



## Introduction to V-Trees Graph Drawing of binary trees using Fractals

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**Abstract:** This paper introduces a new invention which is called as the V-Tree by the author. As the name implies it is named after the geometrical shape of the English alphabet 'V'. Given the great success of recursive nature of H-Trees, it was inspired to invent another Tree which we call it a V-Tree. The basic problem with H-Tree and C-Tree is they have no hierarchical basis to visualize the binary tree. The main objective of this work is to design a set of v-Tree with suitable grammar in L-systems which visually implies the hierarchy. H-Trees which are used for clock pulse distribution in VLSI design, fills a space which is bound by a rectangle. We demonstrate the flexibility of V-Trees which can be bound in a rectangle as well as in a trapezoid which is an advantage in VLSI design in a trapezoidal circuit board. Similarly, these Trees can be visualized as a layout for graph drawing of binary trees with trapezoidal boundaries.

**Index Terms - H-Trees, L-systems, Clock pulse distribution, VLSI design, graph drawing of binary trees**

### I. INTRODUCTION

The Franco-American mathematician Benoît Mandelbrot gave in 1975 the name of "fractals"[1]. The geometry he developed, based on self similarity concept, is better suited to describe nature, complex and irregular, than Euclidian Geometry. As he wrote, "clouds are not spheres, mountains are not cones, coasts are not circles, and lightning does not move in a straight line".

HUNGARIAN biologist, Linden Mayer (1925-1989) first studied the growth of baking yeasts, mushrooms and seaweeds at cell level. In the research center of the university Utrecht, he did teamwork specialized in computer science. Thus the team developed a definite grammar (L) - Systems to represent the growth of plants, but only at cell level. It is only later that idea came to him to model vegetables on large scale as a mathematical tool.

L-System or Linden Mayer System is a definite grammar, which consists of modelling the process of development and proliferation of plants or microbes where the growth is a parallel process.

In their book, 'The Algorithmic Beauty of Plants', which was co-authored by Przemyslaw Prusinkiewicz, they precisely explain the growth of vegetables becoming a big tree. It is observed that the notions of biology and botany enrich L-system models of plants and allow to generate very nice pictures.

L-system is a form of generative grammar. This grammar was graphically implemented which leads to spectacular pictures. A systematic study of a certain formulation was undertaken by Przemyslaw Prusinkiewicz in 1980s.[2][3]

At first, Linden Mayer conceives the formalization as a model of definite languages which allows to represent the development of simple multicellular organisms. In this approach he works on baking yeasts, mushrooms and seaweeds. But under the influence of the theoreticians and the practitioners of computer science, this system drove to families of definite languages and also to methods to generate not only very complex idealized plants graphically but also landscape pictures artificially.

L-system is a group of rules and symbols. The central concept of L-systems is the notion of rewriting. Rewriting is a technology to construct complex objects by replacing parties of a simple initial object by using rules of rewriting.

In biological interpretation, cells are modelled with the aid of symbols. In every generation, cells become divided, what is modelled by operation consisting in replacing a symbol by one or several other successive symbols.

The acquired character string has a graphic interpretation, in two or three dimensions. In two dimensions, they imagine that a hand holds a pencil which moves on the leaf according to instructions: 'mounting of a notch, then turns 20° to the left, move twice notch, memorize your position and still advance of a notch, get up then rely on memorized position' and so on... They introduce therefore variable symbols  $\in V$ , or constant  $\in S$ , to allow to guide the hand. Some of them were normalized, they are part of what they call 'Turtle interpretation'. This name comes from the 'turtle' of the programming language Logo which works on the same principle.[4][5]

## 2. Geometric Interpretation Of L-Systems

F- move forward by a fixed step of length  $d$  and draw a line from the old to the new position

f- move forward (as F) but do not draw the line

+ turn left (counterclockwise) by a fixed angle

$\delta$  – turn right (clockwise) by the angle  $\delta$

Alphabet:

The alphabet is a finite set  $V$  of formal symbols, usually taken to be letters  $a, b, c, \dots$ , or possibly some other characters[3].

