Connected Roman Semi Total Block Domination in Graphs

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ABSTRACT

A connected Roman dominating function $f=(V_0,V_1,V_2)$ *on a semitotalblock graph* $T_b(G)=H$ *is a function* $f:V(H)\to\{0,1,2\}$ *satisfying the condition that every vertex* u *for which* $f(u)=0$ *is adjacent to at least one vertex* v *for which* $f(v)=2$ *such that* $\langle V_1 \cup V_2\rangle$ *or* $\langle V_2\rangle$ *is connected. The weight of a connected Roman dominating function is the value* $f(V(H))=\sum_{v\in V(H)} f(v)$. *The minimum weight of a connected Roman dominating function on a semitotalblock graph* H *is called the connected Roman semitotalblock domination number of* G *and is denoted by* $\gamma_{RCT}(G)$.

In this paper, we study the connected Roman domination number of semitotalblock graph H *and obtain some results on* $\gamma_{RCT}(G)$ *in terms of elements of* G, *but not in terms of* H. *Further we develop its relationship with other different domination parameters of* G.

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INTRODUCTION

All graphs considered here are simple (finite, undirected and loop less). For standard graph theory the terminology not given here we refer [2] and [3]. Let $G=(V,E)$ be a graph with vertex set V and edge set E, the open neighborhood $N(v)$ of the vertex v consists of vertices adjacent to v and the closed neighborhood of v is $N[v]=v\cup N(v)$. For a subset $S\subseteq V(G)$, we define $N(S)=\underset{v\in S}{\cup} N(v)$ and $N[S]=\underset{v\in S}{\cup} N[v]$. The subgraph induced by S is denoted by $\langle S\rangle$.

For any graph $G=(V,E)$, the semitotalblock graph $T_b(G)=H$ is the graph whose set of vertices is the union of the set of vertices and set of blocks of G in which two vertices are adjacent if and only if the corresponding members of G are adjacent or the corresponding members of G are incident, see[6].

A set $D\subseteq V$ is a dominating set if every vertex not in D is adjacent to atleast one vertex in D. The minimum cardinality of a minimal dominating set D is called a domination number of G and is denoted by $\gamma(G)$.

A dominating set D is connected dominating set if an induced subgraph $\langle D\rangle$ is connected. The connected domination number $\gamma_C(G)$ of a graph G is the minimum cardinality of a connected dominating set. See[5].

A dominating set D is a total dominating set if the induced subgraph $\langle D \rangle$ has no isolated vertices. The total domination number $\gamma_t(G)$ of a graph G is the minimum cardinality of a total dominating set. See [5]

A Roman dominating function on a graph $G=(V,E)$ is a function $f:V \to \{0,1,2\}$ Satisfying the condition that every vertex u for which $f(u)=0$ is adjacent to at least one vertex v for which $f(v)=2$. The weight of a Roman dominating function is the value $f(V)=\sum_{v \in V} f(v)$. The minimum weight of Roman dominating function on a graph G is called the Roman domination number of G and is denoted by $\gamma_R(G)$. See[1]

A Roman dominating function $f=(V_0,V_1,V_2)$ on a semitotal block graph $T_b(G)=H$ is a connected Roman dominating function (CRDF) on G if $\langle V_1 \cup V_2 \rangle$ or $\langle V_2 \rangle$ is connected. The connected Roman semitotalblock domination number $\gamma_{RCT}(G)$ of G is the minimum weight of a connected Roman semitotalblock dominating function of G.

We need the following theorems to prove our further results.

Theorem A[8]: For any graph G, $\gamma_R(G) \le \gamma_{RC}(G)$.

Theorem B[8]: For any nontrivial tree T, $\gamma_{RC}(T)=2\gamma(T)$ if and only if every non end vertex of T is adjacent to atleast one end vertex.

Theorem C[4]: If G is a connected graph with $p \ge 3$ vertices then $\gamma_C(G)=p-k$ or $\gamma_C(G)+k=p$ where k be the number of end vertices of T.

Theorem D[7]: For any graph G, $\gamma_{RB}(G) \le \gamma_R(G)$.

Theorem E[9]: Let G be any (p,q) with $p \ge 2$ vertices then $p \le \gamma_{RS}(G)$.

RESULTS

In this section we illustrate the connected Roman semitotal block domination number by determining the value of $\gamma_{RCT}(G)$ for several classes of graphs.

