A NEW APPROACH TO DETECT ISOMORPHISM AMONG KINEMATIC CHAINS USING LINK-JOINT-LOOP ADJACENCY

[1]SANKALP VERMA, [2]Dr. P. B. DESHMUKH
[1]Research Scholar, [2]Director
[1]Department of Mechanical Engineering
[1]Shri Shankaracharya Group of Institution, Bhilai, CG, India

Abstract : Motion transmission to meet the specific requirement is an important aspect of mechanical engineering. Linkages, gear-trains, cams, etc. are commonly adopted to fulfill this need. Kinematic analysis and synthesis of linkages, gear-trains, etc. is a major area of study in mechanical engineering. Structural synthesis of all the distinct possible kinematic chains with the specified number of links and degrees of freedom is necessary in order to select the best possible chain for the specified task at the conceptual stage of design. The structural synthesis should not end only with the generation of distinct chains as it does not lead to logical conclusion unless the capabilities of the structures are explored. The accuracy of generation of kinematic chain depends upon the accuracy for the test for isomorphism. Consequently, some methods are developed to test isomorphism and many of these methods fail at some stage or the other. Most of the methods in structural analysis have been reported to test isomorphism and inversions, but the methods reported so far is either based on link-link adjacency, link-joint adjacency or link-loop adjacency. But all the three important features of kinematic chains viz. link-joint-loop have not been considered for the exact assessment. Keeping in view, quantitative methods are being developed in this paper to detect isomorphism among kinematic chains which includes link-joint-loop adjacency. The method developed for testing isomorphism and other characteristics will result in many aspect e.g. reliability, computational time, efficiency, etc. Since, all the three features of kinematic chains viz. link-joint-loop are considered to analyze the structural aspect and its relation to function; the expected outcome will reveal some kinematic characteristics of the chain with high degree of reliability and efficiency for application in planar manipulators from the viewpoint of workspace.

***IndexTerms* - Kinematic analysis, Synthesis of linkages, Structural synthesis, Isomorphism**

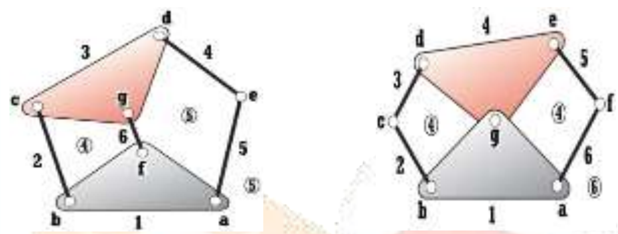
I. INTRODUCTION

In mechanical engineering design, the synthesis and analysis of kinematic chains is an important factor and hence a lot of researchers have made their efforts in studying the various aspects of mechanisms. In that context, detection of isomorphism among kinematic chains is very essential for which time and effort is dedicated for evolving a technique which is trustworthy and proficient. In the early 70’s, Uicker and Raicu [1] presented characteristic polynomial approach for detecting isomorphism of two kinematic chains. Later, Mruthyunjaya and Balasubramanian [2] proposed a degree matrix method while Ambekar and Agrawal [3] developed minimum code for testing of isomorphism. Agrawal and Rao [4] attempted to develop computationally simple and efficient analytical methods using matrices. Rao and Raju [5] developed the hamming number technique while Hwang and Hwang [6] detected isomorphism by degree codes of contracted link adjacency matrices of kinematic chains. Rao and Rao [7] compared loop hamming values of links and chains for testing of isomorphism. Chu and Cao [8] originated link's adjacent-chain table to identify isomorphism. Shende and Rao [9] applied graph theory to convert kinematic chains into their equivalent graphs and compared for isomorphism. Tischler et. al. [10] presented a new orderly method for synthesizing kinematic chains. A computer aided method is proposed by Yadav et. al. [11, 12, 13] for detecting isomorphism using a new invariant called the arranged sequence of modified total distance ranks of all the links using the concept of modified distance, link-link multiplicity distance and link-path code. Quantitative methods based on link assortment, joint assortment and loop type and their adjacency is presented by Rao and Anne [14]. A new method based on an artificial neural network (ANN) technique is presented by Kong and Zhang [15] to identify the isomorphism. Rao [16] utilized fuzzy logic to investigate isomorphism. Rao and Pathapati [17] reported loop concept to reveal simultaneously chain is isomorphic, link is isomorphic, and type of freedom with no extra computational effort. Rao [18] presented genetic algorithm for testing isomorphism. Rao and Deshmukh [19] developed a method which obviates the test of isomorphism thus resulting in saving of time, space and effort to identify the distinct chains. Chang et. al. [20] developed a new method based on eigenvectors and eigenvalues. Mruthyunjaya presented a broad review of the extensive literature available on the subject kinematic structure with a view to trace its history highlighting major trends and discussing significant contributions. Xiao et. al. [22] presented two novel evolutionary approaches – ant algorithm (AA) and artificial immune system (AIS) to identify isomorphism. Srinath and Rao [23] used concept of correlation while Ding and Huang [24] presented perimeter topological graph to detect isomorphism. Bal, Deshmukh and Jagadeesh [25] defined link invariant functions based on distance matrix and loops of a kinematic chain. Zeng et. al. [26] proposed a fast deterministic algorithm called the dividing and matching algorithm (DMA) and representing a kinematic chain uniquely by a graph.to identify isomorphism. Rizvi et. al. [27] presented a new algorithm using adjacency matrices for determining the possible distinct inversions from a kinematic chain. Romaniak [28] reviewed the methods for identifying the isomorphism of kinematic chains suggested by researchers is contained in this study, including hamming number technique, eigenvalues and eigenvectors, perimeter graphs, dividing and matching vertices. Kamesh et. al. [29] proposed a novel and simple algorithm based on graph theory by which elimination of isomorphic chains can be done. Shukla et. al. [30] proposed gradient matrices, based on gradient analogy, to distinctly denote the structure of each kinematic chain which is applied successfully for detection of isomorphism and distinct inversions to all known families of kinematic chains i.e. single degree of freedom, multi degree of freedom, and detection of isomorphism in graphs.

