INVERSE SIGNED DOMINATION OF CORONA OF A CYCLE WITH A COMPLETE GRAPH

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Abstract : Let G be a graph and let $\mathrm{f}: \mathrm{V} \to \{-1,1\}$ be a function such that $\mathrm{f}(\mathrm{N}[\mathrm{v}]) \le 0$ for every $\mathrm{v} \in \mathrm{V}$, where N[v] is the closed neighbourhood of v, then f is an inverse signed dominating function of G. The weight of f is defined as $\mathrm{w}(\mathrm{f}) = \sum_{\mathrm{v} \in \mathrm{V}} \mathrm{f}(\mathrm{v})$. In this paper we determine the inverse signed domination number and inverse signed total domination number of corona of a cycle and a complete graph.

Index Terms - **Corona of two graphs, inverse signed dominating function, inverse signed domination number and inverse signed total domination number.**

I. INTRODUCTION

Let $G = (V, E)$ be a graph. A subset S of V is called a dominating set of G if every vertex in $V - S$ is adjacent to at least one vertex in S.

A function $f: V \to \{0,1\}$ is called a dominating function of G if $\sum_{u \in N[v]} f(u) \ge 1$ for every $v \in V$. Dominating function is a natural generalization of dominating set. If S is a dominating set, then the characteristic function is a dominating function.

Let $G = (V, E)$ be a graph. A function $f: V \to \{-1,1\}$ is called a signed dominating function (SDF) of G if $f(N[v]) = \sum_{u \in N[v]} f(u) \ge 1$, for each $v \in V$. The weight of f is called a signed domination number of G and it is denoted by $\gamma_s(G)$.

The concept of inverse signed total domination numbers of some kinds of graphs was introduced by Huang et.al[2]. Let $G = (V, E)$ be a simple graph. A function is said to be an inverse signed dominating function if the sum of its functional values over any closed neighbourhood is atmost one. If the sum of its functional values over any open neighbourhood is atmost one then the corresponding function is called an inverse signed total dominating function of a G.

II. CORONA OF TWO GRAPHS

Frucht and Harary [1] introduced the concept of corona of two graphs in 1969. It is new and simple operation on two graphs G_1 and G_2 with the property that the group of the two graphs is isomorphic with the wreath product of the groups of G_1 and G_2.

Let $G_1(V_1, E_1)$ and $G_2(V_2, E_2)$ be two graphs. The corona $G_1 \odot G_2$ is defined as the graph G obtained by taking one copy of G_1 and $|V_1|$ copies of G_2 and then joining i^{th} vertex of G_1 to every vertex in the i^{th} copy of G_2.

In this paper we consider the corona of a cycle with a complete graph $C_n \odot K_m$ and some results on inverse signed domination and inverse signed total domination of this graph are discussed.

Siva Parvathi M [3] discussed some properties of a corona graph $C_n \odot K_m$.

Theorem 2.1: The degree of vertex v in $G = C_n \odot K_m$ is given by

$$d(v) = \begin{cases} m+2, & if\ v \in C_n \\ m, & if\ v \in K_m \end{cases}.$$

Theorem 2.2: The number of vertices and edges in $G = C_n \odot K_m$ is given respectively by

1. $|V(G)| = n(m+1)$
2. $|E(G)| = \frac{n}{2}(m^2 + m + 2)$.

Let us denote the vertices of a cycle C_n in $G = C_n \odot K_m$ by $u_1, u_2, \dots, u_n$ and the vertices of i^{th} copy of K_m by $v_{i1}, v_{i2}, \dots, v_{im}$ for $i = 1$ to n.

III. INVERSE SIGNED DOMINATING FUNCTIONS

Let $f: V \to \{-1,1\}$ is said to be an inverse signed dominating function of G if $f(N[v]) = \sum_{u \in N[v]} f(u) \le 0$ for all $v \in V$ and the maximum weight of f is called the inverse signed domination number of G it is denoted by $\gamma_s^0(G)$.

In this section some results on inverse signed dominating functions and of corona of a cycle with a complete graph are discussed and also inverse signed domination number is determined.

