SOME CHARACTERISTICS OF DOMINATION SET IN HAMILTON CYCLES

[1]T.Renuga, Assistant professor, Department of Mathematics

[1]Adhiyaman Arts and science college for women, Srinivasa nagar, Uthangarai, Krishnagiri, Tamil Nadu.

Abstract : In this paper is the work based on the domination on Hamiltonian cycles in a graph. We discussed the nature and impact of domination on Hamiltonian graphs. The work is planned to study the Hamiltonian graphs and domination sets on any graph and type of dominations. We discussed the independent domination on a graph. Then using the properties of hamiltonicity of graphs and domination, we tried to find the domination sets in Hamiltonian graph.

I. INTRODUCTION

We have chosen to present mathematical topics from the field of graph theory. Graph theory to bring a previously unfamiliar scientist to the frontiers of research rather quickly. Graph theory has involved as a collection of seemingly disparate topics.Interest in a graphs and their application continues to grow rapidly, largely due to the usefulness of graphs as a models for the computation and optimization .In the last three decades, graph theory has established itself as a worthwhile mathematical discipline and there are many application of graph theory to a wide variety of subjects which include Operations Research, Physics, Chemistry, Economics, Genetics, sociology, linguistics, Engineering, computer Science, etc…

Graph theory also has been independently discovered many items through some puzzles that arose from the physical world, consideration of chemical isomers, electrical, network, etc..

Domination in graphs is the most fascinating area for many researchers in graph theory all over the world. The reason for this attraction is not very difficult to fathom. The concept of domination can be applied to form various Mathematical models for practical problems. Several areas like communication network, decision making process, administration, social network theory chemical bond structure etc. used in domination theory. Research in domination theory centers around two major ideas.

(i) New types of domination to suit the needs.

(ii) New types of dominating sets to meet the demands of the practical situation.

When different types of domination are introduced and parameters are studied, a natural question that arises in one's mind is "Is there a way to unify them with respect to a particular condition?". The second major problem that plays a vital role in any branch of Mathematics is the problem of representation.

II. Preliminaries

Hamilton cycle:

Hamilton cycle in the graph G is a cycle that passes through each vertex exactly once.

Hamilton walk:

Hamilton walk in a graph G is a walk that passes through each vertex exactly once.

Hamiltonian graph:

If a graph has a Hamiltonian cycle it is called Hamiltonian graph.

Semi Hamiltonian graph:

If a graph has a walk it is called semi Hamiltonian graph.

PROPERTIES OF HAMILTONIAN:

1. The graph G (2n + 2, 3n + 3) for n ≥ 1, which is regular of degree three, non-bipartite and planner, is always Hamiltonian.

2. The graph G(3m + 6, 12 + 6m) for m ≥ 1, which is regular of degree four, non-bipartite and planner, has two edge-disjoint Hamiltonian cycles.

3. If one vertex is added outer side the region of the graph H(3m + 6, 6m + 12) for m ≥ 1, making the degree of added vertex of degree four, the new graph H(3m + 7, 6m + 16) for m ≥ 1, which is planar, non-regular, non-bipartite but always Hamiltonian graph.

4. Intersection graph obtained from Euler diagram is not Hamiltonian.

5. The graph structure G (3m + 7, 6m + 14) for m ≥ 1, which is regular of degree four, non-bipartite and planner, has two- equal path partitions.

6. The graph H(3m + 7, 6m + 14) for m ≥ 1, which is planner, regular of degree four, non-bipartite but Hamiltonian graph , has perfect matching 4 with non- repeated edge for simultaneous changes of m= 2n+1 for n≥0.

7. Let 'G' be a complete graph having n ≥ 3 vertices then L(G) is Complete Hamiltonian.

Let G be a graph in which every vertex has odd degree. Then G contains an even no. of Hamilton Cycles through a fixed edge of G.

