STUDYING BIANCHI TYPE-I COSMOLOGICAL MODELS IN RIEMANNIAN GEOMETRY BY MAPLE

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Abstract

In this paper we study Bianchi type-I model based on Riemannian geometry. The aim of this paper is to get the components of homothetic vector field, killing vector field, conformal killing vector fields in Riemannian geometry for Bianchi type-I, in different cases, using ordinary method and Computer program to get the components of the vectors .

Keywords:

Bianchi type-I; homothetic vector field; Christoffal symbols; Riemannian Geometry.

I) Introduction:

Riemannian geometry is the branch of differential geometry that studies Riemannian manifolds, smooth manifolds with a *Riemannian metric*, i.e. with an inner product on the tangent space at each point that varies smoothly from point to point. This gives, in particular, local notions of angle, length of curves, surface area and volume. From those, some other global quantities can be derived by integrating local contributions. Riemannian geometry originated with the vision of Bernhard Riemann expressed in his inaugural lecture "Ueber die Hypothesen, welche der Geometrie zu Grunde liegen" ("On the Hypotheses on which Geometry is Based"). It is a very broad and abstract generalization of the differential geometry of surfaces in R^3. Development of Riemannian geometry resulted in synthesis of diverse results concerning the geometry of surfaces and the behavior of geodesics on them, with techniques that can be applied to the study of differentiable manifolds of higher dimensions. It enabled the formulation of Einstein's general theory of relativity, made profound impact on group theory and representation theory, as well as analysis, and spurred the development of algebraic and differential topology.

Anisotropic Bianchi type-I universe, which is more general than FRW universe, plays a significant role to understand the phenomenon like formation of galaxies in early universe. Theoretical arguments as well as the recent observations of cosmic microwave background radiation (CMBR) support the existence of anisotropic phase that approaches an isotropic one we propose to study homogeneous and anisotropic Bianchi type-I cosmological models with time dependent gravitational and cosmological “constants”. [1]-[9]

In Sec. II, the metric and basic homothetic equations and killing’s equations have been presented in Riemannian geometry and solved , Section III case (1) of metric where the metric functions be equals and time dependent, case (2) where the metric functions be equals and equal t, case (3) metric functions be constant , case(4) metric functions equals one , get the homothetic, killing and conformal vector fields.

Problem statement and objectives:

Where studying some of the models of metric space times in the Riemannian geometry it is difficult to get homothetic equations as well as to get solved. In this paper we calculate the equations in the ordinary method as well as using a computer program and compare the results to be able to use computer programs in difficult models.

Methods: In this paper we get homothetic equations by equation (II.3). We solve the partial differential equations by separate variables and use Maple 17 program for getting the homothetic vector field.

II) THE METRIC AND BASIC HOMOTHETIC EQUATIONS

We consider the space-time metric of the spatially homogeneous and anisotropic Bianchi-I of the form:

$$ds^2 = dt^2 - A(t)dx^2 - B^2(t)dy^2 - C^2(t)dz^2 \quad \text{(II.1)}$$

where A(t), B(t) and C(t) are the metric functions of cosmic time t.

Where $g_{\mu\nu}$ is a metric tensor as in Riemannian connection $\left\{ {\alpha \atop \mu\nu} \right\}$, but also by a function

$$\left\{ {\alpha \atop \mu\nu} \right\} = \frac{1}{2} g^{\alpha\rho}(g_{\rho\nu,\mu} + g_{\rho\mu,\nu} - g_{\mu\nu,\rho}) \quad \text{(II.2)}$$

As in Riemannian geometry , a global vector field $\zeta = \zeta^{\mu}(t,x,y,z)_{\mu=0}^{3}$ on M is called conformal vector field if the following condition holds:

$$\mathcal{L}_{\eta} g_{\mu\nu} = g_{\rho\nu}\nabla_{\mu}\eta^{\rho} + g_{\mu\rho}\nabla_{\nu}\eta^{\rho} = 2\psi g_{\mu\nu} \ , \ \psi(t,x,y,z) \quad \text{(II.3)}$$

where ψ is constant we get the homothetic equations, where $\psi = 0$ we get the killing equations, $\mathcal{L}$ denote a Lie derivatives and ∇ is the covariant derivative such that:

$$\left. \begin{array}{l} \nabla_{\mu}\zeta^{\rho} = \partial_{\mu}\zeta^{\rho} + \Gamma^{\rho}_{\mu\alpha}\zeta^{\alpha} \\ \nabla_{\mu}\zeta_{\rho} = \partial_{\mu}\zeta_{\rho} - \Gamma^{\alpha}_{\mu\rho}\zeta_{\alpha} \end{array} \right\} \quad \text{(II. 4)}$$

we get the nine non vanishing Christoffel symbols of second kind in Riemannian geometry from (II.2) as :[10]-[12]

$$\Gamma^{t}_{xx} = \dot{A}A,\ \Gamma^{t}_{yy} = \dot{B}B,\ \Gamma^{t}_{zz} = \dot{C}C\ ,\Gamma^{x}_{xt} = \Gamma^{x}_{tx} = \frac{\dot{A}}{A}\ ,\Gamma^{y}_{yt} = \Gamma^{y}_{ty} = \frac{\dot{B}}{B}\ ,\ \Gamma^{z}_{tz} = \Gamma^{z}_{zt} = \frac{\dot{C}}{C} \quad \text{(II.5)}$$

The ten homothetic equations in Riemannian Geometry from equation (II.3):[13],[19]

$$\zeta^1_{,0} + \zeta^0_{,1} - 2\frac{\dot{A}}{A}\zeta^1 = 0 \quad \text{(II.6)}$$

$$\zeta^2_{,0} + \zeta^0_{,2} - 2\frac{\dot{B}}{B}\zeta^2 = 0 \quad \text{(II.7)}$$

$$\zeta^3_{,0} + \zeta^0_{,3} - 2\frac{\dot{C}}{C}\zeta^3 = 0 \quad \text{(II.8)}$$

$$\zeta^1_{,1} - A\dot{A}\zeta^0 = -\psi A^2 \quad \text{(II.9)}$$

$$\zeta^2_{,2} - B\dot{B}\zeta^0 = -\psi B^2 \quad \text{(II.10)}$$

$$\zeta^3_{,3} - C\dot{C}\zeta^0 = -\psi C^2 \quad \text{(II.11)}$$

$$\zeta^0_{,0} = \psi \quad \text{(II.12)}$$

$$\zeta^1_{,2} + \zeta^2_{,1} = 0 \quad \text{(II.13)}$$

$$\zeta^1_{,3} + \zeta^3_{,1} = 0 \quad \text{(II.14)}$$

$$\zeta^3_{,2} + \zeta^3_{,2} = 0 \quad \text{(II.15)}$$

The solution of the homothetic equations see Appendix(1)is: [20]