Most of the methods in structural analysis have been reported to test isomorphism and inversions, but the methods reported so far is either based on link-link adjacency, link-joint adjacency or link-loop adjacency. But all the three important features of kinematic chains viz. link-joint-loop have not been considered for the exact assessment. Keeping in view, an attempt has been made in this paper to develop a method to detect isomorphism among kinematic chains which includes link-joint-loop adjacency thereby increasing the uniqueness.

II. REPRESENTATION OF KINEMATIC CHAIN

Consider Stephenson's and Watt's chain consisting of six links (two ternary and four binary) with one degree of freedom having seven revolute joints as shown in Fig. 1.



(a) **(b)**

Fig. 1: (a) Stephenson's Chain and (b) Watt's Chain

The links and joints are having their usual notations while the loops are identified by considering only the outermost loop and sub-loops of a kinematic chain and are numbered according to the number of links occurring in the loops. Thus, it can be observed that the Stephenson's chain has a sub-loop b-c-g-f having 4 links (two ternary and two binary links), another sub-loop a-f-g-d-e having 5 links (two ternary and three binary links) and an outermost loop a-b-c-d-e having 5 links (two ternary and three binary links) while the Watt's chain has a sub-loop b-c-d-g having 4 links (two ternary and two binary links), another sub-loop a-g-e-f having 4 links (two ternary and two binary links) and an outermost loop a-b-c-d-e-f having 6 links (two ternary and four binary links).

III. STRING VALUES AND LINK-JOINT-LOOP (LJL) VALUE

A three dimensional matrix is being constructed for Stephenson's chain showing the interconnection between all the links with the revolute joints along with the loops resulting in strings values and finally the link-joint-loop (LJL) value for a given kinematic chain. The matrix shows connection between the links and the joints along with the loop values. For each link the joint values are taken for direct joints while for other joints, the shortest distances are taken and entered in the matrix. A binary link has a joint value of 2 since it involves two design parameters and can transfer motion only to two of its adjacent joints. Similarly, a ternary joint has a joint value of 3 since it involves three design parameters and can transfer motion to three of its adjacent joints. Likewise, a quaternary joint will have a joint value of 4. Link 1 is a ternary link which is having its direct joints at a, b and f while the other joints i.e. c, d, e, and g are at various joint distances from link 1. Now since the link 1 (ternary link) is directly connected to link 2 (binary link) at joint b, therefore its joint value is taken as 3 because when a ternary link joins with a binary link it has 3 remaining joints which can be further connected to other links. Hence, the joint value for 1-b is written as 3 in the matrix. Similarly, the joint value for 1-f (ternary link 1 joining with binary link 6) and for 1-a (ternary link 1 joining with binary link 5) also comes out as 3. Now for other remaining joints, the shortest distance from joints of link 1 is taken. In present case, the shortest distance for joints of link 1 with joint c is 2. Similarly, for 1-d is 3, for 1-e is 2 and for 1-g is 2 which are tabulated in the matrix. Now, for loop values of link 1, we have to consider its direct joints a, b and f which are participating in the loops. Now for 1-a, we can observe that joint a is participating in sub-loop a-f-g-d-e having 5 links and also in the outermost loop a-b-c-d-e having 5 links. Hence, the loop value for 1-a comes out to be their addition i.e. 10. Similarly, loop values for 1-b and for 1-f is 9. Since the joints c, d, e and g does not occur in link 1, therefore their loop values are taken as 0. A similar procedure is adopted for all the links in a chain in connection with their joints and their participation in loops and a link-joint-loop adjacency matrix is constructed as shown in Table 1.

Tab. 1: Matrix for finding String Values & LJL Values of Stephenson Chain

		Joints							
		a	**b**	**c**	**d**	**e**	**f**	**g**	**String Value**
Li	**1**	3	3	2	3	2	3	2	**18**
		10	9	0	0	0	9	0	**28**

	2	2	3	3	2	3	2	2	**17**
		0	9	9	0	0	0	0	**18**
	3	3	2	3	3	2	2	3	**18**
		0	0	9	10	0	0	9	**28**
	4	2	3	2	3	2	3	2	**17**
		0	0	0	10	10	0	0	**20**
	5	3	2	3	2	2	2	3	**17**
		10	0	0	0	10	0	0	**20**
	6	2	2	2	2	3	3	3	**17**
		0	0	0	0	0	9	9	**18**

Links String Values						
1	**2**	**3**	**4**	**5**	**6**	**LJL Value**
18	17	18	17	17	17	**104**
28	18	28	20	20	18	**132**

Now the string value of a particular link in relation with the joint and loop values is calculated by summation of the numerators and the denominators separately for computation purpose only as shown below:

$$\text{String Value} = \frac{\text{Summation of joint values}}{\text{Summation of loop values}}$$

i.e. for Link 1, String Value = $\frac{3+3+2+3+2+3+2}{10+9+0+0+0+9+0} = \frac{18}{28}$

Here the numbers 18 and 28 are not computed as fraction but individually shows respective joint and loop values of Link 1.

