	0.5787000



Variation of Load Carrying Capacity w for different value of h & L , at

$$\mu = .5, v = .5$$



Variation of Load Carrying Capacity w for different value of h &L , at $\mu = .5, \nu = .5$

Result and Conclusion:

The research paper proposes a more re-elastic model for explaining the lubrication mechanism occurring a normally pressure and load capacity. The result for axial pressure and load bearing capacity has been examined for different values of h , x and L .

We observe that the axial pressure increases with the increase the value of x and decreases with the increase of value h .

We have also observed that the load carrying capacity increases as with the increase of the value of film thickness. Again we observed that the load carrying capacity depends on the flow behaviour and the gap between the approaching surfaces.

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