Axiom:

The axiom (also called the initiator) is a string of symbols from  $V$ . The set of strings (also called words) from  $V$  is denoted  $W$ . Given  $w$ , some examples of words are  $aabca, caab, b, bbc, \dots$ . The length  $|w|$  of a word  $w$  is the number of symbols in the word.

Productions:

A production (or rewriting rule) is a mapping of a symbol to a word. This will be labeled and written with notation:

## 3. TYPES OF L-SYSTEM

The global process for the Fibonacci system works because this L- system is context-free meaning that the production rules take account only of an individual symbol, and not of what their neighbors are[3]. It is possible to consider context-sensitive L- systems where the production rules apply to a particular symbol only if the symbol has certain neighbors.

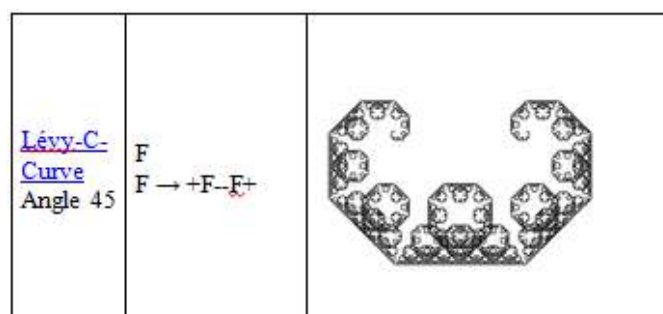
We have not stipulated yet that for each symbol in the alphabet there is exactly one production, although this has been true for our first few examples. If there is indeed exactly one production for each symbol, then the L- system is called deterministic and the sequence of generations is uniquely defined as a sequence of elements of  $W$ . If there is more than one production for a given symbol, say  $a$  and  $b$ , we need a criterion for deciding when to apply which rule. One possibility is to use one of the possible productions with certain probabilities. This is called a stochastic L- system (the word stochastic always connotes an element of randomness). In this section, we will consider only deterministic L- systems. A deterministic context-free L- system is popularly called a D0 L- system. [7][8]

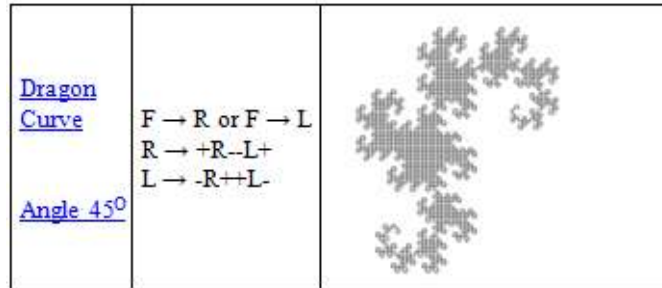
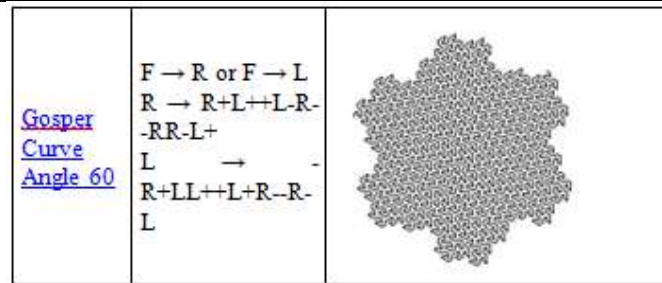
The study of generation of formal subsets of  $W$  by means of processes similar to that underlying L- systems originated in formal language theory, advanced by Chomsky as a mathematical way for discussing the formation and evolution of natural languages. For this reason, any subset  $S$  of  $W$  is called a language. L- systems languages are examples of a much broader class known in theoretical computer science as recursively enumerable languages. Some novel aspects of L- systems are the parallel nature of the evolution of one word from the next and the dynamic nature in that we can think of the language as growing over time.