8.The total no of directed Hamilton in cycles for all simple graph of order n=1,2,3.Thereexists a value of r such that if G is an r-regular Hamilton graph of n vertices,then G contains atleasttwo Hamiltonian cycles.

9. Let G=(V,E) be a simple a graph where |V|=n≥3,if for every two vertices u,v ∈ V, $(u,w) \in E \Rightarrow \deg(u) + \deg(w) \geq n$.

10. If G =(V,E) is a simple graph having n vertices and for every v ∈ V we have deg(v) ≥n/2 then G is Hamiltonian graph.

Domination:

In graph theory, a dominating for a graph G=(V,E) is a subset D of V such that every vertex not in D is adjacent to atleast one member of D. The domination number γ(G) is the number of vertices in a smallest dominating set of G.Or

A subset D ⊂ V is called a dominating set of a graph G if for every v ∈ V, either v ∈ D or v is adjacent to a vertex in D, that is, N [D]= V. The minimum cardinality of a dominating set in G is the domination number of G.

Independent domination:

An independent set in G is a set of pairwise nonadjacent vertices, and the independence number of G, P(G) is the maximum cardinality of an independent set in G. A dominating set which is also an independent set is called an independent dominating set.

III. DOMINATION IN HAMILTONIAN GRAPHS

This chapter provides a summary of results that have previously been obtained for domination in Hamiltonian graphs, and in particular, those results that motivated the new results for independent domination in Hamiltonian graphs and total domination in Hamiltonian graphs. Section 4.1 contains general results on domination in graphs and section 4.2 contains the results on domination in Hamiltonian graphs.

DOMINATION IN GRAPHS

Theorem

If G is a connected 3-γ -critical graph, then G as a dominating cycle.

Proof:

Since a connected 3-γ - critical graph G contains a dominating path, it must contain a longest such path by show in the longest dominating path in G is actually a Hamiltonian path, it shows G as a Hamiltonian cycle.

Theorem

Let G be a graph on n vertices. If $d(x)+d(\mathrm{y})\geq \mathrm{n}$ for every pair of non adjacent vertices x and y, then G is Hamiltonian .

Proof :

Bondy and chvatal generalize ore's theorem by defining the closure of a graph G to be the graph cl (G) obtained from G by recursively joining non adjacent vertices whose degrees sum to at least n. they proved that a graph G is Hamiltonian if and only if its closure is Hamiltonian .

A new closure concept the could be useful in the study of Hamiltonian properties of 3-γ - critical graph. It involves adding an edge uv to G when ever $\{\mathrm{u,v}\} > v-w$ for some $d(w)\geq 3$. The obtained graph is denoted by D*(G). it follows the result.

Theorem

If G is a connected 3-γ - critical graph then G as a dominating cycle.

Theorem

If G is a 3-γ - critical graph on more then six vertices, then G as a Hamiltonian path .

Theorem

The independence number β of a 3- γ - critical graph with minimum degree $\delta\geq 2$ satisfies $\beta\leq\delta+2$ .more over if $\beta=\delta+2$ then every maximum independent set contains every vertex of degree δ.

Lemma

Let W be an independent set of $\mathrm{k}\geq 3$ vertices of a 3-γ - critical graph G such that W $\cup\{\mathrm{x}\}$ is independent for sum $\mathrm{x}\notin W$.

Lemma

$$If\, u_i \in c_i\, and\, u_j \in c_j\, are\, two vertices\, with i \neq j then u_i u_j \notin E(G)$$

Lemma

Let S be a set of vertices of C, and let A be the set of vertices of C which are adjacent on C to a vertex of S. If S is a C-independent set, and every vertex of A is adjacent in G−E(C) to some vertex of S, then G has at least one more Hamiltonian cycle. In particular, if S is a C-independent C-dominating set in G, then G contains at least two Hamiltonian cycles.