Now the link-joint-loop (LJL) value of a particular kinematic chain is calculated by the summation of all the joint values of all the links along with summation of all the loop values of all the links participating in the formation of loop, separately, as shown below:

LJL Value = $\frac{\text{Sum of all joint values of all the Links}}{\text{Sum of all loop values participating in the formation of loops}}$

i.e. LJLV = $\frac{18+17+18+17+17+17}{28+18+28+20+20+18} = \frac{104}{132}$

Hence, for Stephenson's chain, the LJL value comes out to be **104/132**. Alternately, a schematic presentation of LJL Value comprising joint and loop values is shown below:

LJLV = **[104/132]** {4(17), 2(18), 2(18), 2(20), 2(28)}

IV. DETECTION OF ISOMORPHISM:

The above mentioned procedure is applied for all the available chains with fixed number of links and degree of freedom. It is observed that two isomorphic chains shows same LJL values as well as string values however distinct chains have different LJL values and if somehow their LJL values are same, their string values are different. Application of the concept is illustrated with the help of several examples.

The method is applied to well known cases of kinematic chains up to 10-links and having one, two and three degrees of freedom. All the 16 eight bar 1-dof chains, 40 nine-bar 2-dof chains, 230 ten-bar 1-dof chains and 98 ten-bar 3-dof chains have been tested for isomorphism. All of them have yielded distinct *LJL values or string values*. The results in the form schematic presentation of kinematic chains with 8-links 1-dof and with 9-links 2-dof is included in the appendix I and II. An example of chains with higher number of links cited from literature sources is also included in the present work to establish its reliability.

Example 1: It has been reported that there are two distinct chains available with 6 links and 1-dof i.e. Stephenson's chain and Watt's chain [3] shown in Fig. 1. Hence, their string values and LJL values are tabulated below in Table 2 which shows that these two chains have different LJL values and are non-isomorphic.

Tab. 2: String Values & LJL Values of Stephenson & Watt Chain

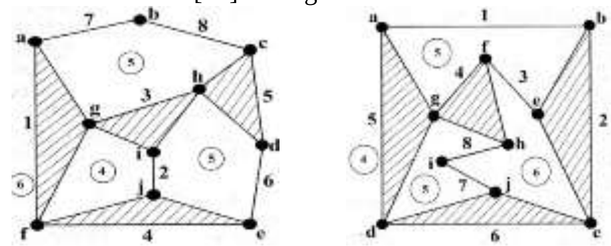
Chain	**String Value**						**LJL Value**
Stephenson Chain	18	17	18	17	17	17	**104**
	28	18	28	20	20	18	**132**
Watt Chain	18	17	17	18	17	17	**104**
	28	20	20	28	20	20	**136**

The schematic presentation of above chains will be:

$LJLV_{SC}$ = **[104/132]** {4(17), 2(18), 2(18), 2(20), 2(28)}

$LJLV_{WC}$ = **[104/136]** {4(17), 2(18), 4(20), 2(28)}

Example 2: Consider two isomorphic kinematic chains [21] having 8 links with 1-dof as shown in Fig. 2.



(a) **(b)**

Fig. 2: A pair of 8-links, 1-dof isomorphic chains [21]

Now the string values and LJL values for pair of kinematic chains shown in Fig. 2 are tabulated in Tables 3 and 4.

Tab. 3: String Values & LJL Values of chain in Fig. 2(a)

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**LJL Value**
27	27	27	27	27	26	26	27	**214**
30	18	28	30	32	22	22	22	**204**

Tab. 4: String Values & LJL Values of chain in Fig. 2(b)

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**LJL Value**
27	27	26	27	27	27	26	27	**214**
18	30	22	32	28	30	22	22	**204**

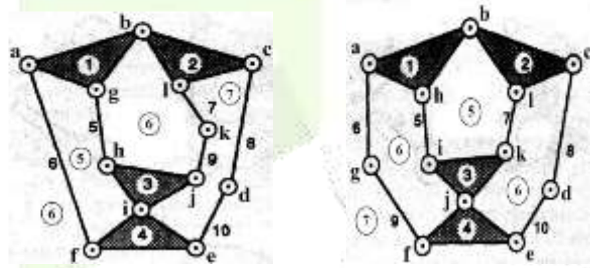
Alternately, schematic presentation will be:

$LJLV_{2a}$ = **[214/204]** {1(18), 3(22), 2(26), 6(27), 1(28), 2(30), 1(32)}

$LJLV_{2b}$ = **[214/204]** {1(18), 3(22), 2(26), 6(27), 1(28), 2(30), 1(32)}

From above tables, it can be observed that both the chains have same LJL values as well as string values and thus are isomorphic.

Example 3: Consider two isomorphic kinematic chains [10] having 10 links with 3-dof as shown in Fig.3.



(a) **(b)**

Fig. 3: A pair of 10-links, 3-dof isomorphic chains [10]

Now the string values and LJL values for pair of kinematic chains shown in Fig. 3 are tabulated in Tables 5 and 6.

