Upper And Lower Bounds On The Chromatic Number Of S (n, m) Graphs.

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Abstract

In this paper we discuss about the lower and upper bounds on Chromatic number of S(n,m) graphs. We have also discussed about the Chromatic number of S(n,m) for n ≥ 2m+2, odd m ≥ 3, S(n,2) and S(n,4) graphs.

Keywords : Lower and Upper bounds, Chromatic number of S(n,m) graphs.

1. Introduction

In this paper we consider the graph S (n,m) which is a quartic graph and also both Eulerian and Hamiltonian.

The graph S (n, m)[1] consists of n vertices denoted as $v_1, v_2, \ldots\ldots v_n$. The edges are defined as follows:

i) v_i is adjacent to v_{i+1} and v_n is adjacent to v_1.
ii) v_i is adjacent to v_{i+m} if $i+m \leq n$.
iii) v_i is adjacent to v_{i+m-n} if $i+m > n$.

Definition 1.1. *A k-vertex coloring or k-coloring for short, of a graph G is an assignment of one of k available colors to each vertex 'x' of G such that adjacent vertices receive different colors. The smallest k for which a graph G admits a k-coloring is called the Chromatic Number of G and is denoted by $\chi(G)$.*

Definition 1.2. *The matrix Q = A + D, where A is the Adjacent matrix of graph G and D is the diagonal matrix whose main entries are the degrees in G, is called the Signless Laplacian of G.*

2. BOUNDS ON CHROMATIC NUMBER OF S(n,m) GRAPHS (n ≥ 2m+2)

Theorem 2. 1:The Chromatic number χ [S(n, m)] , $n \geq 2m + 2$ satisfies $1+ [\frac{4}{4-\delta_n}] \leq \chi\,[S(n, m)] \leq 5$, where δ_n is the eigenvalue of Signless Laplacian of S(n, m).

Proof . In 2011 Lima, Oliveira, Abreu and Nikiforov [2, 3] proved that $\chi[G] \geq 1+[\frac{2q}{2q-p\delta_p}]$, where G is a graph with q edges and p vertices. δ_p is the eigenvalue of Signless Laplacian of G which satisfies $\delta_1 \geq \delta_2 \geq \ldots.\ \delta_p \geq 0$.

In S(n, m) graphs, the number of edges is twice the number of vertices. i.e., Number of edges = 2n. This is illustrated by Fig.1.

No. of vertices = n = 6
No. of edges = 12

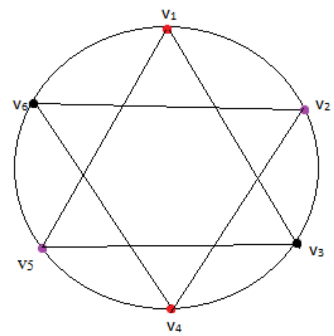

FIG.1 : S(6,2) graph

So, χ [S(n,m)] ≥ 1+[$\frac{2(2n)}{2(2n)-n\delta_n}$] . i.e., χ [S(n,m)] ≥ 1+[$\frac{4n}{4n-n\delta_n}$]. i.e., χ [S(n,m)] ≥ 1+[$\frac{4}{4-\delta_n}$].

By Greedy Coloring Theorem, χ(G) ≤ d+1, where d is the largest degree of the vertex. In S(n,m) graphs the degree of each vertex is 4 [FIG.1]and so χ [S(n,m)] ≤ 5. Therefore, 1+[$\frac{4}{4-\delta_n}$] ≤ χ [S(n,m)] ≤ 5. In particular, since, $\delta_n \geq 0$, $\frac{4}{4-\delta_n} \geq 1$ and so 1+[$\frac{4}{4-\delta_n}$] ≥ 2. So, 2 ≤ χ [S(n,m)] ≤ 5.

3. CHROMATIC NUMBER OF S(n,m) GRAPHS

Theorem 3.1. The chromatic number χ [S(n,m)] is , (i). 2 for even n ≥ 2m+2 and odd m ≥ 3 (ii) 4 for odd n ≥ 2m+2 and odd m ≥ 3.

Proof . Let $v_1, v_2, \ldots\ldots v_n$ be the vertices of the graph S(n,m) and its edges be denoted by $(v_iv_{i+1}),(v_iv_{i+m}),(v_iv_{i+n-m})$ for i= 1,2,3….. and (v_nv_1). Let the coloring set of S(n,m) be {1,2,3,….}. We define the function f from the vertex set of S(n,m) to the coloring set {1,2,3,..}as follows:

Case (i) : Even n ≥ 2m+2 and odd m ≥ 3.

$$f(v_i) = \begin{cases} 1, i-\text{odd}\ 1 \leq i \leq n \\ 2, i-\text{even}\ 1 \leq i \leq n. \end{cases}$$

Using the above pattern the graph S(n,m) for even n ≥ 2m+2 and odd m ≥ 3 admits

vertex coloring. The chromatic number χ [S(n,m)] = 2.

