

# Application of $\{n (\sin \Pi x / \Pi x)\}$ Filter as Denoising filter

Susan John

## Abstract

In this paper,  $\{n (\sin \Pi x / \Pi x)\}$  filter of different percentages is investigated for noise removal. First the image is corrupted with noise of different standard deviation, and then sinc  $\{n (\sin \Pi x / \Pi x)\}$  filter of different percentages is applied on the corrupted image to suppress the noise.  $\{n (\sin \Pi x / \Pi x)\}$  filter find applications in satellite image data processing, color image processing, and multispectral biomedical image processing. In this paper, results are presented for the case of grey images. Experimental works confirm that that  $\{n (\sin \Pi x / \Pi x)\}$  filter achieves very good filtering results for various noises. Experiments conducted on remotely sensed satellite images shows, from careful visual analysis, that the  $\{n (\sin \Pi x / \Pi x)\}$  filters are able to remove noise. Here it is demonstrated that the  $\{n (\sin \Pi x / \Pi x)\}$  filter is both effective and efficient in a great variety of computer vision and computer graphics applications including noise reduction and detail enhancement.

**Keywords:** Filters, noise, PSNR,  $\{n (\sin \Pi x / \Pi x)\}$  and RMSE

## Introduction:

In digital image processing the input image is digital, i.e. the spatial coordinates and amplitude of 2D image function which is finite and discrete quantities. A digital image is composed of picture elements called pixels. While transmission and processing of images, these images are encountered by many types of noises, thus degrading the quality of images. Image filtering is one of biggest challenges in front of computer vision and computer graphics. Some examples of noise are: Gaussian or White, Rayleigh, Shot or Impulse, periodic, sinusoidal or coherent, uncorrelated, and granular. Only method to remove noise is by filtration either by using spatial domain filters or by using frequency domain filters. Filters provide an aid to visual interpretation of images. Filters change a pixel's value taking into account the values of neighboring pixels too.

Image Processing is divided into two categories, Frequency Domain & Spatial Domain. Image processing in Frequency Domain involves first transforming an image into Fourier transform, doing the processing there, and obtaining the inverse transform to bring the result back in spatial domain.[3] This is in contrast to Spatial Domain, which refers to image plane itself and the image processing methods are based on direct manipulation of pixels in an image. Generally, spatial processing techniques are more efficient computationally and require less processing resources to implement [1]. An image is denoted as a 2D function.

$\{n (\sin \Pi x / \Pi x)\}$  filter has adopted frequency domain filtering for the noise removal. Frequency domain filtering is much better than spatial domain filtering because filtering in the frequency domain is much faster especially for large images.

There are various metrics used for objective evaluation of an image. Some of them are mean squared error (MSE), root mean squared error (RMSE), mean absolute error (MAE) and peak signal to noise ratio (PSNR).

Let the original noise-free image, noisy image, and the filtered image be represented by  $f(x, y)$ ,  $g(x, y)$ , and  $\hat{f}(x, y)$ , respectively. Here,  $x$  and  $y$  represent the discrete spatial coordinates of the digital images. Let the images be of size  $M \times N$  pixels, i.e.  $x=1,2,3,\dots,M$ , and  $y=1,2,3,\dots,N$ . Then, MSE and RMSE are defined as:

$$\text{MSE} = \frac{\sum_x \sum_y [\hat{f}(x, y) - f(x, y)]^2}{M \times N}$$

$$\text{RMSE} = \sqrt{\text{MSE}}$$

The MAE is defined as:

$$\text{MAE} = \frac{\sum_x^M \sum_y^N |\hat{f}(x, y) - f(x, y)|}{M \times N}$$

The PSNR is defined in logarithmic scale, in dB. It is a ratio of peak signal power to noise power. Since the MSE represents the noise power and the peak signal power is unity in case of normalized image signal, the image metric PSNR is defined as:

$$\text{PSNR} = 10 \cdot \log_{10} \left( \frac{1}{\text{MSE}} \right) \text{dB}$$

Though these image metrics are extensively used for evaluating the quality of a restored (filtered) image and thereby the capability and efficiency of a filtering process. [1]

In this paper, we focus on the use of the  $\{n (\sin \Pi x / \Pi x)\}$  filter for removal of noise in the images corrupted with noise of different standard deviation. Experiments show that the  $\{n (\sin \Pi x / \Pi x)\}$  filter performs very well in terms of both quality and efficiency in a great variety of applications, such as noise reduction, detail smoothing/enhancement.