Tab. 5: String Values & LJL Values of chain in Fig. 3(a)

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**LJL Value**
33	34	34	34	33	33	34	34	34	34	**337**
34	38	36	36	22	22	26	26	26	26	**292**

Tab. 6: String Values & LJL Values of chain in Fig. 3(b)

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**LJL Value**
34	34	33	34	33	34	33	34	34	34	**337**
36	36	34	38	22	26	22	26	26	26	**292**

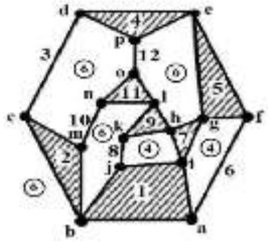
The schematic presentation will be:

$LJLV_{3a}$ = **[337/292]** {2(22), 4(26), 3(33), 8(34), 2(36), 1(38)}

$LJLV_{3b}$ = **[337/292]** {2(22), 4(26), 3(33), 8(34), 2(36), 1(38)}

Again, it can be observed that both the chains have same LJL values as well as string values and thus are isomorphic.

Example 4: Consider two kinematic chains [25] having 12 links with 1-dof as shown in Fig. 4. The chains possess identical characteristic polynomial of their degree matrices.



(a) (b)

Fig. 4: A pair of 12-link, 1-dof kinematic chains possessing identical characteristic polynomial (Degree Matrix)

Now the string values and LJL values for pair of kinematic chains shown in Fig. 4 are tabulated in Tables 7 and 8.

Tab. 7: String Values & LJL Values of chain in Fig. 4(a)

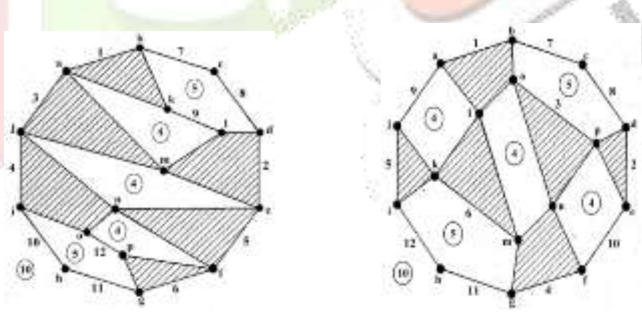
1	2	3	4	5	6	7	8	9	10	11	12	LJL Value
50	48	48	47	47	49	48	49	47	48	47	48	**576**
40	36	24	36	32	20	28	20	32	24	36	24	**352**

Tab. 8: String Values & LJL Values of chain in Fig. 4(b)

1	2	3	4	5	6	7	8	9	10	11	12	LJL Value
50	51	47	45	47	51	47	49	49	49	47	48	**580**
40	20	32	36	32	20	32	24	36	24	32	24	**352**

The schematic presentation will be:
$LJLV_{4a}$ = **[576/352]** {2(20), 3(24), 1(28), 2(32), 3(36), 1(40), 4(47), 5(48), 2(49), 1(50)}
$LJLV_{4b}$ = **[580/352]** {2(20), 3(24), 4(32), 2(36), 1(40), 1(45), 4(47), 1(48), 3(49), 1(50), 2(51)}
From above tables it can be observed that the string values and LJL values of both the chains are different from each other and hence are non-isomorphic.

Example 5: Consider a pair of kinematic chains having 12 links with 1-dof as shown in Fig.5. The chains possess identical characteristic polynomial of their degree matrices.



(a) (b)

Fig. 5: A pair of 12-link, 1-dof kinematic chains

Now the string values and LJL values for pair of kinematic chains shown in Fig. 5 are tabulated in Tables 9 and 10.

Tab. 9: String Values & LJL Values of chain in Fig. 5(a)

1	2	3	4	5	6	7	8	9	10	11	12	LJL Value
52	49	49	49	49	52	55	53	56	53	55	56	**628**
38	46	36	46	36	38	30	30	18	30	30	18	**396**

Tab. 10: String Values & LJL Values of chain in Fig. 5(b)

1	2	3	4	5	6	7	8	9	10	11	12	LJL Value

49	52	50	49	52	50	53	54	53	53	53	54	**622**
46	38	26	46	38	26	30	30	28	28	30	30	**396**

The schematic presentation will be:

$LJLV_{5a}$ = **[628/396]** {2(18), 4(30), 2(36), 2(38), 2(46), 4(49), 2(52), 2(53), 2(55), 2(56)}

$LJLV_{5b}$ = **[622/396]** {2(26), 2(28), 4(30), 2(38), 2(46), 2(49), 2(50), 2(52), 4(53), 2(54)}

From above tables it can be observed that the string values and LJL values of both the chains are different from each other and hence the chains are distinct in nature.

V. CONCLUSION

Unified quantitative methods, simple and less time consuming, are desirable to compare the chains quantitatively at the conceptual stage without carrying the dimensional synthesis of design for characteristics like static and dynamic behavior, workspace, etc. Most of the methods in structural analysis have been reported to test isomorphism, but the methods reported so far are either based on link-link adjacency, link-joint adjacency or link-loop adjacency. But all the three important features of kinematic chains viz. link-joint-loop have not been considered for the exact assessment. Keeping in view, quantitative methods are developed to detect isomorphism and to compare chains for the specified task which includes link-joint-loop adjacency.