Proof

The second statement follows immediately from the first. To prove the first statement, we remove edges from G − E(C) until every vertex of A has degree three, and is adjacent to exactly one vertex of S.
Let this graph be G′. Take a Hamiltonian path P ⊂ C which begins with a vertex s ∈ S and ends (by default) in a vertex of A. In the Hamiltonian path graph H, defined on the Hamiltonian paths in G′ beginning with s.

Then every Hamiltonian path has degree at most two, since Hamiltonian paths in G′ beginning with s end in a vertex of A, which has degree three in G′. Now P has degree one, since P ⊂ C.

It follows that there is a Hamiltonian path P′/⊂ C of degree one in H, which means that P′ is contained in a Hamiltonian cycle in G, which is distinct from C.
Therefore to prove that a Hamiltonian graph G has at least two Hamiltonian cycles, we can find C-independent C-dominating set in G.

We remark that this is only a sufficient condition, not a necessary one. In addition, this is exactly the situation with bipartite graphs in Theorem 4.1.2: any one of the parts is a C-independent C-dominating set. Using this condition, we prove our main theorem.

Theorem

Let G be a Hamiltonian graph of order n such that $\delta(G)\ge 6$. Then $\gamma(\mathrm{G})\le\frac{6n}{17}$.

Proof:

Let V(G)={1,2,……,n} and, without loss of generality, assume C=1,2,…,n,1 is a Hamiltonian cycle of G. if $n\le 16$, then by lemma 3, $\gamma(\mathrm{G})\le\frac{n}{3}\le\frac{6n}{17}$. thus ,$n\ge 17$. Now let $k\ge 6$ and consider the following cases.

Case 1.n=3k-1.

Then D={2,5,….3k-1} is a DS set of G such that |D|=K= $\frac{n+1}{3}$. Since n ≥ 17. It follows that $\gamma(\mathrm{G})\le\frac{n+1}{3}\le\frac{6n}{17}$.

Case 2. n=3k.

Then D={2,5,….3k-1} is a DS set of G such that |D|=K=$\frac{n}{3}$. It follows that $\gamma(\mathrm{G})\le\frac{n}{3}\le\frac{6n}{17}$.

Case 3. n=3k+1.

If $k\le 8$, then $n\le 25$, and by lemma 4, $\gamma(\mathrm{G})\le k\le\frac{6n}{17}$.suppose $k\ge 11$.

Then $n\ge 34$ and D={2,5,….3k-1,3k+1} is a DS of G such that |D|=k+1=$\frac{n+2}{3}$. since n ≥34, it follows that $\gamma(\mathrm{G})\le\frac{n+2}{3}\le\frac{6n}{17}$.

Hence, we only need to verify that if G has order n=28, n=31, respectively, then G has a DS of cardinality 9,10respectively.

Since the proofs are similar, we consider only n=31. The proof is by contradiction ,that is we assume $\gamma(\mathrm{G})\ge 11$.

Since $\delta(G)\ge 6$,each vertex of G is incident with at least four chords of C. we choose a lasso L of G of order 31,obtainable from C , such that the number of vertices comprising the body of L is maximum.

That is, L is a spanning sub graph of the union of C and a chord of C

Let $v \in V(G)$ .suppose without loss of generality, that 1 v is a chord of C such that 1,v,v-1,…1 is the body of L. note that 1 is adjacent to both v and 31. We consider possible values of v. if 1 is adjacent to3i for some $1 \le i \le 10$, then by lemma 1, G can be dominated by 10 vertices , which is a contradiction. Thus we may assume that 1 is not adjacent to 3i for all i. by similar reasoning ,31 is not adjacent to 3i-1 for all i. since the body of L is a maximum, and by re-labeling if necessary, we have that $v \ge 17$. Since 1 is not adjacent to 3i for all I, we have v $\in$ {17,19,20,22,23,25,26,28,29}.