## Sinc filter

The full form of term "sinc" is sinus cardinalis (cardinal sine). It was introduced by Phillip M. Woodward in his 1952 paper "Information theory and inverse probability in. In mathematics the sinc function, denoted by  $\text{sinc}(x)$ , where sinc is any function of  $x$ . In mathematics, unnormalized sinc function is defined by

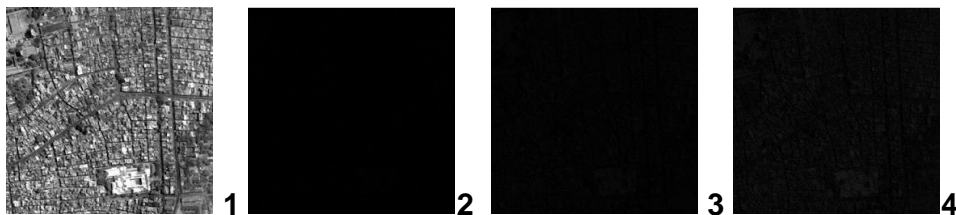
$$\text{sinc}(x) = \frac{\sin(x)}{x}$$

In digital image processing and digital signal processing the normalized sinc function is defined by

$$\text{sinc}(x) = \frac{\sin(\pi x)}{\pi x}$$

The normalization causes the definite integral of the function over the real numbers to equal 1 whereas the same integral of the unnormalized sinc function has a value of  $\pi$ . As a further useful property, all of the zeros of the normalized sinc function are integer values of  $x$ . This function is fundamental in the concept of reconstructing the original continuous band limited signal from uniformly spaced samples of that signal [2] The only difference between the two definitions is in the scaling of the independent variable (the  $x$ -axis) by a factor of  $\pi$ . In both cases, the value of the function at the removable singularity at zero is understood to be the limit value 1. The sinc function is analytic everywhere. In the method filter function  $n \text{sinc}$  i.e.  $n (\sin \Pi x / \Pi x)$ , where  $n=0.1, 0.2, 0.3, \dots$  is operated on image. A graph is plotted for different values of  $n$ .

For subjective evaluation of filter function  $\{n (\sin \Pi x / \Pi x)\}$ , the output images of Bangalore city are shown in the Figure: Figure. 2 To Figure 10. The test images: Cartosat image of Bangalore city of sizes  $256 \times 256$  are used for subjective evaluation. A smooth region and a complex region of Bangalore image are also demonstrated through various Figures for critical analysis.



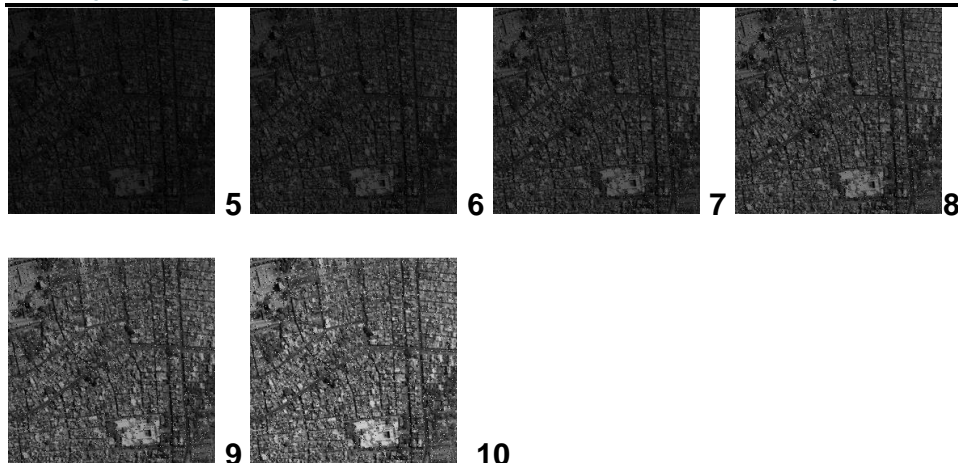


Figure 1: Original Cartosat image of Bangalore i city

Figure 2 to 10: The filtered images of a gray-scale input  $\{n (\sin \Pi x / \Pi x)\}$ , with  $n=0.1, 0.2, \dots, 0.9$ 

**PSNR Results:** We present the PSNR values for our test images in Tables 1.

**Table-1:** Filtering performance of filter function  $\{n (\sin \Pi x / \Pi x)\}$  with various values of  $n$ , in terms of PSNR (dB), operated on Cartosat image of Bangalore city under various noise conditions (standard deviation varies from 5% to 50%)