REFERENCES

[1] Uicker, J. J., and Raicu, A., 1975, "A Method for the Identification of and Recognition of Equivalence of Kinematic Chains," Mech. Mach. Theory, **10**(5), pp. 375−383.

[2] Mruthyunjaya, T. S., and Balasubramanian, H. R., 1987, "In Quest of a Reliable and Efficient Computational Test for Detection of Isomorphism in Kinematic Chains," Mech. Mach. Theory, **22**(2), pp. 131−139.

[3] Ambekar, A. G., and Agrawal, V. P., 1987, "Canonical Numbering of Kinematic Chains and Isomorphism Problem: Min Code," Mech. Mach. Theory, **22**(5), pp. 453−461.

[4] Agrawal, V. P., and Rao, J. S., 1989, "Identification and Isomorphism of Kinematic Chains and Mechanisms," Mech. Mach. Theory, **24**(4), pp. 309−321.

[5] Rao, A. C., and Raju, D. V., 1991, "Application of the Hamming Number Technique to Detect Isomorphism Among Kinematic Chains and Inversions," Mech. Mach. Theory, **26**(1), pp. 55–75.

[6] Hwang, W.-M., and Hwang, Y.-W., 1992, "Computer-Aided Structural Synthesis of Planar Kinematic Chains with Simple Joints," Mech. Mach. Theory, **27**(2), pp. 189–199.

[7] Rao, A. C., and Rao, C. N., 1993, "Loop Based Pseudo Hamming Values−I: Testing Isomorphism and Rating Kinematic Chains," Mech. Mach. Theory, **28**(1), pp. 113−127.

[8] Chu, J.-K., and Cao, W.-Q., 1994, "Identification of Isomorphism among Kinematic Chains and Inversions using Link's Adjacent-Chain-Table," Mech. Mach. Theory, **29**(1), pp. 53−58.

[9] Shende, S., and Rao, A. C., 1994, "Isomorphism in Kinematic Chains," Mech. Mach. Theory, **29**(7), pp. 1065−1070.

[10] Tichler, C. R., Samuel, A. E., and Hunt, K. H., 1995, "Kinematic Chains for Robot Hands-I: Orderly Number Synthesis," Mech. Mach. Theory, **30**(8), pp. 1193−1215.

[11] Yadav, J. N., Pratap, C. R., and Agrawal, V. P., 1996, "Computer Aided Detection of Isomorphism among Kinematic Chains and Mechanisms using the Concept of Modified Distance," Mech. Mach. Theory, **31**(4), pp. 439−444.

[12] Yadav, J. N., Pratap, C. R., and Agrawal, V. P., 1996, "Mechanisms of a Kinematic chain and the Degree of Structural Similarity based on the Concept of Link-Path Code," Mech. Mach. Theory, **31**(7), pp. 865−871.

[13] Yadav, J. N., Pratap, C. R., and Agrawal, V. P., 1996, "Computer Aided Detection of Isomorphism among Binary Chains using the Link-Link Multiplicity Distance Concept," Mech. Mach. Theory, **31**(7), pp. 873−877.

[14] Rao, A. C., and Anne, J., 1998, "Topology based Characteristics of Kinematic Chains: Work Space, Rigidity, Input-Joint and Isomorphism," Mech. Mach. Theory, **33**(5), pp. 625−638.

[15] Kong, F. G., Li, Q., and Zhang, W. J., 1999, "An Artificial Neural Network Approach to Mechanism Kinematic Chain Isomorphism Identification, Mech. Mach. Theory, **34**(2), pp. 271–283.

[16] Rao, A. C., 2000, "Application of Fuzzy Logic for the Study of Isomorphism, Inversions, Symmetry, Parallelism and Mobility in Kinematic Chains," Mech. Mach. Theory, **35**(8), pp. 1103–1116.

[17] Rao, A. C., and Pathapati, V. V. N. R. P. R., 2000, "Loop based Detection of Isomorphism Among Chains, Inversions and Type of Freedom in Multi Degree of Freedom Chain," ASME J. Mech. Des., **122**(1), pp. 31–42.

[18] Rao, A. C., 2000, "A Genetic Algorithm for Topological Characteristics of Kinematic Chains," ASME J. Mech. Des., **122**(2), pp. 228–231.

[19] Rao, A. C., and Deshmukh, P. B., 2001, "Computer Aided Structural Synthesis of Planar Kinematic Chains Obviating the Test for Isomorphism," Mech. Mach. Theory, **36**(4), pp. 489–506.

[20] Chang, Z., Zhang, C., Yang, Y., and Wang, Y., 2002, "A New Method to Mechanism Kinematic Chain Isomorphism Identification," Mech. Mach. Theory, **37**(4), pp. 411–417.

[21] Mruthyunjaya, T. S., 2003, "Kinematic Structure of Mechanisms Revisited," Mech. Mach. Theory, **38**(4), pp. 279–320.

[22] Xiao, R., Tao, Z., and Liu, Y., 2005, "Isomorphism Identification of Kinematic Chains Using Novel Evolutionary Approaches," ASME J. Comp. Inf. Sci. Engg., **5**(1), pp. 18−24.

[23] Srinath, A., and Rao, A. C., 2006, “Correlation to Detect Isomorphism, Parallelism and Type of Freedom,” Mech. Mach. Theory, **41**(6), pp. 646–655.