Before proceeding further, we bound the adjacencies of vertices 31 and 30. Suppose (b,c respectively) is adjacent to (30, 31 respectively). Then we obtain lassos L_1 and L_2 (L'_1 and L'_2, respectively) with cycle lengths b+1 and 32-b (c+2 and 31-c respectively). Thus b+1≤v and 32-b≤v(c+2≤v and 31-c≤v, respectively), and so 32-v≤bv-1(31-v≤c≤v-2, respectively).

Case 3.1.v=17.

Then 31 is possibly adjacent to vertices in {32-17,…,17-1,1,30}={15,16,1,30}, contradicting the fact the deg(v)≥6.

Case 3.2.v∈{19,22,25,28}.

Since 31-v≤c≤v-2, 30 is adjacent to some vertex on the cycle 1,v,v-1,….2,1, as v≡1 and 3, lemma 2 implies that G can be dominated by 10 vertices, a contradiction.

Case 3.3.v=20.

Again we check the possible adjacencies of 31. By reasoning like in case 3.1, we have that 31 is adjacent to 1,30 and possibly 12,13,….19. recall that 31 is not adjacent to 3i-1 for all 1≤i≤10. Thus 31 is not adjacent to 14 or 17. Since deg(31)≥6,31 must be adjacent to at least one of the vertices 12,15 or 18. Then D={3,6,9,1215,18,20,23,26,29} is a DS of G of cardinality 10, a contradiction.

Case 3.4. v=23.

Initially , 31 is adjacent to 1,30, and possibly vertices in {9,10,…21,22}. Let D={3,6,9,1215,18,21,23,26,29}. Then D dominates G if 31 is adjacent to3i for some 1≤i≤7. Hence , we eliminate these possibilities and also vertices of the form 3i-1. We now have that 31 is possibly adjacent to vertices in {10,13,16,19,22}. Since deg(31)≥6,31 must be adjacent to either 19 or 22.

Now consider the adjacencies of 30. Initially, 30 is adjacent to 29,31 and possibly vertices in{8,9,…..,20,21}. Let D'={1,3,6,9,12,15,18,21,25,28}. Then D' dominates G if 30 is adjacent to 3i for some 1≤i≤7.

Hence , we eliminate these possibilities and also the vertices of the form 3i-2. We now have that 30 is possibly adjacent to the vertices in{8,11,14,17,20}. Since deg(30)≥6,30 must be adjacent to either 8 or 11. Then D"={2,5,8,11,14,17,19,22,25,28} is a DS of G of cardinality 10, a contradiction.

Case 3.5.v=26.

Initially , 31 is adjacent 1,30, and possibly vertices in {6,7,…,25}. Let D={3,6,9,12,15,18,21,24,26,29}. Then D dominates G if 31 is adjacent to 3i for some 1≤i≤8.

Hence , we eliminate these possibilities and also vertices of the form 3i-1. We now have that 31 is possibly adjacent to vertices in {7,10,13,16,19,22,25}. Since deg(31)≥6,31 must be adjacent to at least one of the vertices in {16,19,22,25}.

Now consider the adjacencies of 30. Initially, 30 is adjacent to 29,31 and possibly vertices in{5,6,….,24}. Let D'={1,3,6,9,12,15,18,21,24,28}. Then D' dominates G if 30 is adjacent to 3i for some 1≤i≤8.

Hence , we eliminate these possibilities and also the vertices of the form 3i-2. We now have that 30 is possibly adjacent to the vertices in{5,8,11,14,17,20,23}. Since deg(30)$\geq$6,30 must be adjacent to at least one of the vertices in{5,8,11,14}.

Then D"={2,5,8,11,14,16,19,22,25,28} is a DS of G of cardinality 10, a contradiction.

Case 3.6.v=29

Initially , 31 is adjacent 1,30, and possibly vertices in {3,4,…,28}. Let D={3,6,9,12,15,18,21,24,27,29}. Then D dominates G if 31 is adjacent to 3i for some 1$\leq$i$\leq$9.Hence , we eliminate these possibilities and also vertices of the form 3i-1. We now have that 31 is possibly adjacent to vertices in {4,7,10,13,16,19,22,25,28}. Since deg(31)$\geq$6,31 must be adjacent to at least one of the vertices in {16,19,22,25}.