Theorem 1:

1. For any path P_p, $p>3$ vertices,

a. $\gamma_{RCT}(P_p)=p$, if $p=4,5$.

b. $\gamma_{RCT}(P_{2k})=2k+n$, if $p \ne 4,5$. where $k=3,4,.....;\ n=1,2,......$

c. $\gamma_{RCT}(P_{2k+1})=2k+n+1$.

2. For any graph G, if G is a block or G has exactly one cut vertex incident with blocks which are complete, then $\gamma_{RCT}(G)=2$.

Upper bounds for $\gamma_{RCT}(G)$:

We establish the upper bounds for the connected Roman semitotalblock domination number.

Theorem 2: For any non trivial graph G, $\gamma_{RCT}(G) \le 2\gamma_C(G)$.

Proof: Let G be any non trivial connected graph with $V(G)=\{v_1,v_2,.....,v_n\}$. Suppose there exists a set $D_c=\{v_1,v_2,.....,v_i\}$, $1 \le i \le n$ such that $D_c \subset V(G)$ and $\forall v_j \in V(G)-D_c$, $1 \le j \le n$ is adjacent to atleast one vertex of D_c. Also there exists a unique path between every pair of vertices of D_c. Then $|D_c|=\gamma_c(G)$. Let the graph $T_b(G)$ has $\{b_i; 1 \le i \le n\}$ number of block vertices corresponding to the blocks $B_i \in G; i=1,2,....,n$. Then $V[T_b(G)]=V(G) \cup \{b_i\}$. Suppose there exists the sets D_1, D_2 in $T_b(G)$ such that $D_1 \subseteq D_c$ and $D_2 \subseteq \{b_i\}$. If $f(V_1)=\phi$, then some

$v_k \in D_c; 1 \le k \le i$, $v_m \in \{b_i\}$. Hence $\{v_k \cup v_m\}$ forms a dominating set in $T_b(G)$ in which $\{v_k \cup v_m\} \in V_2$ and if $\langle v_k \cup v_m \rangle$ is connected, then $|v_k \cup v_m| = 2|D_c|$, which gives $\gamma_{RCT}(G) = 2\gamma_C(G)$. If $f(V_1) \ne \phi$, for some $v_l \in V(G)$ or $v_l \in D_c$ also belongs to V_1, then there exists $v_k^{'} \subset v_k$ and $v_m^{'} \subset v_m$ such that $\{v_k^{'} \cup v_m^{'}\} \in V_2$. If $\langle v_l \cup v_k^{'} \cup v_m^{'} \rangle$ is connected , then $|v_l \cup v_k^{'} \cup v_m^{'}| \le 2|D_c|$, which gives $\gamma_{RCT}(G) \le 2\gamma_C(G)$.

Theorem 3: For any graph G with n blocks such that each block is complete, then $\gamma_{RCT}(G) \le 2n$.

Proof: Let G be any graph with n blocks such that each block is complete and $f = (V_0, V_1, V_2)$ be a γ_{RC}-function in $T_b(G)$. Let $\{v_1, v_2, ..., v_n\}$ be the number of vertices incident with n blocks which forms a γ_c-set in G. If every end blocks $\{B_i; 1 \le i \le n\}$ of G contains the vertices $\{v_i\} \subseteq \{v_n\}$, then $\{v_i\} \in V_2$. Otherwise there exists some non end blocks $\{B_j; 1 \le j \le n\}$ contains the vertices $\{v_j\} \subseteq \{v_n\}$, such that $\{\{v_k\}, \{v_l\}\} \subseteq \{v_j\}$ and $\{v_k\} \in V_2$, $\{v_l\} \in V_1$. If $\langle \{v_i\} \cup \{v_j\} \rangle$ is connected. Then $\{\{v_i\} \cup \{v_j\}\} \le 2\{\{B_i\} \cup \{B_j\}\} = 2n$. Hence $\gamma_{RCT}(G) \le 2n$.

Theorem 4: Let G be any non trivial graph with $G \ne P_p$, $p \ge 6$ then $\gamma_{RCT}(G) \le p$.

Proof: Let G be any non trivial graph with $G \ne P_p$, $p \ge 6$. Suppose $G = P_p$, $p \ge 6$. Then by Theorem 1, $\gamma_{RCT}(G) > p$, a contradiction. Hence $G \ne P_p$, $p \ge 6$. Suppose $G = P_p$ with $1 \le p \le 5$. Then by Theorem 1, the result is obvious.