[24] Ding, H., and Huang, Z., 2007, “The Establishment of the Canonical Perimeter Topological Graph of Kinematic Chains and Isomorphism Identification,” ASME J. Mech. Des., **129**(3), pp. 915–923.

[25] Bal, J. S., Deshmukh, P. B., and Jagadeesh, A., 2013, “Link Invariant Functions and Detection of Isomorphism and Inversions of Kinematic Chains,” Proceedings of 16th National Conference on Machines and Mechanisms, IIT, Rorkee, pp. 554−561.

[26] Zeng, K., Fan, X., Dong, M., and Yang, P., 2014, “A Fast Algorithm for Kinematic Chain Isomorphism Identification Based on Dividing and Matching Vertices,” Mech. Mach. Theory, **72**(2), pp. 25–38.

[27] Rizvi, S. S. H., Hasan, A. and Khan, R. A., 2016, “An efficient algorithm for distinct inversions and isomorphism detection in kinematic chains,” Perspectives in Science, **8**(1), pp. 251—253.

[28] Romaniak, K., 2016, “Identifying the isomorphism of kinematic chains,” Acta Mechanica Et Automatica, **10**(3), pp. 195–200.

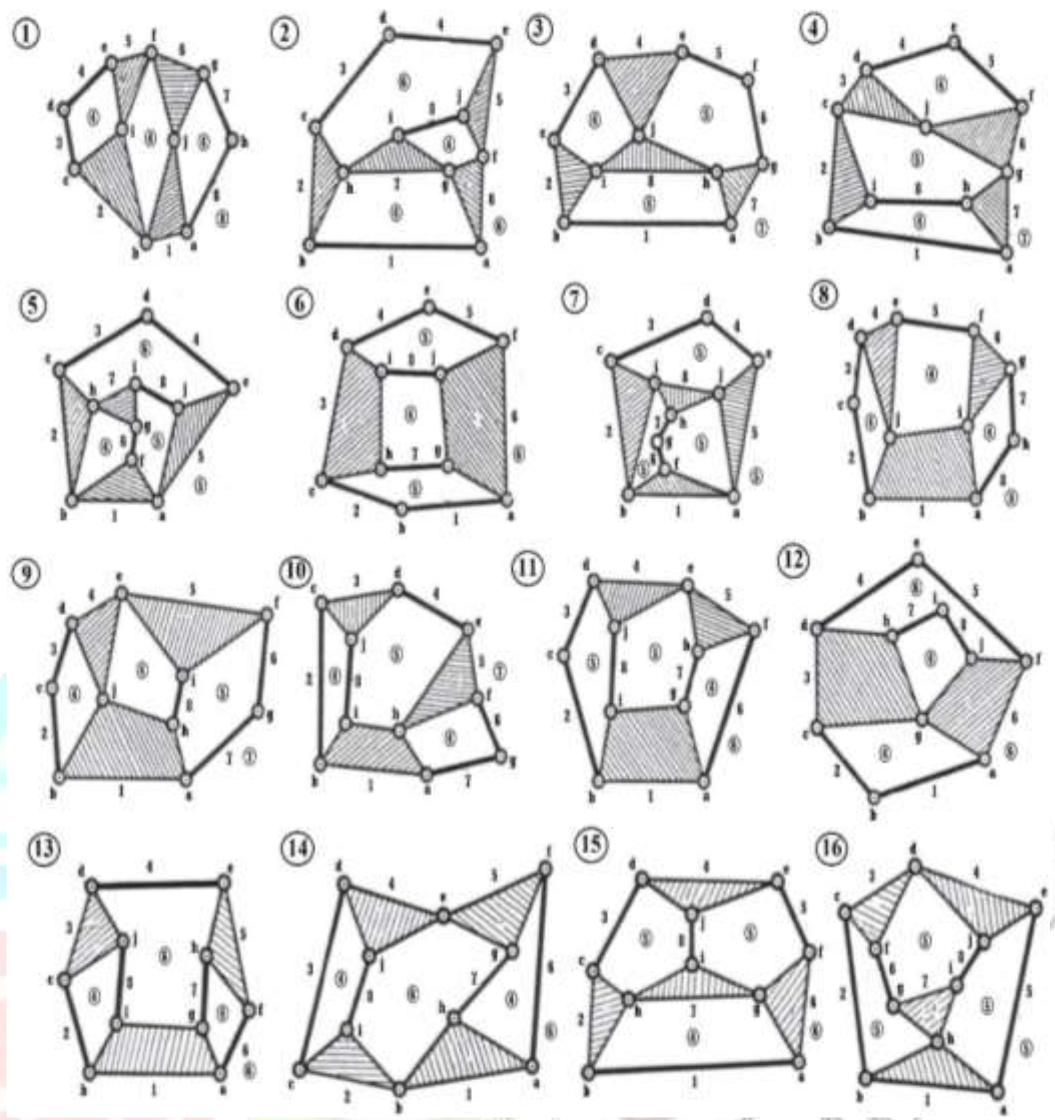
[29] Kamesh, V. V., Rao, K. M., and Rao, A. B. S., 2017, “An Innovative Approach to Detect Isomorphism in Planar and Geared Kinematic Chains Using Graph Theory,” Journal of Mechanical Design, **139**(12), pp. 381–392.

[30] Arvind Shukla, A., and Sanyal, S., 2017, “Gradient method for identification of isomorphism of planar kinematic chains” Australian Journal of Mechanical Engineering, **15**(2), pp. 1–18.