Now consider the adjacencies of 30. Initially, 30 is adjacent to 29,31 and possibly vertices in{2,3,….,27}. Let D'={1,3,6,9,12,15,18,21,25,28}. Then D' dominates G if 30 is adjacent to 3i for some 1$\leq$i$\leq$9. Hence , we eliminate these possibilities and also the vertices of the form 3i-2. We now have that 30 is possibly adjacent to the vertices in{2,5,8,11,14,17,20,23,26}.

Since deg(30)$\geq$6,30 must be adjacent to at least one of the vertices in{5,8,11,14}. Then D"={2,5,8,11,14,16,19,22,25,28} is a DS of G of cardinality 10, a contradiction.

Suppose 31 is adjacent to one of the vertices in {19,22,25,28}. Let D"={2,5,8,11,14,17,19,22,25,28}. Then D" dominates G if 30 is adjacent to 3i-1 for some 1$\leq$i$\leq$6. Hence , we eliminate these possibilities. If follows that 30 is adjacent to 29,31 and possibly to vertices in {20,23,26}, which implies that deg(30)$\leq$5, a contradiction. We conclude that 31 is not adjacent to any of the vertices in {19,22,25,28}.

Suppose 31 is adjacent to 4. Then 2 must be adjacent to some vertex on the cycle 31,4,5,…30,31 of length 28. By lemma 2,G can be dominated by 10 vertices , which is a contradiction. Suppose 31 is adjacent to 7. Then 2 must be adjacent some vertex on the cycle 31,7,8,….30,31 of length 25.

By lemma 2, the vertices on the cycle and the vertices 1,2,3 can be dominated by a set composed of 9 vertices. Adding the vertex 5 to this set yields a DS set of G of cardinality 10, a contradiction.

Thus , 31 is adjacent to 1,30 and possibly to vertices in {10,13,16} which implies that deg(31)$\leq$5, a contradiction.

Corollary:

Let G be a Hamiltonian graph of order n such that $\delta(G) \geq k \geq 3$. Then $\gamma(\mathrm{G}) \leq \dfrac{kn}{3k-1}$

Theorem

Let G be a graph of order n such that $\delta(G) \geq k \geq 7$ then $\gamma(G) = \dfrac{k}{3k-1}$

Proof: Suppose $k \geq 7 and let \delta(G) \geq k$ .We must show that $1-\delta(G)\left(\frac{1}{\delta(G)+1}\right)^{(1+1/\delta(G))} \leq 1-k\left(\frac{1}{k+1}\right)^{1+1/k}$

$$f(x) = 1-x\left(\frac{1}{x+1}\right)^{1+1/x}$$

$$n\left(1-\delta(G)\left(\frac{1}{\delta(G)+1}\right)^{(1+1/\delta(G))}\right) \leq \frac{k}{3k-1}$$

It is suffices to shows that

$$1-\delta(G)\left(\frac{1}{\delta(G)+1}\right)^{(1+1/\delta(G))} \leq \frac{k}{3k-1}$$

Let $f(x)=1-x\left(\frac{1}{x+1}\right)^{1+1/x}$ for x≥7.Then $f^{1}(x)=-\frac{In(x+1)}{x}\left(\frac{1}{x+1}\right)^{1+1/x}<0$ for x>0. Hence x≥7, fis immediately decreasing. Since $\delta(G)\geq k\geq 7$, wehave $f(\delta(G))\leq f(k)$;that is

$$1-\delta(G)\left(\frac{1}{\delta(G)+1}\right)^{(1+1/\delta(G))}\leq 1-k\left(\frac{1}{k+1}\right)^{1+1/k}.$$