Now let G be any non trivial graph with $V(G) = \{v_1, v_2, ..., v_n\}$ and $f = (V_0, V_1, V_2)$ be a γ_{RC}-function in $T_b(G)$. Further $D_c = \{v_1, v_2, ..., v_i\}$ $1 \le i \le n$ be the number of vertices which forms a γ_c-set of G and $\{v_j\} \subseteq \{v_i\}$ be the number of vertices of G such that $\forall$ $v \in \{v_j\}$, $V - v$ has atleast two components. Suppose every vertex of $V - \{v_j\}$ is adjacent to atleast one vertex of v_j. Then $\{v_j\} \in V_1$ or V_2 and $V - \{v_j\} \in V_0$. Otherwise there exists atleast one vertex $\{v_k\} \in V - \{v_j\}$ which is not adjacent to $\{v_j\}$. Let $\{B_k; 1 \le k \le n\}$ be the number of blocks in G such that $v_k \in \{B_k\}$. Then there exists the vertices $\{b_k; 1 \le k \le n\}$ in $T_b(G)$ corresponds to the blocks $\{B_k\}$ of G such that $\{b_k\} \in V_2$ and $N(b_k) \cap \{v_j\} \in V_1$ or V_2 which gives $|V_1| + 2|V_2| \le |v_n|$.
Hence $\gamma_{RCT}(G) \le p$.

Theorem 5: For any nontrivial graph G, $\gamma_{RCT}(G) \le \gamma_{RS}(G)$.

Proof: By Theorem 4 and Theorem E, $\gamma_{RCT}(G) \le p \le \gamma_{RS}(G)$.

Hence $\gamma_{RCT}(G) \le \gamma_{RS}(G)$.

Theorem 6: For any nontrivial graph G, $\gamma_{RCT}(G) \le \gamma_c(G) + \gamma_t(G)$.

Proof: Let $V(G) = \{v_1, v_2, ..., v_n\}$ be the number of vertices of G. Let $D_c = \{v_1, v_2, ..., v_i\}$ $1 \le i \le n$ be the number of vertices of G such that $D_C \subset V$ and $\langle D_c \rangle$ is connected. Further $D_t = \{v_1, v_2, ..., v_k\}$ $1 \le k \le n$ be the number of vertices of G and $D_t \subset V$ such that $\langle D_t \rangle$ has no isolated vertices. Then $|D_c| = \gamma_c(G)$ and $|D_t| = \gamma_t(G)$. Let γ_{RC}-set D_{RCT} such that $|D_{RCT}| = \gamma_{RCT}(G)$. Since D_c is connected. Therefore the set D_c must contain atleast one vertex from each block $\{B_i; 1 \le i \le n\}$. Then the block vertices $\{b_i; 1 \le i \le n\}$ of $T_b(G)$ corresponding to the blocks $\{B_i; 1 \le i \le n\}$ are adjacent to atleast one vertex of D_c. Suppose $f(V_1) \ne \phi$ and $f = (V_0, V_1, V_2)$ be a γ_{RC}-function in $T_b(G)$ with $\forall$ $v_k \in V_1$, $1 \le k \le n$. Then there exists atleast two vertices $(u, w) \in N(v_k)$ such that $(u \cup w) \in V_2$. Also if there exist atleast one vertex $v_m \subseteq D_c \cup D_t$, $1 \le m \le n$. Suppose v_m belongs to only one block B of G. Then there exist a block vertex $b \in V[T_b(G)]$ is adjacent to atleast one vertex of $D_c \cup D_t$ such that $b \in V_2$ in which $\{v_k \cup \{u \cup w\} \cup b\}$ forms a connected component. Then $|D_{RCT}| \le |D_c| + |D_t|$. Suppose $f(V_1) = \phi$. Then $\forall\ v_l \in D_c \cup D_t$, $1 \le l \le n$, $v_l \in V_2$, also $\{v_l\}$ forms a γ_{RC}-set of $T_b(G)$ such that $\langle v_l \rangle$ is connected, which gives $|D_{RCT}| \le |D_c| + |D_t|$. Hence $\gamma_{RCT}(G) \le \gamma_c(G) + \gamma_t(G)$.

Theorem 7: For any tree with n blocks and $c \ge 1$ cut vertices, $\gamma_{RCT}(T) < n + c$.

