Cost Analysis of M/M/1 queueing system in Transient state with server Start-up and Time-Out N policy

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Abstract-This paper is to analyze the cost function of Finite M/M/1 vacation queueing system with server timeout and N-Policy in Transient mode. We consider single server vacation queueing system that operates in the following manner; when the system is empty, the server waits for fixed time C. At the expiration of this time, if no one arrives into the system then server goes for vacation. If there is at least one customer in the system, the server commences service and serves exhaustively. The server resumes service from vacation if at least N- customers accumulate. Various system parameters are calculated by using Runge-Kutta method of order 4. Numerical results are also illustrated.

Keywords: Vacation queueing system, N- Policy, Server timeout, Cost function and Transient State.

I. Introduction

Queueing theory is the mathematical study of waiting lines or queue. The theory enables mathematical analysis of several related processes, including arriving at the queue, waiting in the queue and being served by the server(s) at the front of the queue. The theory permits the derivation and calculation of several performance measures including the average waiting time in the queue or the system, the expected number of customers (waiting or receiving service) and the probability of encountering the system in certain states, such as empty, full, having an available server or having to wait for a certain time to be served.

In queues the situation where a server is unavailable for primary customers in occasional intervals of time is known as vacation. The vacations may represent server's working on some supplementary jobs, performing server maintenance inspection and repairs, or server's failures that interrupt the customer service.Allowing servers to take vacations (such as vacation, dormant, startup, break down etc.) makes the queue models more realistic and flexible in studying real world situations. Therefore, queues with vacations or simply called *vacation models* attracted great attention of queueing researchers and became an active research area.Miller[6] (1964) was the first to study a queueing system in which the server becomes idle and is unavailable during some random length of time for the M/G/1 queueing system. Levy and Yechiali[5] (1975) also studied Queueing models of similar nature which include several types of generalizations of the classical M/G/1 queueing system.

The two fundamental models of vacation systems are the exhaustive and the gated vacation systems. The exhaustive vacation system is that where customers are continuously served until the system becomes empty, while in the gated vacation system, the customers waiting

at the end of a vacation are only served continuously and the customers arriving to the system during a service period will be served in the next service period.

A wide class of policies for governing the vacation mechanism have been discussed in the literature viz. single vacation policy, multiple vacation policy, N-policy, (M,N)-policy, T-policy, (N, T)-policy, min (N,T)-policy, D-policy, (p,T)-policy, Q-policy etc.

N-policy: Under the N-policy vacation models, the server leaves the service channel when it becomes empty and returns when the queue length reaches to a desired level N ($\geq$ 1) and begins service with startup or without startup time. The concept of N-policy was first introduced by Yadin and Naor[10] (1963). Levy , Heyman[3] (1976) etc.. have studied the so called N-policy.

Startup time: The startup time refers to the time required for preparatory work of the service before starting the service. In some actual situations, the server often requires a startup time before starting his each service period. Backer[1] (1973) first proposed the N-policy M/M/1 queueing system with exponential startups.

Vacation queueing systems with server timeout are studied by some of authors. Oliver C.Ibe [7] discussed an M/ G/1 vacation queueing system with server timeout. E.Ramesh Kumar and Y. Praby Loit [9] studied Vacation Bulk Queueing Model with setup time and server timeout.

In many applications one has to find transient solutions in Queueing systems. Transient analysis is dependent on time, it uses different analysis algorithms, control options with different convergence related issues and different initialization parameters. A time-dependent solution for the number in a single-server queueing system with Poisson arrivals and exponential service times is derived in a direct way by P. R. Parthasarathy[8]..Jacob.M.J. and Madhusoodanan.T.P[4]. (1988) [examined the transient behaviour of the infinite capacity M/G/1 model with batch arrivals and server vacations. Dong-Yuh Yang and Ying-Yi Wu (2014) [2] presented Transient Behavior Analysis of a Finite Capacity Queue with Working Breakdowns and Server Vacations.

However, to the best of our knowledge, for M/M/1 queueing systems with N-Policy, server timeout, there is no literature which takes time dependent probabilities and cost function into consideration. This motivates us to present cost analysis of a M/M/1 system with N-policy, server start-up and Timeouts in Transient State. Thus, in this present paper, we consider Cost analysis of M/M/1 queueing system with server Start-up, N-Policy and server timeouts in Transient mode.

