Numerical Simulation of Flow past a Circular Cylinder between Parallel Walls

[1]Amiya Kumar Singha

[1]Associate Professor in Mechanical Engineering
Department of Jute and Fibre Technology,
University of Calcutta,
Kolkata, India

Abstract: This study has been undertaken to investigate the flow past a circular cylinder placed between parallel wall to see the effect of wall confinement on flow patterns. The fluid flow is numerically simulated by finite difference method, based on vorticity-stream function formulation for Reynolds numbers in the range consistent with two dimensional assumption and gap ratios (G/D), ratio of gap to diameter in the range between 0.5 and 2.0. The transitions from steady flow to a periodic vortex shedding regime are investigated. The results are consistent with previous investigations the suppression of vortex shedding for a cylinder placed near one wall. The unsteady vortex shedding regime changes, from a pattern similar to the von Karman vortex street when the body is in about the centre of the walls. The vortex shedding influences by wall opposite sign vorticity as the body approaches one wall. As the cylinder approaches nearest wall, cylinder vorticity elongates with the interactions from wall vorticity for lower Reylonds number. As Reynolds number increases suppression of shedding noticed as the cylinder vortices interacts with wall vorticity.

Keywords: Vorticity, Stream function, vortex shedding, Circular cylinder, Gap ratio, parallel walls.

I. Introduction

The incompressible flow of Newtonian fluid past a circular cylinder in between parallel walls is the subject of this work. The flow past a circular cylinder is characterized by the cylinder diameter based Reynolds number Re=UD/ν, where U is the free stream velocity, D is the cylinder diameter and ν is the kinematic viscosity. Flow pattern and shedding of near wake vortices of an unbounded circular cylinder is an interesting phenomenon. Fluid flow over heat exchanger tubes are such examples.
Between Reynolds numbers 10 to 40 a pair of steady of steady symmetric vortices grow behind the cylinder Beaudan and Moin (1994). In this range, as Reynolds number increases, re circulation zone length grows linearly Beaudan and Moin (1994). At moderate Reynolds number, around 49, the instability of the systematic wake occurs followed by a time-periodic regime characteristics by alternate shedding of vortices at cylinder wake whose dimensional period depends on the Reynolds numbers. The shedding of vortices remains laminar for Reynolds number up to approximate 150 Beaudan and Moin (1994). By further increasing Reynolds numbers transition to three dimensional flow starts at Reynolds numbers of around 180-194 depending on experimental condition and ends at Reynolds numbers equal to about 260 at which fine scale three dimensional eddies appear Williamson (1996). The vortex shedding is regular and the Strouhal number, which represents vortex shedding frequency, remains unchanged.

When a circular cylinder is placed near a plane wall, the separation and wake development depend on the Reynolds number, the gap ratio and the characteristics of the boundary layer of the wall. Various studies, mostly experimental, have been done over the past few decades Goktun (1975), Bearman and Zdravkovich (1978) and Grass et al (1984). Most of the experiments were carried out at Reynolds number in the sub- critical regime. The flow regime that is relatively insensitive to Reynolds number is selected for the experiments. Lei et al (2000) investigated vortex shedding near a plane wall for different gap ratios and for different Reynolds numbers ranging from 80 up to 1000. Singha et al (2008) studied vortex shedding suppression and heat transfer for flow past a single cylinder near a plane wall. They found a critical gap ratio on which vortex shedding suppressed.
Taneda (1965) carried out an experiment at low Reynolds number up to 170. By placing a cylinder between two walls, he observed that the critical Reynolds number, at which vortex street first appears, increases as the distance between two walls decreases.
Zovatto and Pedrizzetti (2001) have also studied the onset of vortex formation and the loss of stability of flow for a cylinder confined in between two walls for blockage ratio of 0.2. The study is based on the parabolic velocity profile of the liquid at the inlet to the channel. Jayavel and Tiwari (2008) established that confining walls and influence in the confining boundaries on vortex generators can also be effectively utilized to control the flow characteristics. Singha & Sinhamahapatra (2010) have studied the effect of wall confinement on vortex shedding and reported increased drag on the cylinder.

The present work is aimed to study the interaction between a laminar stream and a circular cylinder placed between two parallel walls. In this study, the two dimension Navier-Stokes equations are solved in the finite difference implicit method with vorticity stream function formulation. The value of Reynolds number of this study is so selected in consistent with two dimensional flow aspects.

I. MATHEMATICAL FORMULATION

The flow past a circular cylinder is characterized by the cylinder based Reynolds number, Re=UD/ν , where U is the free stream velocity, D is the cylinder diameter and ν is the kinematic viscosity.