The Fibonacci global process could be considered a simple example of what is known as emergent behavior in dynamical systems theory, behavior that is apparent as the system evolves. We will see this sort of phenomenon at various times in the course. This particular global process is a chain or concatenation law, in that several past generations are chained together to form the next generation. In general, we say the L- system satisfies a chain law (or is locally catenative) if there is a sequence of integers

If this holds for one  $n$ , the same argument as that underlying the growth process for the Fibonacci system proves that it holds for all subsequent generations. This leads to a very interesting open problem:

Fractal is a (by mathematician Benoît Mandelbrot in 1975) coined term (Latin fractus, broken, frangere from Latin, (zer in pieces -), break'), referred to the specific natural or artificial structures or geometric patterns. These structures or patterns generally have no integer Hausdorff dimension. This is the case for example, when an object copies of itself several times. Geometric objects of this type vary in material respects from ordinary smooth figures to more complex landscape.





The term fractal can be used both as a noun and an adjective. The field of mathematics, are examined in the fractals and their regularities, is fractal geometry and extends into several other realms, such as function theory, computability theory and dynamical systems. As the name suggests, the classical concept of Euclidean geometry is extended, which is also reflected in the broken and not natural dimensions of many fractals. Besides Mandelbrot include Waław Sierpiński and Gaston Julia at the eponymous mathematicians.

In traditional geometry, a line, an area and a cube is one-dimensional, two-dimensional and three-dimensional spatial structure respectively. But fractal sets can not be the dimensionality specified directly:- If we introduce, for example, an arithmetic operation on a fractal line pattern thousands of times continuously, so filled with time the whole drawing area approaches with lines (around the computer screen), and the one-dimensional structure is a two-dimensional.

Mandelbrot used the term of the generalized dimension to Hausdorff and found that fractal structures usually have a non-integer dimension. It is also called fractal dimension. Therefore, he introduced the following definition:

A fractal is a set whose Hausdorff dimension is greater than its LEBESGUE overlap dimension.

Lots of non-integer dimension is therefore fractals. The converse is not true, fractals may also have integer dimension, for example, the Peano Tree.

The self-similarity need not be perfect, as the successful application of methods of fractal geometry shows natural features such as trees, clouds, coastlines, etc.. The objects referred to are in greater or lesser degree self-similar structure (a tree branch looks vaguely like a scaled-tree), however, the similarity is not strict, but stochastically. Unlike forms of Euclidean geometry, which are at a magnification often flat and therefore easier (about one loop), can appear in fractals increasingly complex and new details.

Examples of fractals in three-dimensional space are the Menger sponge and the Sierpinski pyramid based on the tetrahedron (as the Sierpinski triangle on the equilateral triangle based). Correspondingly also in higher dimensions fractals make by Sierpinski - for example based on the Pentachoron in four-dimensional space.

#### 4.applications

Through its variety of forms and the aesthetic appeal associated play in digital art a role and there have the genre of fractal spawned. They are further used in the computer-aided simulation form rich structures such as realistic landscapes. To receive the wireless technology different frequency ranges, fractal us

Fractal forms can also be found in nature. However, the number of levels of self-similar structures is limited and is often only three to five. Typical examples from biology, the fractal structures in green cauliflower breeding Romanesco and ferns. Even the cauliflower has a fractal structure, where you often do not regard this carbon at first glance. But there are always some cauliflower heads, see the Romanesco fractal in structure very similar.

Widely used are fractal structures without strict but with statistical self-similarity. These include, for example, trees, blood vessels, river systems and coastlines. In the case of the coastline arises as a consequence of the impossibility of determining the exact length of coastline: The more precisely you measure the intricacies of the coastal path, the greater the length obtained. In the case of a mathematical fractal, such as the Koch Tree, it would be unlimited.

Fractals are also found as explanatory models for chemical reactions. Systems such as oscillators (Standard Example Belousov-Zhabotinsky reaction) can be on the one hand to use as a basic diagram, but then explain as fractals. Likewise, we find fractal structures in crystal growth and in the formation of mixtures, eg. As when one drop of color solution in a glass of water.e

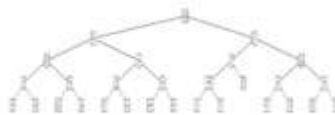
Fractals can be created in many different ways, but all methods include a recursive procedure:

The iteration of functions is the easiest and most popular way to generate fractals; The Mandelbrot set is produced that way. A special form of this method are IFS fractals (Iterated Function Systems) in which several functions are combined. This makes it possible to create natural formations.