Now, let $H(x)=f(x)-\frac{1}{3}$ and notice that ,since f is monotonically decreasing ,H(x) is also. Then for x=8,we have $h(8)\leq H(8)=1-8\left(\frac{1}{9}\right)^{(1+1/8)}-\frac{1}{3}<0$ .Since h is monotonically increasing, it follows that $h(x)\leq h(8)\leq 0$ for $x\geq 8$,more over for x=7 we have $h(7)=1-7\left(\frac{1}{8}\right)^{(1+1/7)}-\frac{7}{3(7)-1}<0$ .Thus,$1-k\left(\frac{1}{k+1}\right)^{(1+1/k)}\leq\frac{k}{3k-1}$ for k≥7, and the result follows.

Theorem

Let G be the Hamiltonian graph ,suppose that P is a longest (x, y) path such that {x∩ {x, y}} is a small as possible and that for this path , $d(x_p)=k\geq 4$ .Let $z\in p_i$ .If

i) $[a_1,z]\to x_p\,([b_k,z])\to x_p$, and $z\notin p_{k-1,}$

ii) There exist a some vertex $a_i\in A$ with $i\neq j$ such that) $[a_1,z]\to x_p\,([b_k,z])\to x_p$, then there is an independent set I such that $x_p\in I$ and $|x|\geq k+1$

Proof:

If $x\notin X$ ($y\notin Y$ respectively) , then $|B|=k$ ($|A|=k$, respectively). Thus by lemma is an independent set is required. Hence we may assume $\{x,y\}\subseteq X$ which implies that for any longest (x,y) path p^{1}.

$\{x,y\}\subseteq N(x_{p^1})$

If $z\notin p$, then . By lemma there is some vertex $u\in p_{i-1}$ such that $b_{j+1}u,a_1u^{+}\in E(G)$ which is contradicts Then $z\notin b_{j+1}$. As sum there is some $a_i\in A$ such that $a_m z^{+}\in E(G)$ If $m\leq j$, then the (x, y) path $x\overline{P}x_m x_p x_{k-1}\overline{p}z^{+}a_m\overline{p}za_{k-1}py$ is Hamiltonian path, a contradiction. If m>j there

a some vertex $u\in p_m$ such that $zu,a_1u^{+}\in E(G)$ .Thus ,the (x ,y) path $xx_p x_m\overset{\mathbf{u}}{p}z^{+}a_m\overline{p}uz\overline{P}a_1u^{+}\overline{P}y$ is Hamiltonian also a contradiction. Therefore , A∪$\{z^{+},x_p\}$ is an independent set as required.

(1) We will consider the following two cases separately .

Case:1 j≥ i, In this case ,we will show A∪$\{z^{+},x_p\}$ an independent set. We will first show that z∉B. If z∈ B ,then $z=b_{j+1}$. That is $[a_i,b_{j+1}]\to x_p$. Thus by lemmas there is a vertex v∈ p_1 such that $a_1v,b_{j+1}v^{1}\in E(G)$, which contracts thus , z∉B and hence $z^{+}x_p\notin E(G)$.

By lemma and $z^{+}x_p\notin E(G)$ in order to prove that A∪$\{z^{+},x_p\}$ is an independent set but we need to show that for any $a_m\in A,a_m z^{+}\notin E(G)$. Suppose to the contrary that there is some $a_m\in A,$ such that $a_m z^{+}\in E(G)$.

If m=1 then by lemma, we have $b_2 z\notin E(G)$ which implies $a_i b_2\in E(G)$ and hence $xx_p x_m\overset{\mathbf{u}}{p}z^{+}a_m\overline{p}uz\overline{P}a_1u^{+}\overline{P}y$ is a Hamiltonian path connecting x and y ,a contradiction. Hence $a_1z\notin E(G)$

If 2≤m≤j, then since $a_1 z \in E(G)$, we can see that $xx_p x_m \overleftarrow{p} z^+ a_m \overline{p} uz \overline{P} a_1 u^+ \overline{P} y$ is a Hamiltonian path connecting x and y ,a contradiction.