$A(t) = -\sqrt{\frac{-f_8(t)}{2\psi}}, B(t) = \sqrt{\frac{-C_1}{2\psi}}, C(t) = \sqrt{\frac{-C_2}{2\psi}}, \zeta^1 = -f_{10,x}(t,x)y - f_{12,x}(t,x)z$

$\zeta^2 = f_8(t)z + c_1y + f_{10}(t,x), \zeta^3 = -f_8(t)y + c_2y + f_{12}(t,x), \zeta^0 = \psi t + f_2(x,y,z)$

Where c_i , i=1,2,…are the integration constants and f_i , i=1,2,…the integration functions

The homothetic vector fields is:

$\zeta = [\psi t + f_2(x,y,z)]\,\partial t + [-f_{10,x}(t,x)y - f_{12,x}(t,x)z]\,\partial x + [f_8(t)z + c_1y + f_{10}(t,x)]\,\partial y + [-f_8(t)y + c_2y + f_{12}(t,x)]\,\partial z$

The solution of the killings equations:

$A(t) = \sqrt{c_5 - 2f_8(t)}, B(t) = c_7\sqrt{f_{12}(t)}\,, C(t) = -\sqrt{c_6 + 4f_7(t)}$

$\zeta^0 = (c_1z + c_2)x + c_3 + c_4\,, \zeta^2 = f_{12}(t)f_{13}(y)$

$\zeta^1 = \frac{1}{6c_1}\left[-3xc_1\dot{f}_8(t)(c_1xz + c_2x + 2c_3z + 2c_4) - 2z\dot{f}_7(t)(c_1^2z^2 + 3c_1c_2 + 3c_2^2) - 6c_1\left(f_9(t)z - f_{11}(t)\right)\right],$

$$\zeta^3 = \frac{1}{c_1^2}\left[(xzc_1^2 + (c_2x + c_3z + 2c_4)c_1 - c_2c_3)(c_2 + c_1z)\,\dot{f}_7(t) + c_1^2\left(\frac{x^2}{6}(c_1x + 3c_3)\dot{f}_8(t) + f_9(t)x + f_{10}(t)\right)\right]$$

III) Case(1):

In the case where $A(t) = B(t) = C(t)$ the metric be as:

$$ds^2 = dt^2 - A^2(t)(dx^2 + dy^2 + dz^2)$$

The homothetic equations will has the solution Appendix(2):

$A(t) = c_4\,, \zeta^0 = c_1x + c_2y + 2\psi t + c_3, \zeta^1 = c_5y + c_6z - c_1t - 2c_4^2\psi x + c_7$

$\zeta^2 = c_8z - c_2t - c_5x - 2c_4^2\psi y + c_9, \zeta^3 = c_{10} - c_8y - c_2t - 2c_4^2\psi z - c_6x$

Homothetic vector fields is:

$\zeta = [c_1x + c_2y + 2\psi t + c_3]\partial t + [c_5y + c_6z - c_1t - 2c_4^2\psi x + c_7]\partial x + [c_8z - c_2t - c_5x - 2c_4^2\psi y + c_9]\partial y + [c_{10} - c_8y - c_2t - 2c_4^2\psi z - c_6x]\partial z$

The killing's equations has the solution:

$A(t) = \frac{c_1}{2}t^2 + c_2t + c_3\,, \zeta^0 = 0, \zeta^1 = (c_9 - c_4y - c_7z)(c_1t^2 + 2c_2t + 2c_3)^2$

$\zeta^2 = (c_6 + c_4x + c_5z)(c_1t^2 + 2c_2t + 2c_3)^2, \zeta^3 = (c_8 - c_5y + c_7x)(c_1t^2 + 2c_2t + 2c_3)^2$

The killing vector field:

$\zeta = (c_1t^2 + 2c_2t + 2c_3)^2\{0\partial t + (c_9 - c_4y - c_7z)\partial x + (c_6 + c_4x + c_5z)\partial y + (c_8 - c_5y + c_7x)\,\partial z\}$

Case(2): In the case where $A(t) = B(t) = C(t) = t$ the metric be as:

$$ds^2 = dt^2 - t^2(dx^2 + dy^2 + dz^2)$$

The solution of homothetic equations see appendix(3) be:

$\zeta^0 = 2\psi t, \zeta^1 = (c_1 y + c_2 z + c_3)t^2, \zeta^2 = (c_4 z - c_1 x + c_5)t^2, \zeta^3 = (c_6 - c_2 x - c_4 y)t^2$

Homothetic vector fields is:

$\zeta = 2\psi t\,\partial t + [(c_1 y + c_2 z + c_3)t^2]\,\partial x + [(c_4 z - c_1 x + c_5)t^2]\,\partial y + [(c_6 - c_2 x - c_4 y +)t^2]\,\partial z$

The solution of killing's equations is:

$\zeta^0 = 0, \zeta^1 = (c_1 y + c_2 z + c_3)t^2, \zeta^2 = (c_4 z - c_1 x + c_5)t^2, \zeta^3 = (c_6 - c_2 x - c_4 y)t^2$

The killing vector field:

$\zeta = 0\,\partial t + [(c_1 y + c_2 z + c_3)t^2]\,\partial x + [(c_4 z - c_1 x + c_5)t^2]\,\partial y + [(c_6 - c_2 x - c_4 y +)t^2]\,\partial z$

