EDGE – ODD GRACEFULNESS OF SOME GRAPHS

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Abstract: A (p, q) connected graph is edge odd graceful if their exists an injective map f: E(G) → {1,3, ---, 2q - 1}so that the induced map $f_+ : V(G) \rightarrow \{0,1, 2, ---, 2k - 1\}$defined by $f_+(x) \equiv \sum\{f(x, y) / xy\in E\}(\text{mod } 2k)$ where the vertex x is incident with other vertex y and k = max {p, q}makes all the edges distinct and odd. When the graph admits the edge odd graceful labeling, the graph is called edge – odd graceful graph. In this article, the edge odd gracefulness of the complete bipartite graph $K_{m,n}$, where m is even and n is odd and the star graph $G = S_{2,n} + P_{m-2}$ is presented.

Index Terms: Graceful labeling, odd graceful labeling, edge odd graceful labeling, complete bipartite graph and star related graph.

I. INTRODUCTION

Many mathematicians have constructed a larger graceful graph from certain standard graphs by using various graph operations. Join and product operations are used extensively among the graphs such as paths, cycles, stars, complete graphs, complete bipartite graphs, complement of complete graphs and graceful trees etc., to get larger graceful or harmonious graph etc [1]. Sethuraman and Dhavamani (2000), Sethuraman and Kishore (1999)) are adjoined at one common edge and the resultant graphs are proved to be graceful.

1.1 Definition: A function f is called an Graceful labeling of a graph G with q edges. If f is an injection from the vertices of G to the set {0, 1, 2, ..., q} such that when each edge uv is assigned the label | f(u) – f(v)|, the resulting edge labels are distinct. A graph which admits a Graceful labeling is called a Graceful Graph.

1.2 Definition: A Graph G with 'q' edges to be Odd Graceful if there is an injection f from V(G) to {0, 1, 2, 3, ..., 2q – 1} such that when each edge xy is assigned the label | f(x) – f(y)|, the resulting edge labels are {1, 3, 5, ..., 2q – 1}. A Graph which admits an Odd Graceful labeling is called an Odd graceful graph.

1.3 Definition: A (p, q) connected graph is edge odd graceful if there exists an injective map f: E(G) → {1, 3, ---, 2q - 1}so that the induced map $f_+ : V(G) \rightarrow \{0,1, 2, ---, 2k - 1\}$defined by $f_+(x) \equiv \sum\{f(x, y) / xy\in E\}(\text{mod } 2k)$ where the vertex x is incident with other vertex y and k = max {p, q}makes all the edges distinct and odd. When the graph admits the edge odd graceful labeling, the graph is called edge – odd graceful graph.

1.4 Definition: A complete bipartite graph is a graph whose vertices can be partitioned into two subsets V_1 and V_2 such that no edge has both endpoints in the same subset, and every possible edge that could connect vertices in different subsets is part of the graph.

1.5 Definition: The m star graph $S_{m, n}$ is a tree obtained from the double star graph $S_{2, n}$ by merging a path $P_{(m-2)}$ to each of the existing n pendent vertices. It has (mn+1) vertices and mn edges.

II. MAIN RESULT

2.1 Theorem: The complete bipartite graph $K_{m,n}$, where m is even and n is odd is edge odd graceful graph.

Proof: Consider the graph $G = K_{m,n}$. It has $|V(G)| = (2^t + 2r + 1)$ vertices and $|E(G)| = 2^t(2r + 1)$ edges, where $m = 2^t$, t =1, 2, …; $n = 2r + 1$, r = 1, 2, …..

Define edge labeling $f: E(K_{m,n}) \rightarrow \{1, 3, ---, (2q - 1)\}$ as follows:

$f(e_i) = 2i - 1$, i = 1, 2, …., (2mn - 1) for $m = 2^r$, r = 1, 2, … and n is odd.