Theorem 3: The inverse signed domination number of a graph $G = C_n \odot K_m$ is

$$\gamma_s^0(C_n \odot K_m) = \begin{cases} -n, & m \text{ is even} \\ 0, & m \text{ is odd} \end{cases}$$

Proof: Let G be a corona graph $C_n \odot K_m$ with vertex set V.
Case I: Suppose that m is even.
Let $f: V \to \{-1,1\}$ be a function defined as

$f(u_i) = -1$ for all i and $f(v_{ij}) = \begin{cases} 1, & m \equiv 1 \ (mod\ 2) \\ -1, & m \equiv 0 \ (mod 2) \end{cases}$.

The summation value taken over $N[v]$of $v \in V$ is as follows.
Case 1: Let $u_i \in C_n$ be such that $d(u_i) = m + 2$.
Here $N[u_i]$ contains 3 vertices of C_n and m vertices of K_m.
Then $\sum_{w \in N[u_i]} f(w) = -1 - 1 - 1 + \left(\frac{m}{2}(1) + \frac{m}{2}(-1)\right) = -3$.
Case 2: Let $v_{ij} \in K_m$ be such that $d(v_{ij}) = m$, where $i = 1$ to n and $j = 1$ to m.
Here $N[v_{ij}]$ contains one vertex of C_n and m vertices of K_m.
Then $\sum_{w \in N[v_{ij}]} f(w) = -1 + \left(\frac{m}{2}(1) + \frac{m}{2}(-1)\right) = -1$.
Therefore all possibilities, we get $\sum_{w \in N[v]} f(w) \leq 0$ for all $v \in V$.
Hence f is an inverse signed dominating function of G.

Since $\sum_{w \in N[v_{ij}]} f(w) = -1$, the labelling is maximum with respect to the vertices $v_{i1}, v_{i2}, \dots, v_{im}$. If at least one vertex $u_i = 1$, then $\sum_{w \in N[v_{ij}]} f(w) = 1$.

It is easy to observe that $\sum_{v \in V} f(v) = -n$ is maximum for this particular inverse signed dominating function.

Therefore $\gamma_s^0(C_n \odot K_m) = n(-1) + n\left(\frac{m}{2}(-1) + \frac{m}{2}(1)\right) = -n$.
Case II: Suppose that m is odd.
Let $f: V \to \{-1,1\}$ be a function defined as

$f(u_i) = -1$ for all i and $f(v_{ij}) = \begin{cases} 1, & m \equiv 1 \ (mod\ 2) \\ -1, & m \equiv 0 \ (mod 2) \end{cases}$.

The summation value taken over $N[v]$of $v \in V$ is as follows.
Case 1: Let $u_i \in C_n$ be such that $d(u_i) = m + 2$.
Then $\sum_{w \in N[u_i]} f(w) = -1 - 1 - 1 + \left(\frac{m+1}{2}(1) + \frac{m-1}{2}(-1)\right) = -2$.
Case 2: Let $v_{ij} \in K_m$ be such that $d(v_{ij}) = m$, where $i = 1\ to\ n$, $j = 1\ to\ m$.
Then $\sum_{w \in N[v_{ij}]} f(w) = -1 + \left(\frac{m+1}{2}(1) + \frac{m-1}{2}(-1)\right) = 0$.
Therefore all possibilities, we get $\sum_{w \in N[v]} f(w) \leq 0$ for all $v \in V$.
Hence f is an inverse signed dominating function of G.

Since $\sum_{w \in N[v_{ij}]} f(w) = 0$, the labelling is maximum with respect to the vertices $v_{i1}, v_{i2}, \dots, v_{im}$. If at least one $u_i = 1$, then $\sum_{w \in N[v_{ij}]} f(w) = 2$. It is easy to observe that $\sum_{v \in V} f(v) = 0$ is maximum for this particular inverse signed dominating function.

Therefore $\gamma_s^0(C_n \odot K_m) = n(-1) + n\left(\frac{m-1}{2}(-1) + \frac{m+1}{2}(1)\right) = 0$.

IV. INVERSE SIGNED TOTAL DOMINATING FUNCTIONS

Let $f: V \to \{-1,1\}$ is said to be a inverse signed total dominating function of G if $f(N(v)) = \sum_{u \in N(v)} f(u) \leq 0$ for all $v \in V$ and the maximum weight of f is called the inverse signed total domination number of G it is denoted by $\gamma_{st}^0(G)$. In this section inverse signed total dominating functions and inverse signed total domination number of $C_n \odot K_m$ is determined.