The solution of conformal equations:

$$\psi(t,x,y,z) = \frac{1}{4}[\ln(t^2)c_1 + 2\ln(t)(c_3 z + c_5 x + c_7 y + c_1 + c_8) + c_1(x^2 + y^2 + z^2) + 2x(c_4 + c_5) + 2y(c_6 + c_7) + 2z(c_2 + c_3) + \frac{1}{2}(c_8 + c)]$$

$$\zeta^0 = t\left[\frac{1}{2}\ln(t^2)c_1 + \ln(t)(c_3 z + c_5 x + c_7 y + c_8) + \frac{c_1}{2}(x^2 + y^2 + z^2) + c_6 y + c_4 x + c_2 z + c_9\right]$$

$$\zeta^1 = \frac{t^2}{2}[\ln(t^2)c_5 + 2x(c_1\ln(t) + c_3 z) + c_5(x^2 - y^2 - z^2) + 2(c_7 xy + c_4\ln(t) - c_{10}y - c_{11}z + c_8 x - c_{12})]$$

$$\zeta^2 = -\frac{t^2}{2}\left[\ln(t^2)c_7 + 2\left(c_1\ln(t) + c_{3y}z + c_5 xy\right) + c_7(y^2 - x^2 - z^2) + 2(c_6\ln(t) + c_{10}x - c_{13}z + c_8 y - c_{14})\right]$$

$$\zeta^3 = \frac{-t^2}{2}\left[\begin{matrix} ln(t^2)c_3 + 2ln(t)c_1 z + c_3(z^2 + x^2 - y^2) - 2y(ln(t) + c_3 z + c_5 x) + +2zc_5(x + y) \\ +c_2 ln(t) + c_{11}x + c_{13}y + c_8 z - c_{15} \end{matrix}\right]$$

Conformal vector field:

$\zeta = \zeta^0\partial t + \zeta^2\partial x + \zeta^2\partial y + \zeta^3\partial z$

Case(3): In the case where $A(t) = B(t) = C(t) = k$, k is a constant the metric be as:

$$ds^2 = dt^2 - k^2(dx^2 + dy^2 + dz^2)$$

The solution of homothetic equations see appendix(4) be:

$\zeta^0 = \psi t + c_1 x + c_2 y + c_3 z + c_4, \zeta^1 = K^2\psi x - c_1 t + c_5 y + c_6 z + c_7,$

$\zeta^2 = K^2\psi y - c_2 t - c_5 x + c_8 z + c_9,, \zeta^3 = K^2\psi z - c_3 t - c_6 x - c_8 y + c_{10}$

The homothetic vector field:

$\zeta = [\psi t + c_1 x + c_2 y + c_3 z + c_4]\,\partial t + [K^2\psi x - c_1 t + c_5 y + c_6 z + c_7]\,\partial x + [K^2\psi y - c_2 t - c_5 x + c_8 z + c_9]\,\partial y + [K^2\psi z - c_3 t - c_6 x - c_8 y + c_{10}]\,\partial z$

The solution of killing's equations is:

$\zeta^0 = c_1 x + c_2 y + c_3 z + c_4, \zeta^1 = -c_1 t + c_5 y + c_6 z + c_7,$

$\zeta^2 = -c_2 t - c_5 x + c_8 z + c_9, \zeta^3 = -c_3 t - c_6 x - c_8 y + c_{10}$

The killing vector field:

$\zeta = [c_1 x + c_2 y + c_3 z + c_4]\,\partial t + [-c_1 t + c_5 y + c_6 z + c_7]\,\partial x + [-c_2 t - c_5 x + c_8 z + c_9]\,\partial y + [-c_3 t - c_6 x - c_8 y + c_{10}]\,\partial z$

The solution of conformal equations:

$$\zeta^0 = (c_2 z + c_4 x + c_6 y + c_8)t + \frac{c_1}{2}(x^2 + y^2 + z^2) + (c_5 x + c_7 y + +c_3 z + c_9) - \frac{c_1}{2K^2}t^2$$

$$\zeta^1 = K^2\left[(c_2 z + c_6 y + c_8)x + \frac{c_4}{2}(x^2 - y^2 - z^2)\right] + (c_{10} y + c_3 z - c_5 t - c_1 tx + c_{12}) - \frac{c_4}{2}t^2$$

$$\zeta^2 = K^2\left[(c_2 z + c_6 x + c_8)y + \frac{c_6}{2}(y^2 - x^2 - z^2)\right] + (c_{10} x + c_3 z - c_7 t - c_1 ty + c_{14}) - \frac{c_6}{2}t^2$$

$$\zeta^3 = K^2\left[(c_4 x + c_6 y + c_8)z + \frac{c_6}{2}(z^2 - x^2 - y^2)\right] + (c_{15} - c_{11} x - c_{13} y - c_3 t - c_1 tz) - \frac{c_2}{2}t^2$$

$$\psi(t,x,y,z) = (c_2 z + c_4 x + c_6 y + c_8) - \frac{c_1 t}{K^2}$$

The conformal killing vector field:

$\zeta = \zeta^0\,\partial t + \zeta^1\,\partial x + \zeta^2\,\partial y + \zeta^3\,\partial z$

Case(4): In the case where $A = B = C = K = 1$, the metric be as:
$ds^2 = dt^2 - (dx^2 + dy^2 + dz^2)$, Minkowski metric space

The solution of homothetic equations see appendix(5) be:

$\zeta^0 = \psi t + c_1 x + c_2 y + c_3 z + c_4$, $\zeta^1 = \psi x - c_1 t + c_6 y + c_6 z + c_7$
$\zeta^2 = \psi y - c_2 t + c_5 x + c_8 z + c_9$, $\zeta^3 = \psi z - c_3 t - c_6 x - c_8 y + c_{10}$

The solution of killing's equations is:

$\zeta^0 = c_1 x + c_2 y + c_3 z + c_4$, $\zeta^1 = -c_1 t + c_6 y + c_6 z + c_7$
$\zeta^2 = -c_2 t + c_5 x + c_8 z + c_9$, $\zeta^3 = -c_3 t - c_6 x - c_8 y + c_{10}$

The solution of conformal equations:

$$\zeta^0 = (c_2 z + c_4 x + c_6 y + c_8)t + \frac{c_1}{2}(x^2 + y^2 + z^2 - t^2) + (c_5 x + c_7 y + +c_3 z + c_9)$$

$$\zeta^1 = \left[(c_2 z + c_6 y - c_1 t + c_8)x + \frac{c_4}{2}(x^2 - y^2 - z^2 - t^2)\right] + (c_{10} y + c_{11} z - c_5 t + c_{12})$$

$$\zeta^2 = \left[(c_2 z + c_4 x - c_1 t + c_8)y + \frac{c_6}{2}(y^2 - t^2 - x^2 - z^2)\right] + (c_{10} x + c_{13} z - c_7 t - c_1 ty + c_{14})$$

$$\zeta^3 = (c_4 x + c_6 y + c_8)z + \frac{c_6}{2}(z^2 - t^2 - x^2 - y^2) + (c_{15} - c_{11} x - c_{13} y - c_3 t)$$

$$\psi(t,x,y,z) = (c_2 z + c_4 x + c_6 y + c_8) - c_1 t$$

V) DISCUSSION

In this paper we get the ten homothetic equations for Bianchi type-I in Riemann geometry and solve it by science of partial differential equation and also by Maple program and get the homothetic vector fields, killing and conformal killing vector fields in the Riemann geometry. [18]

VI) CONCLUSION

where studying Bianchi Type –I by maple and by ordinary method we get the same results but maple give us the solution is more accurate. If the possibilities of the computer are higher, the equations can be calculated and solved for the most difficult .