The above defined edge labeling function will induce the bijective vertex labeling function $f_+ : V(G) \rightarrow \{0, 1, 2, ---, (2k - 1)\}$ such that $f_+(x) \equiv \sum\{f(x, y) / xy \in E\}(\text{mod } 2k)$ where the vertex x is incident with other vertex y and k = max {p, q} makes all the edges distinct and odd. Hence the graph admits the edge odd graceful labeling.

2.2 Example: Edge odd graceful labeling of the complete bipartite graph $K_{2,5}$ is shown in figure 1

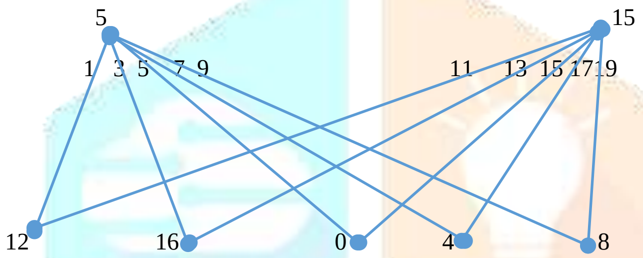

Figure 1

2.3 Lemma: The star graphs $P_9 + S_3$, $P_9 + S_5$ & $P_9 + S_{13}$ are edge odd graceful labelings and the labelings are different from the theorem 3.3.