Appendix -I

Schematic Presentation of LJL Values for 8-Links 1-dof Kinematic Chains

Chain No.	Schematic of LJL Values
1	**[220/224]** {4(24), 4(27), 4(28), 4(32)}
2	**[212/208]** {2(20), 2(24), 4(26), 4(27), 2(28), 2(32)}
3	**[217/212]** {2(22), 2(24), 1(26), 7(27), 1(28), 1(30), 2(32)}
4	**[218/212]** {1(18), 3(22), 6(27), 2(28), 4(32)}
5	**[214/204]** {1(18), 3(22), 2(26), 6(27), 1(28), 2(30), 1(32)}
6	**[230/204]** {2(18), 4(22), 4(28), 2(29), 2(30), 2(40)}
7	**[216/200]** {4(20), 8(27), 4(30)}
8	**[228/224]** {5(24), 1(26), 2(28), 4(29), 1(30), 2(32), 1(40)}
9	**[225/212]** {1(18), 2(22), 2(24), 2(27), 4(28), 1(29), 2(30), 1(32), 1(40)}
10	**[225/212]** {1(18), 3(22), 1(24), 1(26), 5(28), 1(29), 1(30), 2(32), 1(40)}
11	**[221/204]** {1(18), 2(20), 2(22), 5(27), 2(28), 2(30), 1(32), 1(40)}
12	**[228/208]** {4(20), 2(24), 2(27), 4(28), 2(31), 2(40)}
13	**[222/208]** {4(20), 1(24), 1(26), 2(27), 4(28), 1(30), 2(32), 1(40)}
14	**[212/208]** {4(20), 4(26), 4(27), 4(32)}
15	**[213/204]** {2(20), 2(22), 3(26), 5(27), 1(28), 2(30), 1(32)}
16	**[212/200]** {4(20), 4(26), 4(27), 4(30)}