VII) RECOMMENDATIONS

It can be study the same equations for Bianchi type-I in different geometry and using maple to get the homothetic equations and its solution for more difficult models

Appendix(1)

> $with(DifferentialGeometry): with(Tensor):$ **Riemannian Geometry**

> $DGsetup([t, x, y, z], M)$

> $g := evalDG(\,dt \&t\, dt - A^2(t)\, dx \&t\, dx - B^2(t)\, dy \&t\, dy - C^2(t)\, dz \&t\, dz)$

$$g := dt\,dt - A(t)^2\, dx\,dx - B(t)^2\, dy\,dy - C(t)^2\, dz\,dz$$

M > $C2 := Christoffel(g, \text{"SecondKind"})$

$$C2 := A(t)\, A_t D\_t\, dx\, dx + B(t)\, B_t D\_t\, dy\, dy + C(t)\, C_t D\_t\, dz\, dz + \frac{A_t}{A(t)} D\_x\, dt\, dx + \frac{A_t}{A(t)} D\_x\, dx\, dt + \frac{B_t}{B(t)} D\_y\, dt\, dy + \frac{B_t}{B(t)} D\_y\, dy\, dt + \frac{C_t}{C(t)} D\_z\, dt\, dz + \frac{C_t}{C(t)} D\_z\, dz\, dt$$

Killing's equations

M > for _eq in sys1 do _eq end do;

$$\frac{1}{2}\,\frac{\_F2_t A(t) + \_F1_x A(t) - 2\, A_t \_F2(t, x, y, z)}{A(t)} = 0$$

$$\frac{1}{2}\,\frac{\_F3_t B(t) + \_F1_y B(t) - 2\, B_t \_F3(t, x, y, z)}{B(t)} = 0$$

$$\frac{1}{2}\,\frac{\_F4_t C(t) + \_F1_z C(t) - 2\, C_t \_F4(t, x, y, z)}{C(t)} = 0$$

$$-A(t)\, A_t \_F1(t, x, y, z) + \_F2_x = 0$$

$$-B(t)\, B_t \_F1(t, x, y, z) + \_F3_y = 0$$

$$-C(t)\, C_t \_F1(t, x, y, z) + \_F4_z = 0$$

$$\frac{1}{2}\_F3_x + \frac{1}{2}\_F2_y = 0$$

$$\frac{1}{2}\_F4_x + \frac{1}{2}\_F2_z = 0$$

$$\frac{1}{2}\_F4_y + \frac{1}{2}\_F3_z = 0$$

$$\_F1_t = 0$$

$$A(t) = -\sqrt{-2\_F8(t) + \_C5},\ B(t) = \_C7\sqrt{\_F12(t)},\ C(t) = -\sqrt{4\_F7(t) + \_C6},\ \_F1(t,x,y,z) = (\_C1\,z + \_C2)\,x + \_C3\,z + \_C4,\ \_F2(t,x,y,z) = \frac{1}{6}\,\frac{1}{\_C1}\Big(-3\,x\,\_C1\,(\_C1\,x\,z + \_C2\,x + 2\,\_C3\,z + 2\,\_C4)\,\dot{\_F8}(t) + \big(-2\,\_C1^2 z^3 - 6\,\_C1\,\_C2\,z^2 - 6\,\_C2^2 z\big)\,\dot{\_F7}(t) - 6\,\_C1\,(\_F9(t)\,z - \_F11(t))\Big),\ \_F3(t,x,y,z) = \_F12(t)\,\_F13(y),\ \_F4(t,x,y,z) = \frac{1}{\_C1^2}\bigg(\big(x\,z\,\_C1^2 + (\_C2\,x + \_C3\,z + 2\,\_C4)\,\_C1 - \_C3\,\_C2\big)\,(\_C1\,z + \_C2)\,\dot{\_F7}(t) + \_C1^2\Big(\frac{1}{6}\,x^2\,(\_C1\,x + 3\,\_C3)\,\dot{\_F8}(t) + \_F9(t)\,x + \_F10(t)\Big)\bigg)$$

$$A(t) = -\sqrt{-2\_F8(t) + \_C5},\ B(t) = \_C7\sqrt{\_F12(t)},\ C(t) = -\sqrt{4\_F7(t) + \_C6},\ \_F1(t,x,y,z) = (\_C1\,z + \_C2)\,x + \_C3\,z + \_C4,\ \_F2(t,x,y,z) = \frac{1}{6}\,\frac{1}{\_C1}\Big(-3\,x\,\_C1\,(\_C1\,x\,z + \_C2\,x + 2\,\_C3\,z + 2\,\_C4)\,\dot{\_F8}(t) + \big(-2\,\_C1^2 z^3 - 6\,\_C1\,\_C2\,z^2 - 6\,\_C2^2 z\big)\,\dot{\_F7}(t) - 6\,\_C1\,(\_F9(t)\,z - \_F11(t))\Big),\ \_F3(t,x,y,z) = \_F12(t)\,\_F13(y),\ \_F4(t,x,y,z) = \frac{1}{\_C1^2}\bigg(\big(x\,z\,\_C1^2 + (\_C2\,x + \_C3\,z + 2\,\_C4)\,\_C1 - \_C3\,\_C2\big)\,(\_C1\,z + \_C2)\,\dot{\_F7}(t) + \_C1^2\Big(\frac{1}{6}\,x^2\,(\_C1\,x + 3\,\_C3)\,\dot{\_F8}(t) + \_F9(t)\,x + \_F10(t)\Big)\bigg)$$