PEAK-SIGNAL-TO-NOISE RATIO, PSNR (DB)											
S.No	Filters	Standard deviation of salt and pepper noise									
		5%	10%	15%	20%	25%	30%	35%	40%	45%	50%
Test Image: CartoSat Image of Bangalore city.											
1	0.1(sin $x/x$ )	8.207	8.208	8.209	8.21	8.212	8.213	8.214	8.216	8.216	8.218
2	0.2(sin $x/x$ )	8.419	8.425	8.43	8.435	8.437	8.439	8.442	8.444	8.447	8.453
3	0.3(sin $x/x$ )	8.779	8.777	8.774	8.777	8.785	8.797	8.815	8.798	8.803	8.818
4	0.4(sin $x/x$ )	9.424	9.28	9.289	9.279	9.302	9.321	9.351	9.345	9.303	9.32
5	0.5(sin $x/x$ )	9.926	9.916	9.999	9.932	9.999	9.929	9.948	9.909	9.918	9.903
6	0.6(sin $x/x$ )	10.651	10.762	10.663	10.63	10.539	10.575	10.85	10.494	10.52	10.474
7	0.7(sin $x/x$ )	11.694	11.402	12.006	11.249	11.147	11.054	11.492	10.889	10.808	10.841
8	0.8(sin $x/x$ )	12.23	11.949	11.831	11.608	11.363	11.432	11.147	10.849	10.691	10.617
9	0.9(sin $x/x$ )	12.525	12.415	11.506	11.354	10.977	11.2	10.418	10.449	9.934	9.973

**Table-2:** Filtering performance of filter function  $\{n (\sin \Pi x / \Pi x)\}$  with various values of  $n$ , in terms of RMSE, operated on Cartosat image of Bangalore city under various noise conditions (standard deviation varies from 5% to 50%)

ROOT MEAN SQUARED ERROR (RMSE)		
S.No	Filters	Standard deviation of salt and pepper noise

		5%	10%	15%	20%	25%	30%	35%	40%	45%	50%
<b>Test Image: CartoSat Image of Bangalore city.</b>											
1	<b>0.1(sin<math>\pi</math>x/x)</b>	99.13	99.11	99.1	99.09	99.07	99.06	99.05	99.02	99.02	99
2	<b>0.2(sin<math>\pi</math>x/x)</b>	96.74	96.67	96.61	96.56	96.54	96.51	96.44	96.46	96.42	96.36
3	<b>0.3(sin<math>\pi</math>x/x)</b>	92.81	92.83	92.86	92.83	92.74	96.52	92.42	92.61	92.55	92.39
4	<b>0.4(sin<math>\pi</math>x/x)</b>	86.16	87.6	87.52	87.61	87.38	87.19	86.89	86	87.37	87.2
5	<b>0.5(sin<math>\pi</math>x/x)</b>	81.32	81.42	80.64	81.27	80.64	81.3	81.12	81.48	81.4	81.54
6	<b>0.6(sin<math>\pi</math>x/x)</b>	74.82	73.86	74.71	74.99	75.78	75.47	73.12	76.18	75.95	76.35
7	<b>0.7(sin<math>\pi</math>x/x)</b>	66.35	68.62	64.01	69.84	70.66	71.42	67.91	72.79	73.47	73.2
8	<b>0.8(sin<math>\pi</math>x/x)</b>	62.37	64.43	65.31	67.01	68.93	68.38	70.66	73.13	74.47	75.11
9	<b>0.9(sin<math>\pi</math>x/x)</b>	60.3	61.06	67.8	69	72.06	70.23	76.85	76.57	81.25	80.88

## Discussion and Conclusion:

In this paper  $\{n(\sin \pi x / \pi x)\}$  filter is used to remove the noise as shown in the tables. The table describes the above filter effectively reduce the limitations which the classical spatial filters faced. Considering the table, these spatial filters enhance the images which are affected by noise, blurring effect and outliers. This filter that shares the nice property of edge-preserving, smoothing but can be computed efficiently and exactly. Since this filter is frequency domain filter its computational time is less. It not only reduces the noise but also enhance the image. The disadvantage of the frequency sampling technique was that the frequency response gave errors at the points where it was not sampled.

## References

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## About the Author

SUSAN JOHN joined S.F.S College 2006 as Assistant Professor, Department of Physics, Maharashtra State, India. Received P.L. KHARE AWARD, RAMCHANDRA KRISHNA CHANDURKAR MEMORIAL GOLD MEDAL, K.B.L.SETH GOLD MEDAL & XIII NATIONAL SEMINAR ON CRYSTALLOGRAPHY AWARD for scoring highest marks in M.Sc, Physics.