# DISCRETE HEAT EQUATION MODEL WITH 3-VARIABLES

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**ABSTRACT:**We investigate the generalized partial difference equation operator and propose a model of it in discrete heat equation in this paper.The diffusion of heat is studied by the application of newton's law of cooling in dimension up to five and several solutions are postulated for the same. Through numerical simulations solutions are validated and applications are derived.

## 1.GENERALIZED DISCRETE HEAT EQUATION

Consider the temperature distribution of a very long rod

Assume that the rod is so long, that it can be laid on the set R of real numbers. Let $v(k_1, k_2)$ be the temperature at the real time $(k_1)$ and real position $(k_2)$ of a rod at time $(k_1)$

If the temperature $v(k_1, k_2 - l_2)$, $l_2 > 0$ is higher than $v(k_1, k_2)$ heat will flow from the point $k_2 - l_2$ to $k_2$

The amount of increase is $v(k_1 + l_1, k_2) - v(k_1, k_2)$ and

It is reasonable to postulate that the increase is proportional to the difference $v(k_1, k_2 - l_2) - v(k_1, k_2)$say

$$\alpha(v(k_1, k_2 - l_2) - v(k_1, k_2))$$

$v(k_1 + l_1, k_2) - v(k_1, k_2) = \alpha\big(v(k_1, k_2 - l_2) - v(k_1, k_2)\big), \alpha > 0$

(I,e)

$\underset{(l_1,0)}{\Delta} v(k_1, k_2) = \alpha \underset{(0,-l_2)}{\Delta} v(k_1, k_2)$

If $k_1 = k_2$ and $\alpha = \frac{l_1}{-l_2}$

Then $v(k_1, k_2) = (k_1 . k_2)$ is a solution of discrete heat equation

### 1.1: FORMATION OF 3-VARIABLE SIMPLE DISCRETE HEAT EQUATION

Consider the temperature distribution of a very long rod. Assume that the rod is so long, that it can be laid on the set R of real numbers. Let $v(k_1, k_2, k_3)$ be the temperature at the real time $(k_1)$ and real position $(k_2)$ of a rod at time $(k_1)$.If the temperature$v(k_1, k_2 - l_2, k_3 - l_3)$, $l_2 > 0$ is higher than $v(k_1, k_2, k_3)$ heat will flow from the point $k_2 - l_2$ to $k_2$.The amount of increase is $v(k_1 + l_1, k_2 + l_2, k_3) - v(k_1, k_2, k_3)$ and It is reasonable to postulate that the increase is proportional to the difference $v(k_1, k_2 - l_2, k_3 - l_3) - v(k_1, k_2, k_3)$say

$$\alpha(v(k_1, k_2 - l_2, k_3 - l_3) - v(k_1, k_2, k_3))$$

$v(k_1 + l_1, k_2 + l_2, k_3) - v(k_1, k_2, k_3) = \alpha\big(v(k_1, k_2 - l_2, k_3 - l_3) - v(k_1, k_2, k_3)\big), \alpha > 0$

(i,e)

$\underset{(l_1,l_2,0)}{\Delta} v(k_1, k_2, k_3) = \alpha \underset{(0,-l_2,-l_3)}{\Delta} v(k_1, k_2, k_3)$

If $\alpha=\frac{e^{l_1+l_2}-1}{e^{-l_2-l_3}-1}$ discrete a heat equation .where$k_1, k_2, k_3$ are variable and $l_1, l_2, l_3$ are parameters

## 1.2 SOLUTION OF SIMPLE 3- VARIABLE SIMPLE HEAT EQUATION

In this section we derive a solution of equation (4.1), Here we have to obtain a function $v(k_1, k_2)$ satisfying the equation (4.1)

### THEOREM 1.2.1 (first type solution)

Let $u(k_1, k_2, , k_3) = \underset{l_1, l_2, 0}{\Delta} v(k_1, k_2, k_3)$ and integer m such that $u(k_1 - rl_1, k_2 - rl_2, k_3)$ and are known for r=1, 2 ... m then heat equation (3.1) has a solution of the form

$v(k_1, k_2, k_3) - v(k_1 - ml_1, k_2 - ml_2, k_3) = \alpha \sum_{r=1}^{m} u(k_1 - r_1 l_1, k_2 - r_2 l_2, k_3)$(4.2)

### PROOF

From the (1.1), since $\alpha$ is constant

We have

$v(k_1, k_2, k_3) = \alpha \underset{(l_1, l_2, 0)}{\overset{-1}{\Delta}} \left( \underset{(0, -l_2, -l_3)}{\Delta} v(k_1, k_2, k_3) \right)$(4.3)

### REMARK 1.2.2

If $k_1$ is integer multiple of $l_1$ says $m_1 l_1$ then

$$v(k_1, k_2, k_3) - v(k_1 - ml_1, k_2 - ml_2, k_3) = \alpha \sum_{r=0}^{m-1} u(r_1 l_1, r_2 l_2, k_3)$$

### THEOREM 1.2.3

If $1 - 2\alpha \neq 0$, then the second type solution of Heat equation is

$$v(k_1, k_2, k_3) = \frac{1}{(1-\alpha)^m} v(k_1 + ml_1, k_2 + ml_2, k_3) - \sum_{r=1}^{m} \frac{\alpha}{(1-\alpha)^r} v(k_1 + (r-1)l_1, k_2 + (r-2)l_2, k_3 - l_3)$$

### Proof

we have

$$v(k_1 + l_1, k_2 + l_2, k_3) - v(k_1, k_2, k_3) = \alpha[v(k_1, k_2 - l_2, k_3 - l_3) - v(k_1, k_2, k_3)]$$

$$v(k_1, k_2, k_3) = \frac{1}{1-\alpha} v(k_1 + l_1, k_2 + l_2, k_3) - \frac{\alpha}{1-\alpha} v(k_1, k_2 - l_2, k_3 - l_3)$$