The main objective of this paper is to analyze Cost function of M/M/1 vacation queueing system with server timeout and N-Policy of finite capacity .Various system parameters are calculated by using Runge-Kutta method of order 4. Further Numerical results are also illustrated.

This paper is organized in V sections. Section II describes the mathematical model and includes the set of governing differential equations of the model. In Section III, some performance measures are provided using the solution of the system of differential equations. Numerical results are given in Section IV. Section V concludes the paper.

The main objectives of the analysis carried out in this paper for the Cost control policy are:

i. To establish the Transient state equations and obtain the Transient state probability distribution of the number of customers in the system in each state.

ii. To derive values for the expected number of customers in the system when the server is in vacation, in startup, in timeout mode and in service respectively.

iii. To formulate the total expected cost functions for the system and determine its value.

iv. To carry out sensitivity analysis on the System performance measures and the expected cost for various system parameters through numerical experiments.

II. THE SYSTEM AND ASSUMPTIONS

We consider the M/M/1 queueing system with N-policy and server Timeout in Transient state with the following assumptions:

1. Customers are assumed to arrive according to Poisson process with mean arrival rate λ

Customers will get the service in the order in which they arrive.

2. Customers are given service such that Individual service times are assumed to be exponentially distributed with mean $1/\mu$ in exhaustive manner.

3. Whenever the system becomes empty, the server goes on timeout for a period C, After that if at least one customer arrives, he resumes service otherwise he goes for Vacation.

4. As soon as the total number of arrivals in the queue reaches or exceeds the pre-determined threshold N, the server is turned on and is temporarily unavailable for the waiting customers. The server needs a startup time which follows an exponential distribution with mean 1/θ. As soon as the server finishes startup, it starts serving the waiting customers.

It is represented in the following diagrammatic form :
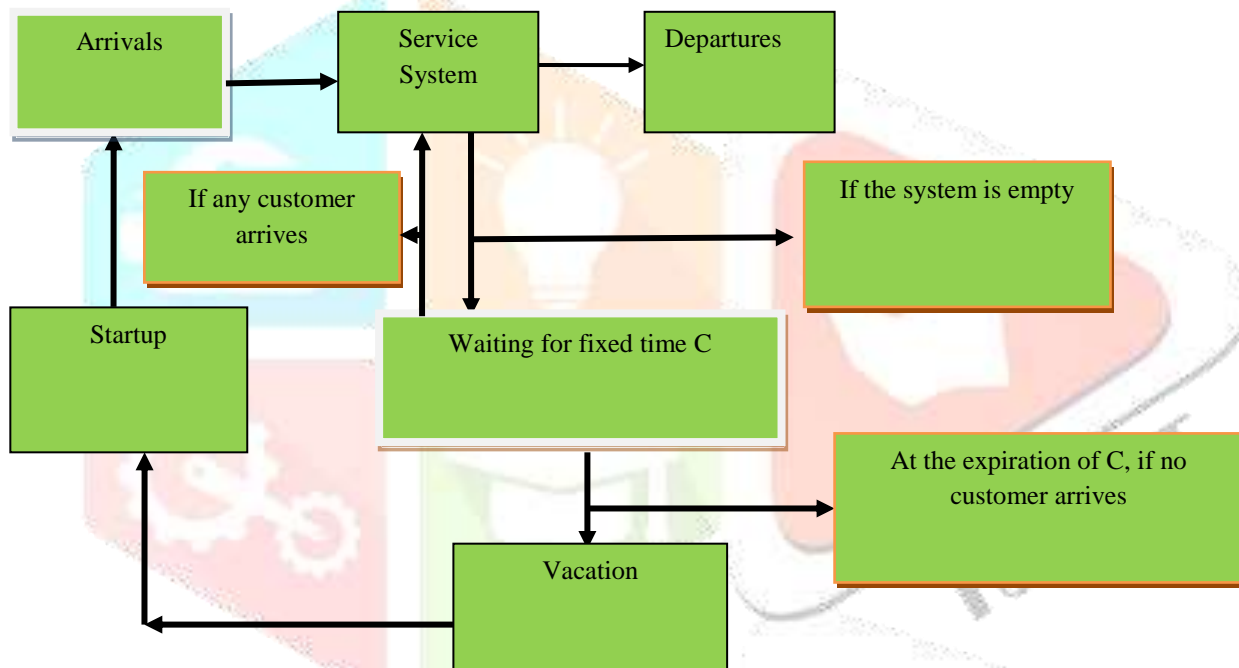
Notations:

$p_{0,i}(t)$ = The probability that there are i customers in the system when the server is on vacation, where i = 0,1,2,3,…,N-1

$p_{1,i}(t)$ = The probability that there are i customers in the system when the server is doing pre-service (startup work), where i = N,N+1,N+2,… ,S

$p_{2,0}(t)$ = The probability that there are 0 customers in the system when the server takes timeout

$p_{3,i}(t)$ = The probability that there are i customers in system when the server is working where i = 1,2,3,… ,S

The Transient state equations governing the system size probabilities at any arbitrary point of time are given by the following Differential Equations:

$$\frac{dp_{0,0}(t)}{dt} = c p_{2,0}(t) - \lambda p_{0,0}(t) \qquad (1)$$

$$\frac{dp_{0,0}(t)}{dt} = \lambda p_{0,i-1}(t) - \lambda p_{0,i}(t); 1 \leq i \leq N-1 \qquad (2)$$

$$\frac{dp_{0,N}(t)}{dt} = \lambda p_{0,N-1}(t) - (\lambda + \theta)p_{1,N}(t) \qquad (3)$$

$$\frac{dp_{1,i}(t)}{dt} = \lambda p_{1,i-1}(t) - (\lambda + \theta)\, p_{1,i}(t); N+1 \le i \le S-1 \tag{4}$$

$$\frac{dp_{1,S}(t)}{dt} = \lambda p_{1,S-1}(t) - (\theta)p_{1,S-1}(t) \tag{5}$$

$$\frac{dp_{2,0}(t)}{dt} = \mu p_{3,1}(t) - (\lambda + c)p_{2,0}(t) \tag{6}$$

$$\frac{dp_{3,1}(t)}{dt} = \lambda p_{2,0}(t) + \mu p_{3,2}(t) - (\lambda + \mu)p_{3,1}(t) \tag{7}$$

$$\frac{dp_{3,I}(t)}{dt} = \lambda p_{3,I-1}(t) + \mu p_{3,I+1}(t) - (\lambda + \mu)p_{3,i}(t); 2 \le i \le N-1 \tag{8}$$

$$\frac{dp_{3,i}(t)}{dt} = \lambda p_{3,i-1}(t) + \mu p_{3,i+1}(t) + \theta p_{1,i}(t) - (\lambda + \mu)p_{3,I}(t); N \le i \le S-1 \tag{9}$$

$$\frac{dp_{3,S}(t)}{dt} = \lambda p_{3,S-1}(t) + \theta p_{1,S}(t) - (\lambda + \mu)p_{3,S}(t) \tag{10}$$

III. Performance measures

Some performance measures are calculated to predict the system behaviour using the probabilities obtained through Runge-Kutta method:

1. Probability of the server being Idle at time t=I(t)= $\sum_{i=0}^{N-1} p_{0,i}(t) + \sum_{i=N}^{S} p_{1,i}(t)$

2. Probability of the server being timeout at time t=T(t)=$p_{2,0}(t)$

3. Probability of the server being busy at time t=B(t)=$\sum_{i=1}^{S} p_{3,i}(t)$

4. L(t)=Expected length of customers in the system at time(t)=$\sum_{n=0}^{S} n * p_n$

5. Waiting time in the system=W(t)= L(t)$\dfrac{L(t)}{(\lambda * (1 - p_{max.customers}(t))}$

Cost function:

Let T (N) be the average cost per unit of time, then

$T(N) = C(t) = C_h * Expected\ Length\ + C_o * p(server\ is\ on\ and\ in\ operation) + C_m * p(startup) + C_t * p(Timeout) +$
$C_s * (\lambda * p_{0,0}) - C_r * (Np_{0,0})$

Where

C_h= Holding cost per unit time for each customer present in the system,

C_o = Cost per unit time for keeping the server on and in operation,

C_m= Startup cost per unit time,

C_s = Setup cost per cycle,

C_t= Timeout cost per unit time for the unavailable server,

C_r= Reward per unit time as the server is doing secondary work in vacation.