Check the solution:

M > $pde7 := \frac{1}{2}\,\frac{\partial}{\partial x}\,\_F3(t,x,y,z) + \frac{1}{2}\,\frac{\partial}{\partial y}\,\_F2(t,x,y,z) = 0$

M > $LHS := \frac{1}{2}\,\frac{\partial}{\partial x}\,\{\_F12(t)\,\_F13(y)\} + \frac{1}{2}\,\frac{\partial}{\partial y}\left\{\frac{1}{6}\,\frac{1}{\_C1}\Big(-3\,x\,\_C1\,(\_C1\,x\,z + \_C2\,x + 2\,\_C3\,z + 2\,\_C4)\,\dot{\_F8}(t) + \big(-2\,\_C1^2 z^3 - 6\,\_C1\,\_C2\,z^2 - 6\,\_C2^2 z\big)\,\dot{\_F7}(t) - 6\,\_C1\,(\_F9(t)\,z - \_F11(t))\Big)\right\}$

$$LHS := \{0\}$$

The homothetic equations in Riemannian geometry

$$\frac{1}{2}\,\frac{\_F2_t\,A(t) + \_F1_x\,A(t) - 2\,A_t\,\_F2(t,x,y,z)}{A(t)} = 0$$

$$\frac{1}{2}\,\frac{\_F3_t\,B(t) + \_F1_y\,B(t) - 2\,B_t\,\_F3(t,x,y,z)}{B(t)} = 0$$

$$\frac{1}{2}\,\frac{\_F4_t\,C(t) + \_F1_z\,C(t) - 2\,C_t\,\_F4(t,x,y,z)}{C(t)} = 0$$

$$-A(t)\,A_t\,\_F1(t,x,y,z) + \_F2_x = -\psi\,A(t)^2$$

$$-B(t)\,B_t\,\_F1(t,x,y,z) + \_F3_y = -\psi\,B(t)^2$$

$$-C(t)\,C_t\,\_F1(t,x,y,z) + \_F4_z = -\psi\,C(t)^2$$

$$\frac{1}{2}\,\_F3_x + \frac{1}{2}\,\_F2_y = 0$$

$$\frac{1}{2}\,\_F4_x + \frac{1}{2}\,\_F2_z = 0$$

$$\frac{1}{2}\,\_F4_y + \frac{1}{2}\,\_F3_z = 0$$

$$\_F1_t = \psi$$

M > $\_F1(t,x,y,z)=\psi t+\_F2(x,y,z)$, $A(t)=-\frac{1}{2}\frac{\sqrt{-2\psi\_F8(t)}}{\psi}$

$B(t)=\frac{1}{2}\frac{\sqrt{-2\psi\_C1}}{\psi}$, $C(t)=\frac{1}{2}\frac{\sqrt{-2\psi\_C2}}{\psi}$, $\_F2(t,x,y,z)=-\frac{\partial}{\partial x}\_F10(t,x)\,y-\frac{\partial}{\partial x}\_F12(t,x)\,z+\_F13(t,x)$, $\_F3(t,x,y,z)=\_F8(t)\,z+\_C1\,y+\_F10(t,x)$, $\_F4(t,x,y,z)=-\_F8(t)\,y+\_C2\,z+\_F12(t,x)$

Appendix(2):

> with(DifferentialGeometry) : with(Tensor) : **Where A(t)=B(t)=C(t)**

> DGsetup([t, x, y, z], M)

> $g := evalDG(dt \&t\, dt - A^2(t)\, dx \&t\, dx - A^2(t)\, dy \&t\, dy - A^2(t)\, dz \&t\, dz)$

$$g := dt\,dt - A(t)^2\,dx\,dx - A(t)^2\,dy\,dy - A(t)^2\,dz\,dz$$

Killing equations

M > for_eq in sys1 do_eq end do;

$$\frac{1}{2}\frac{\_F2_t A(t)+\_F1_x A(t)-2A_t\_F2(t,x,y,z)}{A(t)}=0$$

$$\frac{1}{2}\frac{\_F3_t A(t)+\_F1_y A(t)-2A_t\_F3(t,x,y,z)}{A(t)}=0$$

$$\frac{1}{2}\frac{\_F4_t A(t)+\_F1_z A(t)-2A_t\_F4(t,x,y,z)}{A(t)}=0$$

$$-A(t)\,A_t\_F1(t,x,y,z)+\_F2_x=0$$

$$-A(t)\,A_t\_F1(t,x,y,z)+\_F3_y=0$$

$$-A(t)\,A_t\_F1(t,x,y,z)+\_F4_z=0$$

$$\frac{1}{2}\_F3_x+\frac{1}{2}\_F2_y=0$$

$$\frac{1}{2}\_F4_x+\frac{1}{2}\_F2_z=0$$

$$\frac{1}{2}\_F4_y+\frac{1}{2}\_F3_z=0$$

$$\_F1_t=0$$

Solution of killig's equations:

$A(t)=\frac{1}{2}\_C1\,t^2+\_C2\,t+\_C3$, $\_F1(t,x,y,z)=0$, $\_F2(t,x,y,z)=-(\_C1\,t^2+2\_C2\,t+2\_C3)^2(\_C4\,y+\_C7\,z-\_C9)$, $\_F3(t,x,y,z)=(\_C4\,x+\_C5\,z+\_C6)(\_C1\,t^2+2\_C2\,t+2\_C3)^2$, $\_F4(t,x,y,z)=(\_C1\,t^2+2\_C2\,t+2\_C3)^2(-\_C5\,y+\_C7\,x+\_C8)$

$\{A(t)=\_C5, \_F1(t,x,y,z)=\_C1\,x+\_C2\,y+\_C3\,z+\_C4, \_F2(t,x,y,z)=-\_C1\,t-\_C6\,y-\_C9\,z+\_C11, \_F3(t,x,y,z)=-\_C2\,t+\_C6\,x+\_C7\,z+\_C8, \_F4(t,x,y,z)=-\_C3\,t-\_C7\,y+\_C9\,x+\_C10\}$