Proof: Suppose T be a tree with n blocks and $A = \{v_1, v_2, ..., v_k\}$ be the number of cut vertices of T such that $|A| = c$. Let $\{b_1, b_2, ..., b_n\}$ be the number of block vertices of $T_b(T)$ with respect to the blocks $\{B_1, B_2, ..., B_n\}$ of T such that $|b_n| = n$. Since every block is K_2 in T and each $v_k \in A; 1 \le k \le n$ is adjacent to its corresponding block vertex in $T_b(T)$ such that $v_k \in V_1$ or V_2, then the $\langle V[T_b(T)] - A \rangle$ is not connected it follows that $\{N(b_i) \cap A\}$ forms γ_{RC}-set in $T_b(T)$ such that $\gamma_{RCT}(T) < n + c$. Therefore $\gamma_{RCT}(T) < n + c$.

Again as the number of blocks B_n of T increased by atleast one block of T then atleast one cut vertex and one block vertex are increased in $T_b(T)$, which gives $\gamma_{RCT}(T) < n + c$.

The connected domination, Roman domination and connected Roman semitotalblock domination are related by the following inequality.

Theorem 8: For any graph G, $\gamma_{RCT}(G) \le \gamma_c(G) + \gamma_R(G) - 1$.

Proof: Let G be any graph and $f = (V_0, V_1, V_2)$ be a γ_{RC}-function with connected Roman dominating set D_{RCT} in $T_b(G)$ such that $|D_{RCT}| = \gamma_{RCT}(G)$. Now we consider the following cases.

Case1: Suppose G be any tree and k be the number of end vertices in G. Let $f = (V_0', V_1', V_2')$ be a γ_R-function in G and $f(V_1') = \phi$. Then $\forall\ u_1 \in (V_2' \cap D_c)$ such that $u_1 \in V_2$. Suppose there exist a vertex $w \in (V_0' \cap D_c)$. Then $w \in V_1$ or there exists the vertices $w_1, w_2 \in V_0' \cap D_c$. Clearly $w_1 \in V_1$, $w_2 \in V_2$ or $w_1 \in V_2$, $w_2 \in V_1$. Suppose some $u_2 \in (V_2' \cap D_c)$. Then $u_2 \notin V_2$ and $u_2 \in V_1$. Also there exists $u_2', u_2'' \in V_0' \cap D_c$ where as $(u_2' \cup u_2'') \in V_2$. Hence $\langle V_1 \cup V_2 \rangle$ forms a connected Roman dominating set in $T_b(G)$, which gives $|D_{RCT}| \le |D_c| + |D_R| - 1$.

Thus $\gamma_{RCT}(G) \le \gamma_c(G) + \gamma_R(G) - 1$.

Case2: Suppose G is not a tree. Then we consider the following subcases.

Subcase2.1: Assume G be a graph with $\{B_i; 1 \le i \le n\}$ blocks and C be the number of cut vertices in G such that every vertex of B_i are adjacent to atleast one cut vertex c of G. Then $\forall\ v \in (V_2' \cap D_c)$, $v \in V_2$ and if there exist a vertex $u \in V_0' \cap D_c$, then $u \in N(v)$ and hence $u \in V_1$. Also if there exists $w \in V_1' \cap D_c$ and $N(w) \in V_0' \cap D_c$. Then $w \in V_2$ and $N(w) \in V_1$, which gives $|D_{RCT}| \le |D_c| + |D_R| - 1$.

Hence $\gamma_{RCT}(G) \le \gamma_c(G) + \gamma_R(G) - 1$.

Subcase2.2: Assume G be a graph with $\{B_j;1\le j\le n\}$ blocks such that atleast one vertex of each block is not adjacent to a cut vertex c. Then $\forall\ B_j$ in G there exist the corresponding block vertices $v_j\in b_j$ in $T_b(G)$ and hence $v_j\in V_2$ also $N(v_j)\cap D_c\in V_1$ or V_2, which gives $|D_{RCT}|\le|D_c|+|D_R|-1$. Hence $\gamma_{RCT}(G)\le\gamma_c(G)+\gamma_R(G)-1$.

In the following theorem, we establish the relation of $\gamma_{RCT}(G)$ with $\gamma_c(G)$ and blocks of G.

Theorem 9: For any graph G with n blocks $\gamma_{RCT}(G)\le\gamma_c(G)+n$.

Proof: Let G be any graph with $\{B_i;1\le i\le n\}$ be the number of blocks of G and $f=(V_0,V_1,V_2)$ be a γ_{RC}-function in $T_b(G)$.