Theorem 4: The inverse signed total domination number of a graph $G = C_n \odot K_m$ is

$$\gamma_{st}^{0}(C_n \odot K_m) = \begin{cases} -n, m \text{ is even} \\ -2n, m \text{ is odd} \end{cases}$$

Proof: Let G be a corona graph $C_n \odot K_m$ with vertex set V.
Case I: Suppose that m is even.
Let $f: V \to \{-1,1\}$ be a function defined as
$f(u_i) = -1$ for all i and $f(v_{ij}) = \begin{cases} 1, & m \equiv 1 \ (mod\ 2) \\ -1, & m \equiv 0 \ (mod 2) \end{cases}$.
The summation value taken over $N(v)$ of $v \in V$ is as follows.
Case 1: Let $u_i \in C_n$ be such that $d(u_i) = m + 2$.
Here $N(u_i)$ contains 2 vertices of C_n and m vertices of K_m.
Then $\sum_{w \in N(u_i)} f(w) = -1 - 1 + \left(\frac{m}{2}(1) + \frac{m}{2}(-1)\right) = -2$.
Case 2: Let $v_{ij} \in K_m$ be such that $d(v_{ij}) = m$, where $i = 1\ to\ n, j = 1\ to\ m$.
Here $N(v_{ij})$ contains one vertex of C_n and $(m-1)$ vertices of K_m.
If j is odd, then $\sum_{w \in N(v_{ij})} f(w) = -1 + \left(\frac{m}{2}(1) + \frac{m}{2}(-1)\right) - 1 = -2$.
If j is even, then $\sum_{w \in N(v_{ij})} f(w) = -1 + \left(\frac{m}{2}(1) + \frac{m}{2}(-1)\right) + 1 = 0$.
Therefore all possibilities, we get $\sum_{w \in N(v)} f(w) \leq 0$ for all $v \in V$.
Hence f is an inverse signed total dominating function of G.

Since $\sum_{w \in N(v_{ij})} f(w) = 0$, the labelling is maximum with respect to the vertices $v_{i2}, v_{i4}, \ldots, v_{im}$. If at least one $u_i = 1$, then $\sum_{w \in N(v_{ij})} f(w) = 2$. It is easy to observe that $\sum_{v \in V} f(v) = -n$ is maximum for this particular inverse signed total dominating function.

Therefore $\gamma_{st}^{0}(C_n \odot K_m) = n(-1) + n\left(\frac{m}{2}(-1) + \frac{m}{2}(1)\right) = -n$.
Case II: Suppose that m is odd.
Let $f: V \to \{-1,1\}$ be a function defined as
$f(u_i) = -1$ for all i and $f(v_{ij}) = \begin{cases} -1, & m \equiv 1 \ (mod\ 2) \\ 1, & m \equiv 0 \ (mod 2) \end{cases}$.
The summation value taken over $N(v)$ of $v \in V$ is as follows.
Case 1: Let $u_i \in C_n$ be such that $d(u_i) = m + 2$.
Then $\sum_{w \in N(u_i)} f(w) = -1 - 1 + \left(\frac{m+1}{2}(-1) + \frac{m-1}{2}(1)\right) = -3$.
Case 2: Let $v_{ij} \in K_m$ be such that $d(v_{ij}) = m$, where $i = 1\ to\ n, j = 1\ to\ m$.
If j is odd, then $\sum_{w \in N(v_{ij})} f(w) = -1 + \left(\frac{m+1}{2}(-1) + \frac{m-1}{2}(1)\right) + 1 = -1$.
If j is even, then $\sum_{w \in N(v_{ij})} f(w) = -1 + \left(\frac{m+1}{2}(-1) + \frac{m-1}{2}(1)\right) - 1 = -3$.
Therefore all possibilities, we get $\sum_{w \in N(v)} f(w) \leq 0$ for all $v \in V$.
Hence f is an inverse signed total dominating function of G.

Since $\sum_{w \in N(v_{ij})} f(w) = -1$, the labelling is maximum with respect to the vertices $v_{i1}, v_{i3}, \ldots, v_{im}$. If at least one $u_i = 1$, then $\sum_{w \in N(v_{ij})} f(w) = 1$. It is easy to observe that $\sum_{v \in V} f(v) = -2n$ is maximum for this particular inverse signed dominating function.

Therefore $\gamma_{st}^{0}(C_n \odot K_m) = n(-1) + n\left(\frac{m-1}{2}(1) + \frac{m+1}{2}(-1)\right) = -2n$.

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