Homothetic equations

$$\frac{1}{2}\frac{\_F2_t A(t)+\_F1_x A(t)-2A_t\_F2(t,x,y,z)}{A(t)}=0$$

$$\frac{1}{2}\,\frac{\_F3_t\,B(t)+\_F1_y\,B(t)-2\,B_t\_F3(t,x,y,z)}{B(t)}=0$$

$$\frac{1}{2}\,\frac{\_F4_t\,C(t)+\_F1_z\,C(t)-2\,C_t\_F4(t,x,y,z)}{C(t)}=0$$

$$-A(t)\,A_t\_F1(t,x,y,z)+\_F2_x=-\psi\,A(t)^2$$

$$-B(t)\,B_t\_F1(t,x,y,z)+\_F3_y=-\psi\,B(t)^2$$

$$-C(t)\,C_t\_F1(t,x,y,z)+\_F4_z=-\psi\,C(t)^2$$

$$\frac{1}{2}\_F3_x+\frac{1}{2}\_F2_y=0$$

$$\frac{1}{2}\_F4_x+\frac{1}{2}\_F2_z=0$$

$$\frac{1}{2}\_F4_y+\frac{1}{2}\_F3_z=0$$

$$\_F1_t=\psi$$

The solutions:

$$\{A(t)=\_C4,\ \_F1(t,x,y,z)=\_C1\,x+\_C2\,y+2\,\psi\,t+\_C3,\ \_F2(t,x,y,z)=-2\_C4^2\,\psi\,x-\_C1\,t+\_C5\,y+\_C6\,z+\_C7,\ \_F3(t,x,y,z)=-2\_C4^2\,\psi\,y-\_C2\,t-\_C5\,x+\_C8\,z+\_C9,\ \_F4(t,x,y,z)=-2\_C4^2\,\psi\,z-\_C6\,x-\_C8\,y+\_C10\}$$

The conformal equations

$$A(t)=\_C2\sqrt{\_F8(t)}$$

$$\_F1(t,x,y,z)=\frac{1}{2}\,\frac{1}{\_F8(t)}\Big(\big((x^2+y^2+z^2)\_F7(t)+2\,x\_F9(t)+2\_F11(t)\,y+2\_F12(t)\,z\big)\dot{\_F8}(t)+2\Big(\Big(-\frac{1}{2}y^2-\frac{1}{2}z^2-\frac{1}{2}x^2\Big)\dot{\_F7}(t)-\dot{\_F9}(t)\,x-\dot{\_F11}(t)\,y-\dot{\_F12}(t)\,z+\_F13(t)\Big)\_F8(t)\Big)$$

$$\_F2(t,x,y,z)=\_F7(t)\,x+\_F8(t)\,y+\_F9(t),$$

$$\_F3(t,x,y,z)=(\_C1\,z-x)\_F8(t)+y\_F7(t)+\_F11(t)$$

$$\_F4(t,x,y,z)=\_F7(t)\,z-\_F8(t)\_C1\,y+\_F12(t)$$

$$\psi(t,x,y,z)=\frac{1}{4}\,\frac{1}{\_F8(t)^{3/2}\_C2}\Big(\Big(\Big(\Big(\frac{1}{2}x^2+\frac{1}{2}y^2+\frac{1}{2}z^2\Big)\_F7(t)+\_F12(t)\,z+x\_F9(t)+\_F11(t)\,y\Big)\_C2^2\dot{\_F8}(t)^2+\_F8(t)\_C2^2\Big(\Big(-\frac{1}{2}y^2-\frac{1}{2}z^2-\frac{1}{2}x^2\Big)\dot{\_F7}(t)-\dot{\_F9}(t)\,x-\dot{\_F11}(t)\,y-\dot{\_F12}(t)\,z+\_F13(t)\Big)\dot{\_F8}(t)-2\_F8(t)\_F7(t)\Big)$$

Appendix(3): **Where A(t)=B(t)=C(t)=*t***

> *with*(*DifferentialGeometry*) : *with*(*Tensor*) :

> *DGsetup*([*t*, x, y, *z*], *M*)

> $g := evalDG(\,dt\,\&t\,dt - t^2\,dx\&t\,dx\ - t^2dy\,\&t\,dy - t^2\,dz\&t\,dz)$

$$g := dt\,dt - t^2\,dx\,dx - t^2\,dy\,dy - t^2\,dz\,dz$$

M > $C2 := Christoffel(g, \text{"SecondKind"})$

$$C2 := t\,D\_t\,dx\,dx + t\,D\_t\,dy\,dy + t\,D\_t\,dz\,dz + \frac{1}{t}\,D\_x\,dt\,dx + \frac{1}{t}\,D\_x\,dx\,dt + \frac{1}{t}\,D\_y\,dt\,dy + \frac{1}{t}\,D\_y\,dy\,dt + \frac{1}{t}\,D\_z\,dt\,dz + \frac{1}{t}\,D\_z\,dz\,dt$$

M > killing's equations

$$\frac{1}{2}\,\frac{\_F1_x\,t + \_F2_t\,t - 2\,\_F2(t,x,y,z)}{t} = 0$$

$$\frac{1}{2}\,\frac{\_F1_y\,t + \_F3_t\,t - 2\,\_F3(t,x,y,z)}{t} = 0$$

$$\frac{1}{2}\,\frac{\_F4_t\,t + \_F1_z\,t - 2\,\_F4(t,x,y,z)}{t} = 0$$

$$-t\,\_F1(t,x,y,z) + \_F2_x = 0$$

$$-t\,\_F1(t,x,y,z) + \_F3_y = 0$$

$$-t\,\_F1(t,x,y,z) + \_F4_z = 0$$

$$\frac{1}{2}\,\_F3_x + \frac{1}{2}\,\_F2_y = 0$$

$$\frac{1}{2}\,\_F4_x + \frac{1}{2}\,\_F2_z = 0$$

$$\frac{1}{2}\,\_F4_y + \frac{1}{2}\,\_F3_z = 0$$

$$\_F1_t = 0$$

M > $pdsolve(pde10)$

$$\_F1(t,x,y,z) = \_F2(x,y,z)$$

M > $pdsolve(sys1)$

$$\{\_F1(t,x,y,z) = 0,\ \_F2(t,x,y,z) = (\_C1\,y + \_C2\,z + \_C3)\,t^2,\ \_F3(t,x,y,z) = -t^2\,(\_C1\,x - \_C4\,z - \_C5),\ \_F4(t,x,y,z) = -t^2\,(\_C2\,x + \_C4\,y - \_C6)\}$$