Further $S=\{v_1,v_2,.....,v_k\}\subseteq V(G)$ be the set of all vertices with $\deg(v)\ge 2$. Then there exists a minimal vertex set $S^{'}\subseteq S$, which covers all the vertices of G. Clearly $S^{'}$ forms a minimal γ-set of G. Suppose the subgraph $\langle S^{'}\rangle$ has only one component. Then $S^{'}$ itself is a connected dominating set of G. Otherwise, if the subgraph $\langle S^{'}\rangle$ has more than one component, then attach the minimum number of vertices $\{w_i\}\in V(G)-S^{'}$, where $\deg(w_i)\ge 2$, which are between the vertices of $S^{'}$ such that $S_1=S^{'}\cup\{w_i\}$ forms exactly one component in the subgraph $\langle S_1\rangle$. Clearly, S_1 forms a minimal γ_c-set of G. Further k be the number of end vertices of G. Then we consider the following cases.

Case1: Suppose $k=\phi$. Then $k=0<n$. By Theorem C, $\gamma_c(G)+k=p$. Also by Theorem 4, $\gamma_{RCT}(G)\le p$.

Hence $\gamma_{RCT}(G)\le p=\gamma_c(G)+k<\gamma_c(G)+n$. Therefore $\gamma_{RCT}(G)\le\gamma_c(G)+n$.

Case2: Suppose $k\ne\phi$. Then consider the following subcases.

Subcase2.1: Assume $G\ne K_2$. Then $\forall\ \{v_k\}\in K;1\le k\le n$ there exist the blocks $n\in B_i$ such that $k\le n$. By Theorem C, $p=\gamma_c(G)+k\le\gamma_c(G)+n$. Also by Theorem 4, $\gamma_{RCT}(G)\le p$. Hence $\gamma_{RCT}(G)\le\gamma_c(G)+n$.

Subcase2.2: Assume $G=K_2$. Then by Theorem 1, $\gamma_{RCT}(G)=2=1+1=\gamma_c(G)+n$. Therefore $\gamma_{RCT}(G)\le\gamma_c(G)+n$.

Lower bound for $\gamma_{RCT}(G)$:

In the following theorem, we obtain the lower bounds for $\gamma_{RCT}(T)$.

Theorem 10: For any non trivial tree T, $\gamma(T)+c\le\gamma_{RCT}(T)$, where c be the number of cut vertices of T.

Proof: Suppose G be a non trivial tree with $V=\{v_1,v_2,.....,v_n\}$. Then there exists a set $V^{'}\subset V$ such that $V^{'}=\gamma(T)$. Now for any non trivial tree T with $B_i;1\le i\le n$ be the number of blocks of T. Then in $T_b(T)$, let $\{b_1,b_2,.....,b_n\}$ be the set of block vertices corresponding to B_i in $T_b(T)$. We consider a function $f=(V_0,V_1,V_2)$ be a γ_{RC}-function in $T_b(T)$. Now $C_1\subseteq C$ and $\forall\ \{v_i\}\in C_1$. Then $\{v_i\}\in V_2$. Otherwise $\forall\ \{v_i\}\in C_1$ if $\{v_i\}\notin V^{'}$. Then $\{v_i\}\in V_1$. It follows that the connected induced subgraph $\langle V_1\cup V_2\rangle$ or $\langle V_2\rangle$ is $\gamma_{RCT}(T)$. Hence $\gamma(T)+c\le\gamma_{RCT}(T)$.

In the following theorem, we obtain $\gamma_{RCT}(T)$ for which the lower bond is attained with connected Roman domination number of T.

Theorem 11: Let T be any non trivial tree. Then $\gamma_{RC}(T) \leq \gamma_{RCT}(T)$.

Proof: Let T be any non trivial tree and $f = (V_0, V_1, V_2)$ be a γ_{RC}-function in $T_b(T)$. Suppose $n_1 = \{v_1, v_2, ..., v_n\}$ be the number of non end vertices of T adjacent to atleast one end vertex and $n_2 = \{v_1, v_2, ..., v_i\}$ $1 \leq i \leq n$ be the number of end vertices of T not adjacent to end vertices. Then we consider the following cases.

Case1: Suppose $n_2 = \phi$. Then every non end vertex of T is adjacent to atleast one vertex. By Theorem 14, $\gamma_{RCT}(T) = 2\gamma(T)$. Also by Theorem B, $\gamma_{RC}(T) = 2\gamma(T)$. Hence $\gamma_{RC}(T) = \gamma_{RCT}(T)$.

