# ON THE SUGENO INTEGRAL OF NON-MONOTONIC FUZZY MEASURES

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**Abstract:**

In this paper ,we discusses about the concept of non monotonic fuzzy measures.and also studied some special properties with respect to sugeno integral with examples.

**Keywords:**

Non-motonic fuzzy measures ,non – monotonic continuous fuzzy measures,measureable functions,sugeno Integral,μ-Integrals.

**I Introduction:**

The term fuzzy integral has been introduced by sugeno (58) in 1974 and is now most commonly called the sugeno integral. We study in detail the sugeno integral which can be considered as the most representative fuzzy integral.The non-monotonic fuzzy measures which are set functions with out monotonicity and continuity were introduced at the first time.and the sugeno Integral were being defined from this with the help of the concepts of bounded variations and total variations by the way in functional analysis,But the fuzzy measures referred to above have not monotonicity and continuity ,therefore the corresponding Sugeno Inegral has not monotonicity and continuity too, thus being directed against the fuzzy valued functions taken valued fuzzy numbers fuzzy valued functional in the sense of sugeno's fuzzy measure were defined in (1) and the general properties of convergence theorem were studied.

The main purpose of this paper is to re define a sugeno integral on non negative bounded measureable function spaces by defined non-monotonic fuzzy measures.

The Sugeno Integral defined in this paper has not still monotonicity thus some results ,dominated convergence theorem r emain to be discussed further in the future.

**II Non -Monotonic fuzzy measure and Sugeno Integral:**

Let X be an arbitrary fixed set, $\mathcal{A}$ be a $\sigma$ – algebra formed by the subsets of X,and (X,$\mathcal{A}$) be a measurable space ,$R^+$ denote the interval [0,-∞ **)**

ie) $\mathbf{R^+= [0 ,- \infty )}$

**2.1Definition:**

Let (X,) be an arbitrary measurable space a set function

$\mu : \mathcal{A} \rightarrow [0,1]$

(i) $\mu(\boldsymbol{\phi}) = 0, \mu(x) = 1$

(ii) If $\{A_n\}$ is an arbitrary mono tonic sequence of sets in $\mathcal{A}$ then

$$\lim_{n\to\infty} \mu(A_n) = \mu\left(\lim_{n\to\infty} A_n\right)$$

If set functions μ only satisfies (1)

Then μ is called a non monotonic fuzzy measure,If set functions μ satisfies both (1) and (2) then μ is called a non-monotonic continuous fuzzy measure.At this time ,corresponding $(X,\mathcal{A},\mu)$ is called a non-monotonic fuzzy measure space and non monotonic continuous fuzzy measure space respectively.

**2.2 Definition:**

Let $(X, \mathcal{A})$ be an arbitrary measurable space, f be a real valued function on X, then f is called a measurable function on $(X,\mathcal{A})$ , if $f_\lambda \triangleq \{x \in X / f(x) \geq \lambda\} \in \mathcal{A}$ for all $\lambda \in R^+$.

Let BM(X,) = { f / f is real valued bounded measurable function on $(X,\mathcal{A})$ }

Defining the norm :$\| f \| = \mathrm{Sup}_{x\in X} |f(x)|$

On the space BM (X, ) then we are easy to be proved that $BM(X,\mathcal{A})$ constitutes a Banach space,please to refer (1).

In this paper for convenience sake,we only convenience sake , we only discuss problems on non-negative bounded measurable function spaces.

Let $BM^+$ (X,) = {f ∈ BM(X,)} f(x) ≥ 0 for any x∈ X}

**Definition:2.3**

Suppose that μ is a fuzzy normalized measure on X, the Sugeno Integral of a function

f: X→ [0,1]

w.r.to fuzzy measure μ is defined as $\int f(x)dx = \max\limits_{1\leq i\leq n} \min(f(x_i), \mu(A_i))$

where { $f(x_1),f(x_2),\ldots..f(x_n)$ } are the ranges they are defined as $f(x_1) \leq f(x_2) \leq f(x_3) \leq \ldots\ldots..f(x_n)$.