Homothetic equations

M > for _eq in sys2 do _eq end do;

$$\frac{1}{2}\,\frac{\_F1_x\,t + \_F2_t\,t - 2\,\_F2(t,x,y,z)}{t} = 0$$

$$\frac{1}{2}\,\frac{\_F1_y\,t + \_F3_t\,t - 2\,\_F3(t,x,y,z)}{t} = 0$$

$$\frac{1}{2}\,\frac{\_F4_t\,t + \_F1_z\,t - 2\,\_F4(t,x,y,z)}{t} = 0$$

$$-t\,\_F1(t,x,y,z) + \_F2_x = -2\,\psi\,t^2$$

$$-t\,\_F1(t,x,y,z) + \_F3_y = -2\,\psi\,t^2$$

$$-t\,\_F1(t,x,y,z) + \_F4_z = -2\,\psi\,t^2$$

$$\frac{1}{2}\,\_F3_x + \frac{1}{2}\,\_F2_y = 0$$

$$\frac{1}{2}\_F4_x + \frac{1}{2}\_F2_z = 0$$

$$\frac{1}{2}\_F4_y + \frac{1}{2}\_F3_z = 0$$

$$\_F1_t = 2\psi$$

M > *pdsolve*(*sys2*)

$$\{\_F1(t,x,y,z) = 2\psi t, \_F2(t,x,y,z) = (\_C1\,y + \_C2\,z + \_C3)\,t^2, \_F3(t,x,y,z) = -t^2(\_C1\,x - \_C4\,z - \_C5), \_F4(t,x,y,z) = -t^2(\_C2\,x + \_C4\,y - \_C6)\}$$

Conformal equations

M > *pdsolve*(*sys3*)

$$\_F1(t,x,y,z) = t\left(\frac{1}{2}\ln(t)^2\_C1 + (\_C3\,z + \_C5\,x + \_C7\,y + \_C8)\ln(t) + \left(\frac{1}{2}x^2 + \frac{1}{2}y^2 + \frac{1}{2}z^2\right)\_C1 + \_C6\,y + \_C4\,x + \_C2\,z + \_C9\right)$$

$$\_F2(t,x,y,z) = -\frac{1}{2}t^2\left(\_C5\ln(t)^2 + 2\ln(t)\_C1\,x + 2\_C3\,x\,z + \_C5\,x^2 - y^2\_C5 - z^2\_C5 + 2\_C7\,x\,y + 2\_C4\ln(t) - 2\_C10\,y - 2\_C11\,z + 2\_C8\,x - 2\_C12\right)$$

$$\_F3(t,x,y,z) = -\frac{1}{2}t^2\left(\_C7\ln(t)^2 + 2\ln(t)\_C1\,y + 2\_C3\,y\,z + 2\_C5\,x\,y - \_C7\,x^2 + \_C7\,y^2 - z^2\_C7 + 2\_C6\ln(t) + 2\_C10\,x - 2\_C13\,z + 2\_C8\,y - 2\_C14\right)$$

$$\_F4(t,x,y,z) = -\frac{1}{2}t^2\left(\_C3\ln(t)^2 + 2\ln(t)\_C1\,z - \_C3\,x^2 - \_C3\,y^2 + \_C3\,z^2 + 2\_C5\,x\,z + 2\_C7\,y\,z + 2\_C2\ln(t) + 2\_C11\,x + 2\_C13\,y + 2\_C8\,z - 2\_C15\right)$$

$$\psi(t,x,y,z) = \frac{1}{4}\ln(t)^2\_C1 + \frac{1}{4}(2\_C3\,z + 2\_C5\,x + 2\_C7\,y + 2\_C1 + 2\_C8)\ln(t) + \frac{1}{4}(x^2 + y^2 + z^2)\_C1 + \frac{1}{4}(2\_C4 + 2\_C5)\,x + \frac{1}{4}(2\_C6 + 2\_C7)\,y + \frac{1}{4}(2\_C2 + 2\_C3)\,z + \frac{1}{2}\_C8 + \frac{1}{2}\_C$$

Appendix(4): Where A=B=C=*K*

> *with*(*DifferentialGeometry*) : *with*(*Tensor*) :

> *DGsetup*([*t*, x, y, *z*], *M*)

frame name: M

> $g := evalDG(\text{dt \&t dt} - \text{K}^2\text{dx\&t dx} - \text{K}^2\text{dy \&t dy} - \text{K}^2\text{dz\&t dz})$

$$g := dt\,dt - K^2\,dx\,dx - K^2\,dy\,dy - K^2\,dz\,dz$$

M > *C2* := *Christoffel*(*g*, "SecondKind")

$$C2 := 0\,D\_t\,dt\,dt$$

Homothetic equations

M > for *_eq* in *sys2* do *_eq* end do;

$$\frac{1}{2}\_F2_t + \frac{1}{2}\_F1_x = 0$$

$$\frac{1}{2}\_F3_t + \frac{1}{2}\_F1_y = 0$$

$$\frac{1}{2}\_F3_x + \frac{1}{2}\_F2_y = 0$$

$$\frac{1}{2}\_F4_t + \frac{1}{2}\_F1_z = 0$$

$$\frac{1}{2}\_F4_x + \frac{1}{2}\_F2_z = 0$$

$$\frac{1}{2}\_F4_y + \frac{1}{2}\_F3_z = 0$$

$$\_F1_t = \psi$$

$$\_F2_x = K^2\,\psi$$

$$\_F3_y = K^2\,\psi$$

$$\_F4_z = K^2\,\psi$$

M > *pdsolve*(*sys2*)

$$\{\_F1(t,x,y,z) = \_C1\,x + \_C2\,y + \_C3\,z + \psi\,t + \_C4,\ \_F2(t,x,y,z) = K^2\,\psi\,x - \_C1\,t + \_C5\,y + \_C6\,z + \_C7,\ \_F3(t,x,y,z) = K^2\,\psi\,y - \_C2\,t - \_C5\,x + \_C8\,z + \_C9,\ \_F4(t,x,y,z) = K^2\,\psi\,z - \_C3\,t - \_C6\,x - \_C8\,y + \_C10\}$$