IV Numerical results

The obtained results are numerically evaluated based on specific parameters. The effect of various parameters on the system performance measures such as expected number of customers in the system and mean waiting time in the system are studied and also the corresponding cost function is analyzed. MATLAB software is used to develop the computational program. The effect of different parameters in the system on performance measures is summarized in Tables 1-4

In all numerical computations, the model parameters are taken as

N = 4, S=6, λ =0.4, μ =0.6, θ = 1 , T=2 and h=0.5 .

Costs are assumed as $C\square = 5, Co = 50, Cm = 20, Cs = 1000, Ct = 2, Cr = 15$

From Tables 1-4, it leads to the following observations:

> ➤ With regard to the measure L(*t*) for a particular value of *t* , we observe an increasing trend as λ increases, θ increases, C increases and a decreasing trend with the increase of μ

> ➤ With regard to the measure W(*t*) for a particular value of *t, we observe* an increasing trend as λ increases,and,C increases a decreasing trend with the increase of θ and μ

> ➤ With regard to the measure C(*t*) for a particular value of *t* , we observe an increasing trend as λ increases, θ increases, C increases and a decreasing trend with the increase of μ .

V. Conclusions and further Scope of study

Cost analysis of M/M/1 queueing system with Start-up and finite capacity in transient state is presented. Sensitivity analysis is also performed to know the influence of non monetary parameters on system performance measures. In the above study Gating and Customer impatient behavior can also be included.

REFERENCES

[1] Baker, K. R. (1973). A note on operating policies for the queue M/M/1 with exponential start-up. *INFOR,* 11, 71–72.

[2] Dong-Yuh Yang and Ying-Yi Wu (2014). Transient Behavior Analysis of a Finite Capacity Queue with Working Breakdowns and Server Vacations, Proceedings of the International Multi Conference of Engineers and Computer Scientists 2014, Vol II, March 12 - 14, 2014, Hong Kong.

 [3] Heyman, D.P. (1968). Optimal operating policies for M/G/1 queueing systems, *Operations Research,* 16, 362 – 382.

[4] Jacob, M.J. and Madhusoodanan, T.P. (1988). Transient solution of an M/G/1 queueing system with group arrivals and vacations to the server. *OPSEARCH,* 25(4), 279-284.

[5] Levy, Y. and Yechiali, U. (1975). Utilization of idle time in an M/G/1 queueing system

[6] Miller, L.W. (1964). Alternating priorities in multi-class queue. Ph. D. Dissertation, Cornell University, Ithaca, New York.

 [7] Oliver C. Ibe M/G/1 Vacation Queueing Systems with Server Timeout, American Journal. 2015; 5:77-88.

 [8]. P. R. Parthasarathy, (1987). ” A Transient Solution of an M/M/1 Queue: A simple Approach”, *Adv. Appl. Prob.* 19, 997-998.

[9] Ramesh Kumar E, Praby Loit Y. A Study on Vacation Bulk Queueing Model with setup time and server timeout. IJCMS.2016; 5(12):81-89.

 [10] Yadin, M. and Naor, P. (1963). Queueing Systems with a removable service station. *Operational Research Quarterly,* 14, 4, 393 –405.

APPENDIX

Table1: Effect of λ on system performance measures under values of 't'

	Parameters/Time	0	1	2	3	4
λ=0.4	I(t)	1	1	0.999862	0.999139	0.997055
	B(t)	0	0	0.000138	0.000861	0.002944
	T(t)	0	0	0	8.99E-08	1.03E-06
	L(t)	0	0.2	0.399987	0.599858	0.799316
	W(t)	0	0.500533	1.007078	1.52913	2.074203
	C(t)	340	279.3695	229.9184	189.6451	156.9299
λ =.42	I(t)	1	1	0.999834	0.998973	0.996514
	B(t)	0	0	0.000166	0.001027	0.003484
	T(t)	0	0	0	1.07E-07	1.21E-06
	L(t)	0	0.21	0.419984	0.62983	0.839184
	W(t)	0	0.500609	1.008026	1.532692	2.08234
	C(t)	360	292.8617	238.6486	194.9418	159.8029
λ =.44	I(t)	1	1	0.999802	0.998788	0.995912
	B(t)	0	0	0.000198	0.001212	0.004087
	T(t)	0	0	0	1.26E-07	1.40E-06
	L(t)	0	0.22	0.439981	0.659798	0.879034
	W(t)	0	0.500692	1.009039	1.536437	2.090727
	C(t)	380	306.0589	246.948	199.7731	162.2371