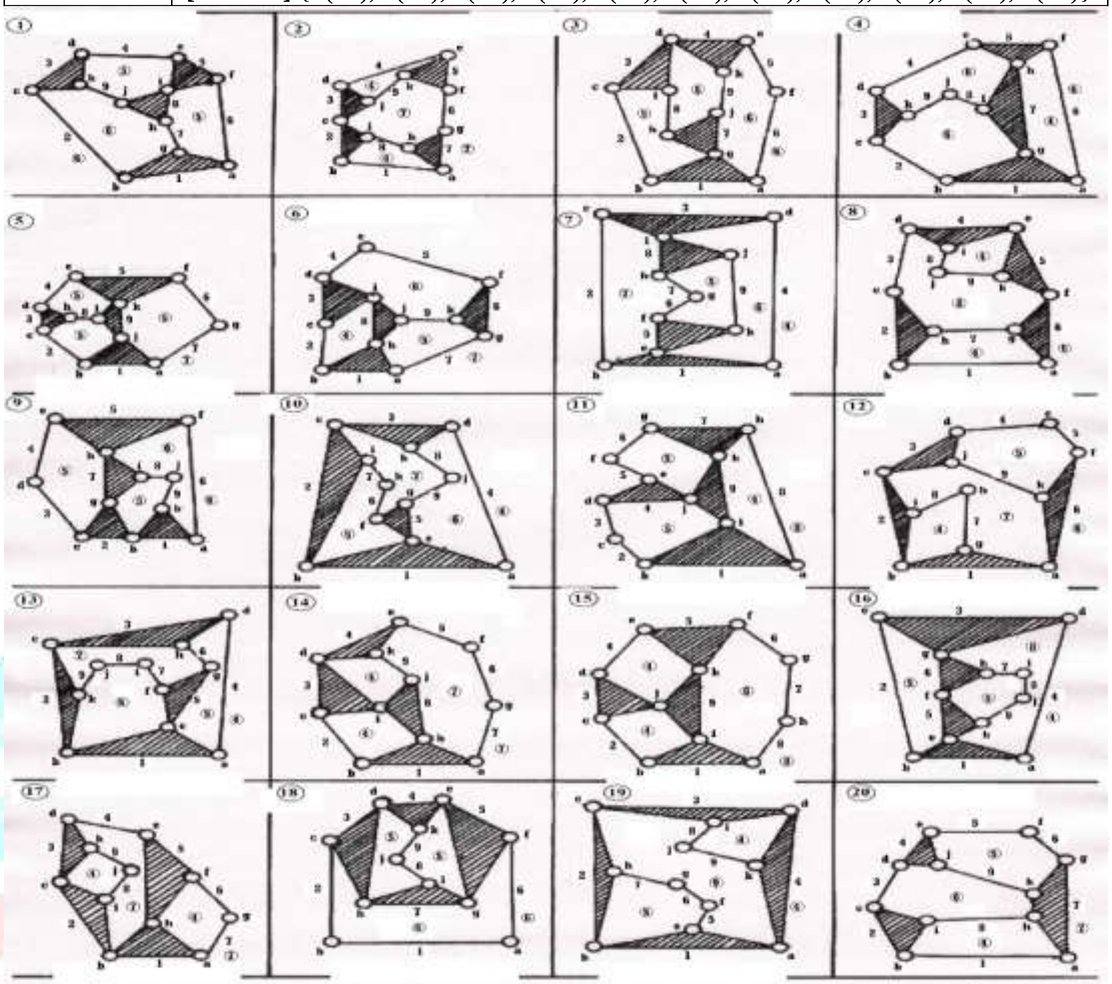


Appendix - II

Schematic Presentation of LJL Values for 9-Links 2-dof Kinematic Chains

Chain No.	Schematic of LJL Values
1	**[270/244]** {4(22), 1(24), 9(30), 2(32), 2(34)}
2	**[280/260]** {4(22), 1(28), 1(30), 6(31), 2(32), 4(36)}
3	**[268/244]** {1(20), 2(22), 2(24), 2(29), 7(30), 2(32), 2(34)}
4	**[269/248]** {1(20), 4(24), 3(29), 4(30), 2(31), 3(32), 1(36)}
5	**[273/248]** {1(20), 4(24), 7(30), 3(31), 3(34)}
6	**[274/252]** {2(22), 1(24), 2(26), 1(29), 4(30), 5(31), 1(32), 1(34), 1(36)}
7	**[273/252]** {1(20), 2(22), 2(24), 1(29), 4(30), 4(31), 2(34), 2(36)}
8	**[278/264]** {1(20), 3(24), 1(28), 1(29), 3(30), 2(31), 2(32), 1(33), 4(36)}
9	**[271/244]** {4(22), 1(24), 1(29), 6(30), 2(31), 2(32), 2(34)}
10	**[272/252]** {1(20), 2(24), 2(26), 8(30), 2(31), 1(32), 1(34), 1(36)}
11	**[277/260]** {1(24), 4(26), 1(28), 2(30), 7(31), 2(34), 1(36)}
12	**[276/252]** {4(22), 1(24), 5(30), 2(31), 2(32), 2(34), 2(36)}
13	**[275/252]** {1(18), 1(24), 3(26), 6(30), 3(31), 2(32), 1(34), 1(36)}
14	**[278/260]** {2(22), 3(28), 4(30), 6(31), 1(32), 2(36)}
15	**[278/264]** {2(24), 4(28), 3(30), 4(31), 3(32), 2(36)}
16	**[279/260]** {1(18), 1(24), 3(26), 2(30), 6(31), 1(33), 2(34), 2(36)}
17	**[283/260]** {4(22), 1(28), 1(29), 2(30), 2(31), 2(32), 2(34), 4(36)}
18	**[275/244]** {2(20), 3(24), 6(30), 2(31), 2(32), 1(33), 2(34)}
19	**[284/276]** {2(26), 3(28), 2(30), 4(31), 2(33), 3(34), 2(36)}

20	**[284/252]** {1(20), 2(22), 2(24), 1(26), 1(30), 3(31), 4(32), 1(33), 1(34), 1(36), 1(44)}



contd….

Chain No.	Schematic of LJL Values
21	**[288/260]** {4(22), 2(28), 2(31), 6(32), 1(34), 2(36), 1(44)}
22	**[277/248]** {2(20), 4(24), 4(30), 4(31), 1(32), 1(33), 1(36), 1(44)}
23	**[283/248]** {2(20), 4(24), 4(30), 4(31), 1(32), 1(33), 1(36), 1(44)}
24	**[299/258]** {1(18), 4(24), 1(26), 1(31), 1(32), 4(33), 4(34), 1(36), 1(50)}
25	**[282/252]** {1(18), 2(22), 3(26), 2(30), 3(31), 4(32), 1(33), 1(36), 1(44)}
26	**[286/252]** {4(22), 1(24), 1(26), 1(29), 4(31), 3(33), 2(34), 1(36), 1(44)}
27	**[286/264]** {1(20), 3(24), 2(28), 1(30), 2(31), 4(32), 2(33), 2(36), 1(44)}
28	**[300/248]** {2(20), 3(22), 1(26), 1(31), 2(32), 3(33), 2(34), 2(35), 1(36), 1(48)}
29	**[287/260]** {1(18), 1(24), 4(26), 1(30), 2(31), 4(32), 1(33), 2(34), 1(36), 1(44)}
30	**[387/260]** {1(18), 5(26), 4(31), 2(32), 3(33), 2(34), 1(44)}
31	**[295/260]** {2(20), 3(24), 1(26), 1(31), 2(32), 5(33), 1(35), 2(36), 1(50)}
32	**[281/252]** {4(22), 2(26), 2(30), 5(31), 1(32), 2(33), 1(36), 1(44)}
33	**[305/276]** {1(18), 4(26), 2(28), 1(31), 1(32), 2(33), 3(34), 2(36), 1(38), 1(62)}
34	**[284/248]** {2(20), 4(24), 2(30), 4(31), 1(32), 2(33), 1(34), 1(36), 1(44)}
35	**[289/276]** {4(26), 2(28), 2(31), 4(32), 3(33), 2(36), 1(44)}
36	**[288/252]** {2(20), 3(24), 1(26), 1(30), 2(31), 3(32), 2(33), 2(34), 1(36), 1(44)}
37	**[288/244]** {1(20), 4(22), 2(24), 2(31), 5(32), 2(33), 2(44)}
38	**[291/252]** {1(18), 1(20), 2(24), 3(26), 6(32), 3(33), 2(44)}
39	**[292/252]** {2(20), 3(24), 2(26), 2(31), 4(32), 3(34), 2(44)}
40	**[302/248]** {2(20), 3(22), 2(26), 1(31), 1(32), 4(33), 2(34), 1(35), 1(38), 1(56)}