Case (ii) : Odd n ≥ 2m+2 and odd m ≥ 3.

$$f(v_i) = \begin{cases} 1, i-\text{odd}, & 1 \leq i \leq n-m \\ 2, i-\text{even}, & 1 \leq i \leq n-m \\ 3, i-\text{odd}, & n-(m-1) \leq i \leq n \\ 4, i-\text{even}, & n-(m-1) \leq i \leq n. \end{cases}$$

Using the above pattern S(n,m) for odd n ≥ 2m+2 and odd m ≥ 3, admits vertex coloring. The chromatic number χ [S(n,m)] = 4.

Theorem 3.2. The chromatic number of S(n,2) for n ≥ 6 is

(i) 3 for n ≡ 0 (mod 6) and n ≡ 3 (mod 6)

(ii) 4 for $n \equiv 1 \pmod 6$ and $n \equiv 4 \pmod 6$
(iii) 5 for $n \equiv 2 \pmod 6$ and $n \equiv 5 \pmod 6$.

Proof. Let $v_1, v_2, \ldots\ldots v_n$ be the vertices of the graph S(n,2) and its edges be denoted by $(v_iv_{i+1}),(v_iv_{i+2}),(v_iv_{i+n-2})$ for i= 1,2,3….. and (v_nv_1). Let f be a function that maps vertex set of S(n,2) to the coloring set {1,2,3….}.

Case (i): $n \equiv 0 \pmod 6$ and $n \equiv 3 \pmod 6$

$$f(v_i) = \begin{cases} 1, & \text{for all } i \equiv 1 \pmod 3)\ 1 \le i \le n \\ 2, & \text{for all } i \equiv 2 \pmod 3)\ 1 \le i \le n \\ 3, & \text{for all } i \equiv 0 \pmod 3)\ 1 \le i \le n \end{cases}$$

By using above pattern S(n,2) admits vertex coloring. The chromatic number χ [S(n,2)] = 3.

Case (ii) : $n \equiv 1 \pmod 6$ and $n \equiv 4 \pmod 6$

$$f(v_i) = \begin{cases} 1, & \text{for all } i \equiv 1 \pmod 3)\ 1 \le i \le n-1 \\ 2, & \text{for all } i \equiv 2 \pmod 3)\ 1 \le i \le n-1 \\ 3, & \text{for all } i \equiv 0 \pmod 3)\ 1 \le i \le n-1. \end{cases}$$

$f(v_n) = 4$. Using the above pattern S(n,2) admits vertex coloring. The chromatic number χ [S(n,2)] = 4.

Case (iii) : $n \equiv 2 \pmod 6$ and $n \equiv 5 \pmod 6$.

$$f(v_i) = \begin{cases} 1, & \text{for all } i \equiv 1 \pmod 3)\ 1 \le i \le n-2 \\ 2, & \text{for all } i \equiv 2 \pmod 3)\ 1 \le i \le n-2 \\ 3, & \text{for all } i \equiv 0 \pmod 3)\ 1 \le i \le n-2. \end{cases}$$

$f(v_{n-1}) = 4$ and $f(v_n) = 5$. Using the above pattern S(n,2) admits vertex coloring. The chromatic number χ [S(n,2)] = 5.

Theorem 3.3. The chromatic number of S(n,4), $n \ge 10$ is 3 for $n \equiv 0 \pmod 3$, $n \equiv 2 \pmod 3$ and $n \equiv 1 \pmod 3$.

Proof. Let $v_1, v_2, \ldots\ldots v_n$ be the vertices of the graph S(n,4) and its edges be denoted by $(v_i v_{i+1}),(v_i v_{i+4}),(v_iv_{i+n-4})$ for i= 1,2,3….. and (v_nv_1). Let f be a function that maps vertex set of S(n,4) to the coloring set {1,2,3….}.

Case (i) : $n \equiv 0 \pmod 3$ and $n \equiv 2 \pmod 3$.

$$f(v_i) = \begin{cases} 1, & \text{for all } i \equiv 1 \pmod 3)\ 1 \le i \le n \\ 2, & \text{for all } i \equiv 2 \pmod 3)\ 1 \le i \le n \\ 3, & \text{for all } i \equiv 0 \pmod 3)\ 1 \le i \le n \end{cases}$$

Using the above pattern S(n,4) admits vertex coloring. The chromatic number χ [S(n,4)] = 3.

Case (ii) : $n \equiv 1 \pmod 3$

$$f(v_i) = \begin{cases} 1, & \text{for all } i \equiv 1 \pmod 3)1 \le i \le n-4 \\ 2, & \text{for all } i \equiv 2 \pmod 3)1 \le i \le n-4 \\ 3, & \text{for all } i \equiv 0 \pmod 3)1 \le i \le n-4 \end{cases}$$

$f(v_{n-3}) = 2$, $f(v_{n-2}) = 3$, $f(v_{n-1}) = 1$ and $f(v_n) = 2$. Using the above pattern S(n,4) admits vertex coloring. The chromatic number χ [S(n,4)] = 3.

4. **Conclusion.** We have found the lower and upper bounds on chromatic number of S(n,m),n≥2m+2. In general χ [S(n,m)],n≥2m+2 satisfies 2≤ χ [S(n,m)] ≤ 5. The chromatic number of S(n,m),n≥2m+2, when m = 2,3,4 are also discussed.

5. References

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