Table2: Effect of μ on system performance measures under values of 't'

	Parameters/Time	0	1	2	3	4
μ =0.6	I(t)	1	0.999861507	0.999138517	0.997055	0.992678
	B(t)	0	0.000138493	0.000861394	0.002944	0.007316

	T(t)	0	0	8.99E-08	1.03E-06	5.70E-06
	L(t)	0.2	0.399987078	0.599858257	0.799316	0.997787
	W(t)	0.500533	1.007078204	1.529130247	2.074203	2.644644
	C(t)	279.3695	229.9184107	189.6450955	156.9299	130.4593
μ =.62	I(t)	1	1	0.999861507	0.999138517	0.997055
	B(t)	0	0	0.000138493	0.000861381	0.002944
	T(t)	0	0	0	1.02E-07	1.16E-06
	L(t)	0	0.2	0.399986652	0.599853641	0.799294
	W(t)	0	0.500533	1.00707714	1.529118706	2.074148
	C(t)	340	279.3695	229.9184085	189.6450721	156.9298
μ =.64	I(t)	1	1	0.999861507	0.999138517	0.997055
	B(t)	0	0	0.000138493	0.000861368	0.002944
	T(t)	0	0	0	1.14E-07	1.30E-06
	L(t)	0	0.2	0.399986227	0.599849025	0.799272
	W(t)	0	0.500533	1.007076076	1.529107166	2.074093
	C(t)	340	279.3695	229.9184064	189.6450487	156.9297

Table3: Effect of c on system performance measures under values of 't'

Parameters/Time		0	1	2	3	4
C=0.2	I(t)	1	1	0.999862	0.999138517	0.997054847
	B(t)	0	0	0.000138	0.000861394	0.002944127
	T(t)	0	0	0	8.99E-08	1.03E-06
	L(t)	0	0.2	0.399987	0.599858257	0.799316354
	W(t)	0	0.500533	1.007078	1.529130247	2.074203341
	C(t)	340	279.3695	229.9184	189.6450955	156.9298761

	I(t)	1	1	0.999861507	0.999138518	0.997054867
	B(t)	0	0	0.000138493	0.000861393	0.002944125
C=.3	T(t)	0	0	0	8.88E-08	1.01E-06
	L(t)	0	0.2	0.399987078	0.599858257	0.799316354
	W(t)	0	0.50053319	1.007078204	1.529130247	2.074203342
	C(t)	340	279.369467	229.9184107	189.6450958	156.9298821
	I(t)	1	1	0.999862	0.999139	0.997055
	B(t)	0	0	0.000138	0.000861	0.002944
C=.4	T(t)	0	0	0	8.78E-08	9.89E-07
	L(t)	0	0.2	0.399987	0.599858	0.799316
	W(t)	0	0.500533	1.007078	1.52913	2.074203
	C(t)	340	279.3695	229.9184	189.6451	156.9299

Table4: Effect of **θ** on system performance measures under values of 't'

	Parameters/Time	0	1	2	3	4
	I(t)	1	1	0.999862	0.999139	0.997055
	B(t)	0	0	0.000138	0.000861	0.002944
θ=1	T(t)	0	0	0	8.99E-08	1.03E-06
	L(t)	0	0.2	0.399987	0.599858	0.799316
	W(t)	0	0.500533	1.007078	1.52913	2.074203
	C(t)	340	279.3695	229.9184	189.6451	156.9299
	I(t)	1	1	0.999838	0.99901	0.996657
	B(t)	0	0	0.000162	0.00099	0.003342
θ=1.2	T(t)	0	0	0	1.06E-07	1.19E-06
	L(t)	0	0.2	0.399985	0.599836	0.799221

	W(t)	0	0.500533	1.007073	1.529076	2.073964
	C(t)	340	279.3695	229.9195	189.6512	156.9485
θ=1.4	I(t)	1	1	0.999816	0.998892	0.996303
	B(t)	0	0	0.000184	0.001108	0.003696
	T(t)	0	0	0	1.21E-07	1.35E-06
	L(t)	0	0.2	0.399983	0.599816	0.799134
	W(t)	0	0.500533	1.007067	1.529025	2.073748
	C(t)	340	279.3695	229.9206	189.6567	156.9651