Killing's equations

M > *pdsolve*(*sys1*)

$$\{\_F1(t,x,y,z) = \_C1\,x + \_C2\,y + \_C3\,z + \_C4,\ \_F2(t,x,y,z) = -\_C1\,t + \_C5\,y + \_C6\,z + \_C7,\ \_F3(t,x,y,z) = -\_C2\,t - \_C5\,x + \_C8\,z + \_C9,\ \_F4(t,x,y,z) = -\_C3\,t - \_C6\,x - \_C8\,y + \_C10\}$$

Conformal equations

M > *pdsolve*

$$\_F1(t,x,y,z) = \frac{1}{2}\,\frac{1}{K^2}\Big(\big(\big((2\_C2\,z + 2\_C4\,x + 2\_C6\,y + 2\_C8)\,t + (x^2+y^2+z^2)\,\_C1 + 2\_C5\,x + 2\_C7\,y + 2\_C3\,z + 2\_C9\big)\,K^2 - \_C1\,t^2\Big)$$

$$\_F2(t,x,y,z) = \frac{1}{2}\big(\_C4\,x^2 + (2\_C2\,z + 2\_C6\,y + 2\_C8)\,x - \_C4\,(y^2+z^2)\big)\,K^2 - \frac{1}{2}\_C4\,t^2 - \_C1\,t\,x + \_C10\,y + \_C11\,z - \_C5\,t + \_C12,$$

$$\_F3(t,x,y,z) = \frac{1}{2}\big(\_C6\,y^2 + (2\_C2\,z + 2\_C4\,x + 2\_C8)\,y - \_C6\,(x^2+z^2)\big)\,K^2 - \frac{1}{2}\_C6\,t^2 - \_C1\,t\,y - \_C7\,t - \_C10\,x + \_C13\,z + \_C14$$

$$\_F4(t,x,y,z) = \frac{1}{2}\big(\_C2\,z^2 + (2\_C4\,x + 2\_C6\,y + 2\_C8)\,z - \_C2\,(x^2+y^2)\big)\,K^2 - \frac{1}{2}\_C2\,t^2 - \_C1\,t\,z - \_C11\,x - \_C13\,y - \_C3\,t + \_C15$$

$$\psi(t,x,y,z) = \frac{(\_C2\,z + \_C4\,x + \_C6\,y + \_C8)\,K^2 - \_C1\,t}{K^2}$$

Appendix(5)

> $with(DifferentialGeometry) : with(Tensor) :$

> $DGsetup([t, x, y, z], M)$

frame name: M

> $g := evalDG(dt \&t\, dt - dx \&t\, dx - dy \&t\, dy - dz \&t\, dz)$

$$g := dt\, dt - dx\, dx - dy\, dy - dz\, dz$$

M > $C2 := Christoffel(g, \text{"SecondKind"})$

$$C2 := 0\, D\_t\, dt\, dt$$

M > $K1 := KillingVectors(g);$

$$K1 := [-z\, D\_t - t\, D\_z,\ D\_t,\ z\, D\_x - x\, D\_z,\ z\, D\_y - y\, D\_z,\ -D\_z,\ -y\, D\_t - t\, D\_y,\ y\, D\_x - x\, D\_y,\ -D\_y,\ -x\, D\_t - t\, D\_x,\ -D\_x]$$

M > $LD := LieDerivative(K1, g)$

$$LD := [0\, dt\, dt,\ 0\, dt\, dt,\ 0\, dt\, dt,\ 0\, dt\, dt,\ 0\, dt\, dt,\ 0\, dt\, dt,\ 0\, dt\, dt,\ 0\, dt\, dt,\ 0\, dt\, dt,\ 0\, dt\, dt]$$

Homothetic equation's solution

$\_F1(t, x, y, z) = \_C1\, x + \_C2\, y + \_C3\, z + \psi\, t + \_C4$

$\_F2(t, x, y, z) = -\_C1\, t + \_C5\, y + \_C6\, z + \psi\, x + \_C7$

$\_F3(t, x, y, z) = -\_C2\, t - \_C5\, x + \_C8\, z + \psi\, y + \_C9$

$\_F4(t, x, y, z) = -\_C3\, t - \_C6\, x - \_C8\, y + \psi\, z + \_C10$

Killing equation's solution

$\_F1(t, x, y, z) = \_C1\, x + \_C2\, y + \_C3\, z + \_C4$

$\_F2(t, x, y, z) = -\_C1\, t + \_C5\, y + \_C6\, z + \_C7$

$\_F3(t, x, y, z) = -\_C2\, t - \_C5\, x + \_C8\, z + \_C9$

$\_F4(t, x, y, z) = -\_C3\, t - \_C6\, x - \_C8\, y + \_C10$

Conformal Killing equation's solution

$$\_F1(t, x, y, z) = -\frac{1}{2}\_C1\, t^2 + \frac{1}{2}(2\_C2\, z + 2\_C4\, x + 2\_C6\, y + 2\_C8)\, t + \frac{1}{2}(x^2 + y^2 + z^2)\_C\ldots + \_C7\, y + \_C5\, x + \_C3\, z + \_C9$$

$$\_F2(t, x, y, z) = \frac{1}{2}\_C4\, x^2 + \frac{1}{2}(-2\_C1\, t + 2\_C2\, z + 2\_C6\, y + 2\_C8)\, x + \frac{1}{2}(-t^2 - y^2 - z^2)\_C4 - \_C5\, t + \_C10\, y + \_C11\, z + \_C12,$$

$$\_F3(t, x, y, z) = \frac{1}{2}\_C6\, y^2 + \frac{1}{2}(-2\_C1\, t + 2\_C2\, z + 2\_C4\, x + 2\_C8)\, y + \frac{1}{2}(-t^2 - x^2 - z^2)\_C6 - \_C7\, t - \_C10\, x + \_C13\, z + \_C14,$$

$$\_F4(t, x, y, z) = \frac{1}{2}\_C2\, z^2 + \frac{1}{2}(-2\_C1\, t + 2\_C4\, x + 2\_C6\, y + 2\_C8)\, z + \frac{1}{2}(-t^2 - x^2 - y^2)\_C2 - \_C3\, t - \_C11\, x - \_C13\, y + \_C15,$$

$\psi(t, x, y, z) = -\_C1\, t + \_C2\, z + \_C4\, x + \_C6\, y + \_C8$

$$\frac{1}{2}\{-\_C2\, y + \_C6\, z - \_C13\} + \frac{1}{2}\{\_C2\, y - \_C6\, z + \_C13\} = 0$$

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