Replace $k_1$ by $k_1 + l_1$ and $k_2$ by $k_2 + l_2$ in

$$v(k_1, k_2, k_3) = \frac{1}{1-\alpha} [\frac{1}{1-\alpha} v(k_1 + 2l_1, k_2 + 2l_2, k_3) - \frac{\alpha}{1-\alpha} v(k_1 + l_1, k_2, k_3 - l_3)] - \frac{\alpha}{1-\alpha} v(k_1, k_2 - l_2, k_3 - l_3)$$

In general

$$v(k_1, k_2, k_3) = \frac{1}{(1-\alpha)^m} v(k_1 + ml_1, k_2 + ml_2, k_3) - \sum_{r=1}^{m} \frac{\alpha}{(1-\alpha)^r} v(k_1 + (r-1)l_1, k_2 + (r-2)l_2, k_3 - l_3)$$

### EXAMPLE 1.2.4

Taking $v(k_1, k_2, k_3) = e^{k_1+k_2+k_3}$, we get

$$e^{k_1+k_2+k_3} = \frac{1}{(1-\alpha)^m} e^{k_1+ml_1+k_2+ml_2+k_3} - \sum_{r=1}^{m} \frac{\alpha}{(1-\alpha)^r} e^{k_1+(r-1)l_1+k_2+(r-2)l_2+k_3-l_3}$$

Taking $k_1 = 2.5, k_2 = 3.5, k_3 = 4.3, l_1 = 2.2, l_2 = 3.3, l_3 = 1.5$in , we get

$$e^{2.5+3.5+4.3} = \frac{1}{(1+245.7146971)^2} e^{2.5+4.4+3.5+6.6+4.3} - \frac{-245.7146971}{1+245.7146971} e^{2.5+2.2+3.5+4.3-1.5} - \frac{-245.7146971}{1+245.7146971} e^{2.5-3.3+3.5+4.3-1.5}$$

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## THEOREM 1.2.5

If $1 - 2\alpha \neq 0$, then the third type solution of Heat equation is

$$v(k_1,k_2,k_3) = \frac{1}{(1-\alpha)^m} v(k_1 + ml_{1,}k_2 + ml_2, k_3) - \frac{\alpha}{(1-\alpha)^{m+1}} v(k_1 + (m-1)l_{1,}k_2 + ml_2, k_3 - l_3) + \sum_{m=1}^{r} \frac{\alpha^2}{(1-\alpha)^{r+1}} v(k_1, k_2 - (3-r)l_2, k_3 - 2l_3)$$

## Proof

From we have

$$v(k_1 + l_{1,}k_2 + l_2, k_3) - v(k_1,k_2,k_3) = \alpha[v(k_1, k_2 - l_2, k_3 - l_3) - v(k_1,k_2,k_3)]$$

$$v(k_1,k_2,k_3) = \frac{1}{1-\alpha} v(k_1 + l_{1,}k_2 + l_2, k_3) - \frac{\alpha}{1-\alpha} v(k_1, k_2 - l_2, k_3 - l_3)$$

Replace $k_2$ by $k_2 - l_2$ and $k_3$ by $k_3 - l_3$ in

$$v(k_1,k_2,k_3) = \frac{1}{1-\alpha} v(k_1 + l_{1,}k_2 + l_2, k_3) - \frac{\alpha}{(1-\alpha)^2} v(k_1 + l_1, k_2, k_3 - l_3) + \frac{\alpha^2}{(1-\alpha)^2} v(k_1, k_2 - 2l_2, k_3 - 2l_3)$$

In general

$$v(k_1,k_2,k_3) = \frac{1}{(1-\alpha)^m} v(k_1 + ml_{1,}k_2 + ml_2, k_3) - \frac{\alpha}{(1-\alpha)^{m+1}} v(k_1 + (m-1)l_{1,}k_2 + ml_2, k_3 - l_3) + \sum_{m=1}^{r} \frac{\alpha^2}{(1-\alpha)^{r+1}} v(k_1, k_2 - (3-r)l_2, k_3 - 2l_3)$$

## EXAMPLE 1.2.6

Taking $v(k_1,k_2,k_3) = e^{k_1+k_2+k_3}$, we get

$$e^{k_1+k_2+k_3} = \frac{1}{1-\alpha} e^{k_1+l_1+k_2+l_2+k_3} - \frac{\alpha}{(1-\alpha)^{m+1}} e^{k_1+(m+1)l_1+k_2+ml_2+k_3-l_3} + \sum_{m=1}^{r} \frac{\alpha^2}{(1-\alpha)^{r+1}} e^{k_1+k_2-(3-r)l_2+k_3-2l_3}$$

Again taking $k_1 = 2.5, k_2 = 3.5, k_3 = 4.3, l_1 = 2.2, l_2 = 3.3, l_3 = 1$ in we get

$$e^{2.5+3.5+4.3} = \frac{1}{(1+245.7146971)^2} e^{2.5+2.2+3.5+3.3+4.3} - \frac{-245.7146971}{(1+245.7146971)^2} e^{2.5+2.2+3.5+4.3-1.5} - \frac{(-245.7146971)^2}{(1+245.7146971)^2} e^{2.5+3.5-6.6+4.3-3}$$

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## CONCLUSION

In this project, we derived several types of solution of discrete heat equation for one, two and three dimensional system

Here we obtained several results and the theorems by introducing partial difference operator with and without variable coefficient All the theorem are nearly derived and suitable examples are provide and valuate the finding verify the results

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