Case2: Suppose $n_2 \neq \phi$. Then $\forall$ $\{v_i\} \in n; i = 1,2,...n$, $\{v_i\} \subseteq V_2$. Now consider $\{n_3, n_4\} \subseteq \{n_2\}$. Suppose $\forall$ $u \in n_3$, $u \in V_2$. Then there exists atleast one vertex $w \in n_4$ and $w \subseteq N(u)$ which gives $w \in V_1$. Suppose $u = \phi$. Then there exist $\{h_i; i = 1,2,..., n\} \in n_4$ such that $\{h_i\} \in V_1$. But $\{v_i\} \in V_2^{'}$ and $\{u \cup w\} \in V_1^{'}$, which gives $\gamma_{RC}(T) < \gamma_{RCT}(T)$. Hence $\gamma_{RC}(T) \leq \gamma_{RCT}(T)$.

Theorem 12: For any non trivial tree T, then $\gamma_R(T) \leq \gamma_{RCT}(T)$.

Proof: From Theorem A and Theorem 11, $\gamma_R(T) \leq \gamma_{RC}(T) \leq \gamma_{RCT}(T)$. Hence $\gamma_R(T) \leq \gamma_{RCT}(T)$.

Theorem 13: For any tree T. Then $\gamma_{RB}(T) \leq \gamma_{RCT}(T)$.

Proof: By Theorem D, $\gamma_{RB}(T) \leq \gamma_R(T)$ and by Theorem 12, $\gamma_R(T) \leq \gamma_{RCT}(T)$. Hence $\gamma_{RB}(T) \leq \gamma_{RCT}(T)$.

In the following theorem we establish the equality result.

Theorem 14: For any non trivial tree T, if every non end vertex of T is adjacent to atleast one end vertex. Then $\gamma_{RCT}(T) = 2\gamma(T)$.

Proof: Let T be any non trivial tree . Further $f = (V_0, V_1, V_2)$ be a γ_{RC}-function with connected Roman dominating set D_{RCT} in $T_b(T)$ such that $|D_{RCT}| = \gamma_{RCT}(T)$ and k be the number of end vertices of T. Suppose $n_1 = \{v_1, v_2, ..., v_n\}$ be the number of non end vertices of T adjacent to atleast one end vertex. Then there exists a minimal γ-set D of T such that $|D| = \gamma(T) = n_1$. Let $\{B_i; 1 \leq i \leq n\}$ be the number of blocks of T corresponding to the block vertices $\{b_i; 1 \leq i \leq n\}$ of $T_b(T)$. Since each $v \in n_1$ is incident with atleast two blocks and is adjacent to $\{k \cup b_i\}$ in $T_b(T)$ with the property $|k \cup b_i| = |V_0|$, $|n_1| = |V_2|$ and $|V_1| = \phi$, which gives $|D_{RCT}| = |V_1| + 2|V_2| = \phi + 2n_1 = 2|D|$. Hence $\gamma_{RCT}(T) = 2\gamma(T)$.

REFERENCES

[1] E. J. Cockyane, P. A. Dreyer, S. M. Hedetniemi, and S. T. Hedetniemi, Roman domination in Graphs, Disc. Math.278 (2004), 11-22.

[2] F. Haray , Graph Theory, Adison Wesley, reading Mass(1972).

[3] T.W. Haynes, S. T. Hedetniemi and P. J. Slater, Fundamentals of Domination in graphs, Marcel Dekker, New York, (1998).

[4] S. T. Hedetniemi and R. C. Laskar, connected domination in graphs, In B. Bollobas editor Graph Theory and Combinatorics, Academic Press London (1984),209-218.

[5] V. R. Kulli, Domination in Graphs, Vishwa International Publications(2010).

[6] V. R. Kulli, Semitotal Block Graph and Total Block Graph of a Graph, Indian J. Pure and Appl. Math. 7,(1976),625-630.
[7] M. H. Muddebihal and Sumangaladevi, Roman Block Domination in Graphs, Int. J. for Research in Sci. and Adv. Tech., Issue-2, Vol. 6(2013), 267-275.
[8] M. H. Muddebihal and Sumangaladevi, Connected Roman Domination in Graphs, International Journal of Research in Engg.Tech.Vol.2 (10), (2013).
[9] M. H. Muddebihal and Sumangaladevi, Roman Subdivision Domination in Graphs, Int. J., Engg, Sciences and Research Tech., Vol. 3(3), (2014), 1441-1447.