### 5. Euclidean geometry

In Euclidean geometry, the main difficulty to represent hierarchies is to determine the best placement of the entities in the tree. For this, many algorithms exist [Herman et al., 2000]. Most of these algorithms are designed to highlight the hierarchical structure of the information space and / or optimize board space.

The diagrams below correspond to the results obtained with some tree drawing algorithms.



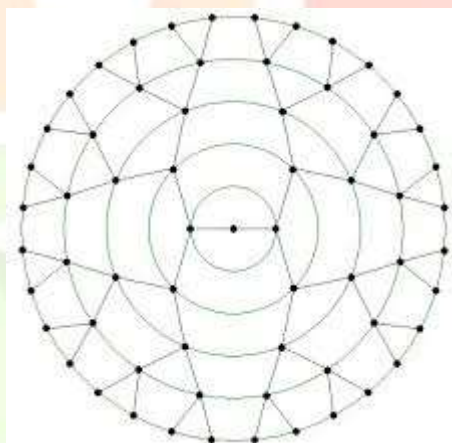
Also as part of Euclidean geometry, techniques map the entities of the hierarchy in a space of three dimensions rather than two. The best known of these systems is that the cones of trees [10].

Hyperbolic geometry is based on the negation of Euclid's fifth postulate: by one point, he spent several non-intersecting parallel lines. The Poincaré model is an application of this geometry to a disc. Thus, we can define hyperbolic geometry as follows: the plan is still within strict a disc whose boundary is called horizon. The lines are arcs of circles orthogonal to the boundary of the disk.

Hyperbolic geometry properties have meant that the visual distance between two points tends to zero as they approach the boundary of the disk. Thus, the map gives the impression that the visual structures are magnified in the center. Conversely, they are more distant from the more they are reduced center.

In the case of a tree, nodes are distributed radially and the user has the impression that the size of the knots and the distance between each node are inversely proportional to their distance from the center of the disc. Thus, nodes are always visible if available, and users simply drag the center those he wants to see more detail.

Here's an example image below of a binary tree with a hyperbolic geometry. This technique is known as radial (or circular) drawing of binary tree



Hyperbolic trees are recommended to search for information in a tree [14]. Indeed, they have the ability to help users "guess and anticipate" the sub-elements of each element.

If at first the high interactivity of hyperbolic trees seduced, it suffers from several flaws that may limit its actual use. Due to the effects of the deformation, the tags associated with the nodes are not aligned and sometimes overlap. But it is mainly its use problematic. Indeed, during handling of the structure, the elements on the border of the viewing space find themselves thrown so "unpredictable". These effects tend to disrupt the user who continually seeks to restore the situation generates a greater cognitive effort and a delicate grip. These projection effects are due to the geometry used. Indeed, the elements are represented in a hyperbolic plane that is not common to our senses. Therefore, the result of the transformations applied to the plan is not foreseeable "naturally".

Later work has extended mapping techniques with a geometry to move from one card within two to three dimensions as in the following example.

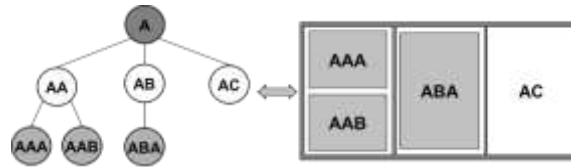
Representations by paving of tree representation of the paving techniques (sometimes also called surface) are the source of techniques developed to allow viewing of large trees while avoiding occlusion problems (as is sometimes the case with representations type of link-node). The principle of these techniques is then to take the maximum board space by superimposed visual surface structures.

The oldest and best known of these techniques is that of Tree-Maps.