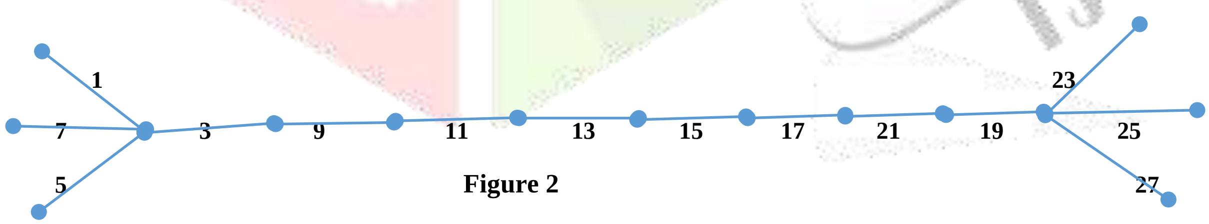

Figure 2

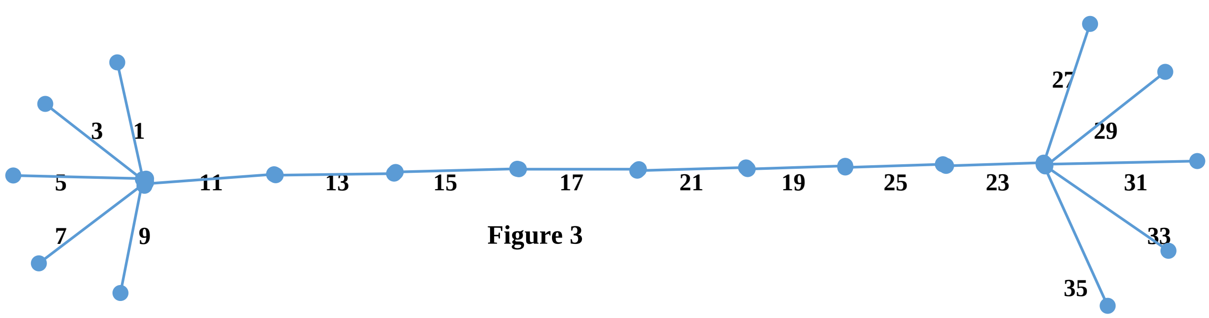

Figure 3

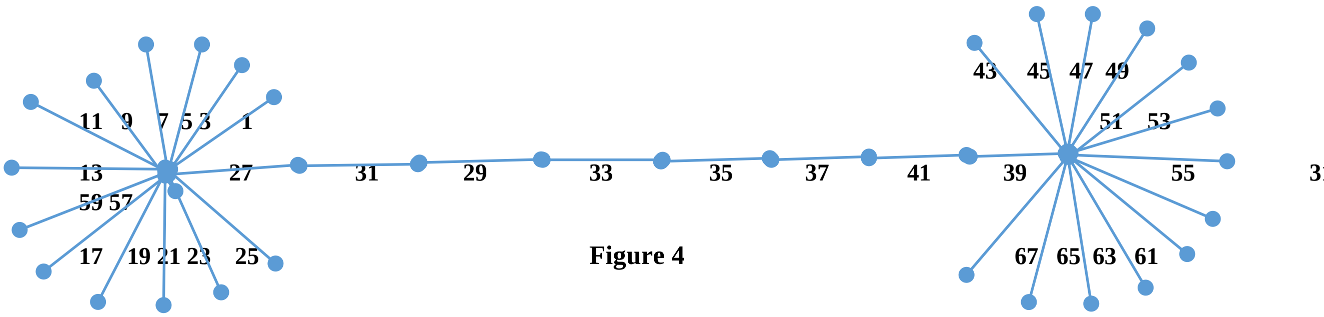

Figure 4

2.4 Theorem: The graph $G = S_{2,n} + P_{m-2}$ where m is nine and n is odd is edge odd graceful graph.

Proof: The graph $G = S_{2,n} + P_{m-2}$ has $|V(G)| = (4n + 11)$ vertices and $|E(G)| = (4n + 10)$ edges.

To define edge labeling $f: E(S_{2,n} + P_{m-2}) \rightarrow \{1, 3, ---, (2q - 1)\}$ as follows:

$f(e_i) = 2i - 1, i = 1, 2, \ldots, (4n + 11)$ for $m = 9$, and n is odd

The above defined edge labeling function will induce the bijective vertex labeling function $f_+ : V(G) \rightarrow \{0, 1, 2, ---, (2k - 1)\}$ such that $f_+(x) \equiv \sum\{f(x, y) / xy \in E\}(\text{mod } 2k)$ where the vertex x is incident with other vertex y and $k = \max\{p, q\}$ makes all the edges distinct and odd. Hence the graph admits the edge odd graceful labeling.

2.5 Example: Edge odd graceful labeling of the star graph $P_9 + S_9$ is shown in figure 5

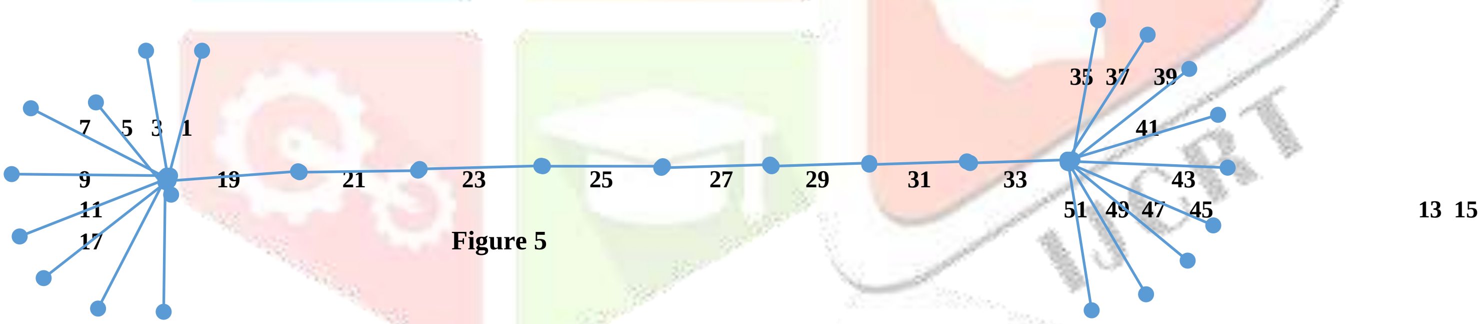

Figure 5

CONCLUSION:

In this paper Edge – Odd gracefulness of complete bipartite graph $K_{m,n}$, where m is even and n is odd and the star graph $G = S_{2,n} + P_{m-2}$ is presented.

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