Now Let $j < m \leq k-1$. If $b_2 z \in E(G)$ then $xx_p x_m \overleftarrow{p} z^+ a_m \overline{p} uz \overline{P} z^+ a_m \overline{P} y$ is a Hamiltonian path connecting x and y a contradiction.

Thus $b_2 z \in E(G)$ which implies that $b_2 a_i \in E(G)$. There exist a Vertex $v \in p_1$ such that $vz, v^+ a_i \in E(G)$.Thus,the (x,y)-path $xx_p x_m \overleftarrow{p} z^+ a_m \overline{p} uz \overline{P} a_1 u^+ \overline{P} y$ is a Hamiltonian path contraction

Case2: $j < i$: In this case, we will show $B \cup \{z^-, x_p\}$ is an independent set. Since $k \geq 4$, we have $z \notin A$ and hence $z^-, x_p \notin E(G)$. Since $z^- x_p \notin E(G)$, it follows from that to prove $B \cup \{z^-, x_p\}$ is an independent set it is enough to show that for every $b_m \in B$, $z^- b_m \notin E(G)$.Suppose to the contrary that there is some$b_m \in B$ such that $z^- b_m \in E(G)$.

We have $A - \{a_i\} \subseteq N(z)$. Thus, We have $z^- b_1 \notin E(G)$. For any $l \in \{2, \ldots, k, -1, k\}$ with $l \neq j+1, i+1$.This implies $m \in \{j+1, i+1\}$. in the following, we will show that $m \in \{j+1, i+1\}$ is also impossible.

If $m = j+1$, that is $z^- b_{j+1} \in E(G)$, then z is a B- vertex. Since $[a_i, z] \rightarrow x_p$ we have$B - \{b_{j+1}\} \subseteq N(a_i)$.

If $j < i-1$, then since $[a_i, z] \rightarrow x_p$, z is a vertex and a_i is a A- vertex, we have $za_{i-1}, a_i b_i \in E(G)$.This implies there is some vertex $v \in p_{i-1}$ such that $av, a_i v^+ \in E(G)$.

Thus ,$x\vec{p}z^- b_{j+1} \vec{p} zv \overleftarrow{p} x_{j+1} x_p x_i \overleftarrow{p}_v + ai^{\overline{p}y}$ is a Hamiltonian path connecting x and y, a contradiction.

Hence we have j=i-1. If j=i-1>1, then Q=$xx_p x_2 \vec{p} z^- b_{j+1} \vec{p}$ $za1pb2ai\overleftarrow{p}yisan$ (x,y)-path of length n-2 with $xQ = x_i$.

We have $xx_i \in E(G)$. Thus, the (x,y)-path $xx_i x_p \overrightarrow{Q} Y$ is Hamiltonian. If $j = i-1 = 1$, then$R = x\vec{p}z^{-1} b_{i+1} \overleftarrow{p} za_{k-1} \vec{p} b_{ka_i} \vec{p} x_{k-1} xpy$ is an (x,y)-path of length n-2 with $x_R = x_i$.

We have $x_i y \notin E(G)$. Thus, the (x,y)-path $x\vec{R}x_p x_i y$ is Hamiltonian, also a contradiction.

Now let m=i+1. If $j < i-1$,then , We have $a_{i-1} \notin E(G)$ which contradicts that $A - \{a_i\} \subseteq N(z)$.

Conclusion:

This dissertation is the work based on the domination on Hamiltonian cycles in a graph. We discussed the nature and impact of domination on Hamiltonian graphs.The work is planned to study the Hamiltonian graphs and domination sets on any graph and type of dominations.We discussed the independent domination on a graph. Then using the properties of hamiltonicity of graphs and domination, we tried to find the domination sets in Hamiltonian graph.This research work is framed with four chapters by taking the properties of Hamilton graphs and the properties of Domination in which the work step in to study the construction of domination sets on the Hamilton graph.

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