### 6. Tree-Maps and its variants

Tree-Maps are from the early work that offered an approach paving[11][12][13]. Their principle is to cut rectangular area in the map space in proportion to each sub-tree. Visible rectangles correspond to leaves of the tree. Each node of the tree is represented by a rectangle enclosing all of its sub-elements. Thus each element graphically subsumes its sub-elements.

Here is a diagram of one tree represented in technical link-node (left) and technical Tree-Maps (right):



With performances by paving, the structure is not explicitly shown and the user's attention is focused on the leaves of the tree (AAA, AAB and AC in our example).

This feature is most evident with a screen shot of the first embodiment of the Tree-Maps [Johnson & Shneiderman 1991] Front view, the map is a two dimensional projection and a similar map is obtained in "Cushion Tree maps" but with the rectangles (projected face roller) placed perpendicularly relative to each other.

The Internet metasearch engine Grokker offers a mapping which uses the same principle as the Tree-Maps but with circular rather than rectangular surfaces. This system is used to represent a hierarchy of 'clusters' websites during a search.

### EXPERIMENTAL RESULTS

Now, We show the screen output as follows:

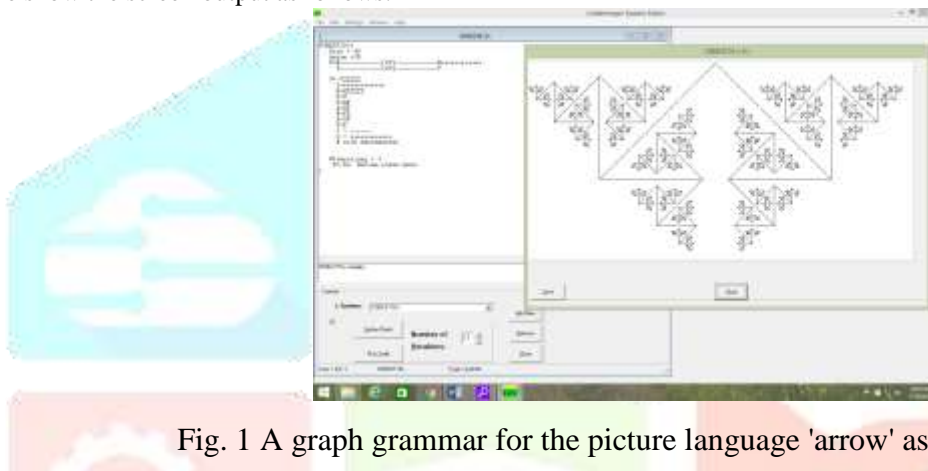


Fig. 1 A graph grammar for the picture language 'arrow' as binary tree



Fig.2: V-TREE WITH POLYGONAL BOUNDARY with angular rotations

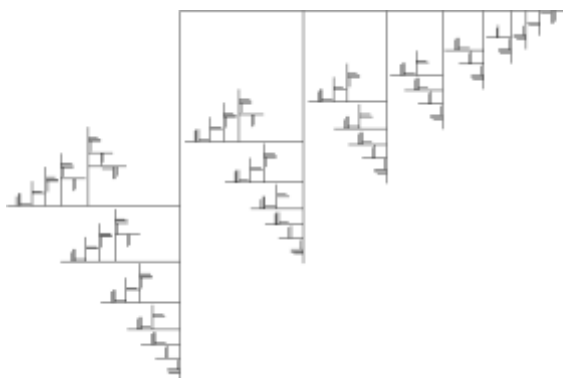


Fig. 2 This Tree has a orthogonal grid drawing of a binary tree

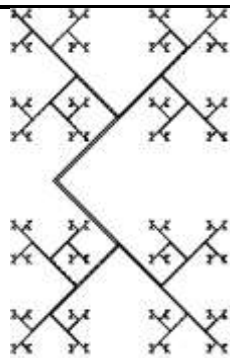


Fig. 3 This Tree is also an orthogonal grid drawing which has rectangular boundaries

### 7 Conclusion

We have introduced a new Tree which we call it a V-Tree. We have given the context free grammar for the same. We have demonstrated some variations with Treed boundaries. These Trees may be treated as a generalized approach for graph drawing of binary trees with implied hierarchy.

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