**Examples:2.4**

Consider a set X={$x_1, x_2, x_3$ } and the range is defined as $f(x_1) = 0.6$ ,$f(x_2) = 0.8$

$f(x_3) = 0.9$ such that the fuzzy measure (non-mnotonic) is defined as

| Functions | Values |
|---|---|
| $\mu(\boldsymbol{\phi})$ | 0 |
| $\mu\{x_1\}$ | 0.9 |
| $\mu\{x_2\}$ | 1 |
| $\mu\{x_3\}$ | 0.6 |
| $\mu\{x_1, x_2\}$ | 0.8 |
| $\mu\{x_2, x_3\}$ | 0.9 |
| $\mu\{x_1, x_3\}$ | 0.6 |
| $\mu\{x_1,x_2,x_3\}$ | 1.2 |

T normalize the fuzzy measures we are dividing all of them by largest value in measure

| Functions | Values |
|---|---|
| $\mu(\boldsymbol{\phi})$ | 0 |
| $\mu\{x_1\}$ | 0.75 |
| $\mu\{x_2\}$ | 0.83333 |
| $\mu\{x_3\}$ | 0.5 |
| $\mu\{x_1, x_2\}$ | 0.66667 |
| $\mu\{x_2, x_3\}$ | 0.83333 |
| $\mu\{x_1, x_3\}$ | 0.5 |
| $\mu\{x_1,x_2,x_3\}$ | 1 |

The Sugeno Integral

$$\int f(x)d\mu = \max_{1\leq i\leq n} \min\{ \min(f(x_i), \mu (A_i)\}$$

$$= \max ( \min (0.6,1), \min(0.8, 0.8333), \min(0.9,0.5)\}$$

= max (0.6, 0.8,0.5)
= 0.8.

**2.5 Definition** :

Consider a positive measurable function f : X→ [0,∞). The fuzzy integral was defined in (2) as

$(S)\int_A f d\mu = \vee_{\alpha\geq 0}[\alpha \wedge \mu(\{f \geq \alpha\} \cap A)]$ ,A∈$\mathcal{A}$

Where $\{ f \geq \alpha\} = \{ x\in X / f(x)\geq \alpha \}$ and we denoted by " ∨ " and "∧ " the operations " sup" and " inf" in [0,∞ ].

Sugeno may also be denoted by $\int f\, d\mu$

**2.6 Lemma:**

(1) Both $F_\alpha$ and $F_{\alpha+}$ are non increasing with respect to α and $F_{\alpha+} \supset F_\beta$ when $\alpha < \beta$

**(2)** $\lim\limits_{\beta\to\alpha-} F_\beta = \lim\limits_{\beta\to\alpha-} F_{\beta+} = F_\alpha \supset F_{\alpha+} = ) \lim\limits_{\beta\to\alpha+} F_\beta = \lim\limits_{\beta\to\alpha+} F_{\beta+}$

Proof:

(1) Is evident.

(2) Follws from the following facts,

$$\cap_{\beta<\alpha}\{x / f(x) \geq \beta\} = \cap_{\beta<\alpha}\{x / f(x) > \beta\}$$
$$= \cap_{\beta<\alpha}\{x / f(x) \geq \alpha\} \supset \{ x / f(x) > \alpha \}$$
$$= \cap_{\beta>\alpha}\{x / f(x) \geq \beta\}$$
$$= \cap_{\beta>\alpha}\{x / f(x) > \beta\} \quad .$$

**3. Monotone Convergence theorem of the Sugeno Integral:**

Let A∈ F ,If $f_n \searrow f$ on A and there exist $n_0$ such that

$\mu ( \{ x / f_{n0}(x) > \int_A f d\mu \} \cap A ) < \infty$

or if $f_n \nearrow f$ then

$\lim\limits_n \int f_n d\mu = \int_A f d\mu$

Proof:

We can assume that A=X without any loss of generality, write $\int f d\mu$ =c

And let $f_n \searrow f$ with $n_0$ such that

$\mu \{x / f_{no}(x) > C\} < \infty$

If c = ∞ by the monotoncity of sugeno integral we have,

$\int f_n d\mu \geq \int f d\mu = \infty$

That is the conclusion of this theorem holds if $c < \infty$ $\int f_n d\mu \geq C$

For any n = 1,2,3,4………. And therefore

$\lim\limits_n \int f_n d\mu \geq C$

Now we use reduction to absurdity to prove that the equality holds .If we assume that $\lim\limits_n \int f_n d\mu > C$

Then there exist $C^1 > C$ such that

$\lim\limits_n \int f_n d\mu > C^1$

And therefore

$\int f_n d\mu > C$

For any n. we know that

$\mu (F_C^{11}) > C^1$

for any n since there exist $n_0$ such that

$\mu(F_{C^1}{}^{n0}) = \mu(\{x / f_{n0}(x) \geq C^1\}) \leq \mu(\{x / f_{n0}(x) > C\}) < \infty$

by applying the continuity from above of μ from known lemma we get

$\mu(F_C{}^1) = \lim_n (F_{C^1}{}^n) \geq C^1$

we know that

$⨍ f d\mu \geq C^1 > C$

This contradicts $\int f d\mu = C$

Consequently, $\lim_n \int f_n\, d\mu = C = \int f\, d\mu$ –

When $f_n \nearrow f$ ,the proof is similar to the above.

**3.1 Theorem:**

Let μ be non monotonic fuzzy measure on measurable space (X,A)

$f \in BM^+(X, \mathcal{A})$ and f be a μ integrable ,If for all $A \in \mathcal{A}$,

$f \leq f^1$

$⨍ f\, d\mu \leq \int f\, d\mu$

$(S) \int_A f d\mu = \bigvee_{\alpha \geq 0}(\alpha \wedge \mu(A \cap \{f \geq \alpha\}))$

put A=X ,

$(S)\int_X f d\mu = (S)\int f d\mu$

$= \bigvee_{\alpha \geq 0}(\alpha \wedge \mu(f \geq \alpha)$

**3.2 Examples:**

f and g are real valued function, $f(x) = \sqrt{x}$ and $g(x) = x$ , $x \in [0,1]$

p =3 ,q =2

(i) $(S)\int_0^1 f d\mu = \bigvee_{\alpha \in [0,1]}(\alpha \wedge \mu(x \geq \alpha))$

$= \bigvee_{\alpha \in [0,1]}(\alpha \wedge (1-\alpha))$

$= 0.5$

(ii) $(S)\int_0^1 g f^3\, d\mu = \bigvee_{\alpha \in [0,1]}(\alpha \wedge \mu(\{x\sqrt{x}\}^3 \geq \alpha))$

$= \bigvee_{\alpha \in [0,1]}(\alpha \wedge (1-\sqrt[5]{\alpha})^2)$

$= 0.2$

(iii) $\int_0^1 fg d\mu = \bigvee_{\alpha \in [0,1]}(\alpha \wedge \mu(\{x\sqrt{x}\} \geq \alpha))$

$= \bigvee_{\alpha \in [0,1]}(\alpha \wedge (1-\sqrt[3]{\alpha})^2)$

$= 0.43016$

Because $0.25 = ((S)\int_0^1 g d\mu)^2 = ((S)\int_0^1 g d\mu)^{p-1}$

$= ((S)\int_0^1 fg d\mu)^{pq}$

$= ((S)\int_0^1 fg d\mu)$

$= 0.43016$

**3.4 Properties of Sugeno Integral:**

**(i)** $\int |_A \circ \mu = \mu(A)$

**(ii)** If $f < g$ ,$\int f \circ \mu < \int g \circ \mu$

**(iii)** $\int(\alpha \vee f) \circ \mu = \alpha \vee (\int f \circ \mu)$

(iv) $\int f \circ (\mu \vee v) = (\int (f \circ \mu) \vee (\int f \circ v))$

(v) N is the null set

$f(x) = g(x)\ .x \notin N$

$\int f \circ \mu = \int g \circ \mu.$

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