In Cartesian co-ordinate systems, the non-dimensional governing equations, expressed in terms of vorticity and stream function, are written as,

$$\omega_t + (\mathbf{u}.\nabla)\omega = \frac{1}{Re}\nabla^2\omega \qquad (1)$$

$$\nabla^2\psi = -\omega \qquad (2)$$

where ω represents vorticity, ψ denotes stream function, and $\mathbf{u} = (u, v)$, denote the components of velocity, which can be calculated from the stream function:

$$u = \partial_y\psi, \quad v = -\partial_x\psi \qquad (3)$$

In order to solve equations (1), (2), and (3) in a curvilinear co-ordinate system, the following co-ordinate transformations are used:

$$\xi = \xi(x, y), \quad \eta = \eta(x, y) \qquad (4)$$

in which (ξ, η) is the co-ordinate system in the computational plane. With the above transformation, the non-dimensional equations (1), (2), and (3) are transformed into the computational co-ordinates (ξ, η) and written as:

$$\alpha\omega_{\xi\xi} + 2\beta\omega_{\xi\eta} + \gamma\omega_{\eta\eta} + \delta\omega_{\xi} + \varepsilon\omega_{\eta} = Re[u(\omega_{\xi}\xi_x + \omega_{\eta}\eta_x) + v(\omega_{\xi}\xi_y + \omega_{\eta}\eta_y)] + Re\,\omega_t \qquad (5)$$

$$\alpha\psi_{\xi\xi} + 2\beta\psi_{\xi\eta} + \gamma\psi_{\eta\eta} + \delta\psi_{\xi} + \varepsilon\psi_{\eta} = -\omega \qquad (6)$$

$$u = \xi_y\psi_{\xi} + \eta_y\psi_{\eta} \quad , \; v = -(\xi_x\psi_{\xi} + \eta_x\psi_{\eta}) \qquad (7)$$

where, subscripts denote differentiation and coefficients are given by

$$\alpha = \xi_x^2 + \xi_y^2 \,, \; \beta = \xi_x\eta_x + \xi_y\eta_y \,, \; \gamma = \eta_x^2 + \eta_y^2 \,, \; \delta = \xi_{xx} + \xi_{yy} \,, \; \varepsilon = \eta_{xx} + \eta_{yy} \qquad (8)$$

The solution of these transformed equations in the computational plane for unsteady flow starts with known uniform velocity at inlet and always constrained by the boundary conditions.

II. PHYSICAL FLOW FIELD AND NUMERICAL METHODS

Two parallel rectilinear walls separated by a distance H, contains a circular cylinder of diameter D, whose position is denoted by G the minimal distance from the cylinder surface to the nearest wall. The problem can be made dimensionless by taking cylinder diameter D as the unit length. The problem is governed by three dimensionless parameters: the Reynolds number Re, the blockage ratio d = D/H which is fixed in this case and the gap ratio G/D. The gap is a positive number which, for symmetry reasons, takes its maximum value = (1 − d)=2d when the cylinder is placed in the centre of between the walls.

The rectangular flow field, considered in this study is depicted in Fig.1. In the fixed wall distance of 5D, four types of G/D ratio, i.e. 0.5, 1.0, 1.5 and 2.0 are considered in this study. G/D of 2.0 is the cylinder placed at centre of the flow field. Horizontal position of the cylinder is considered at a distance of 5D from the inlet boundary. The upper boundary wall of the computational domain is set at a distance of 5D from bottom wall and is fixed and outlet boundary is set at 27D distance from cylinder center.

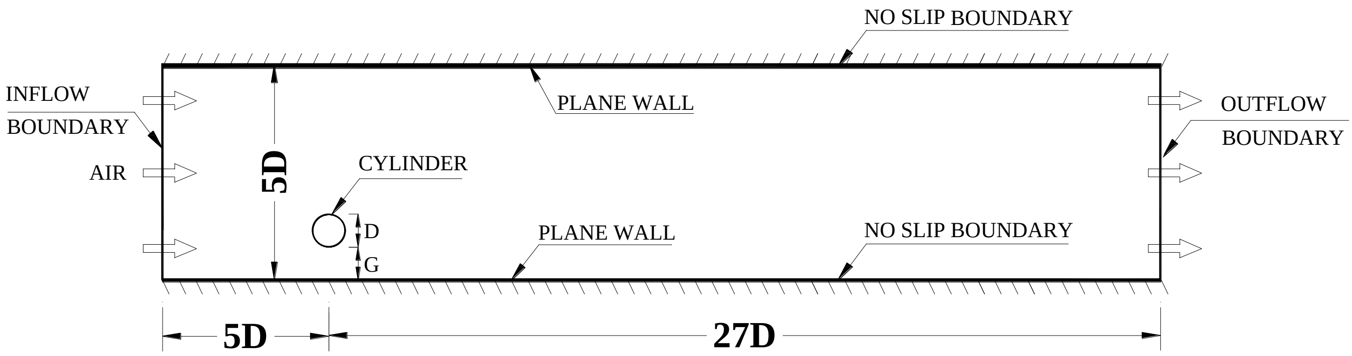


Fig.1. Configuration and boundary conditions of the flow.

Thompson, Thames, Mastin (TTM) method of generation of automatic boundary fitted co-ordinate generation system is used to construct the grids of the physical flow field. In this study, finite difference implicit method is used to solve the transformed governing equations.

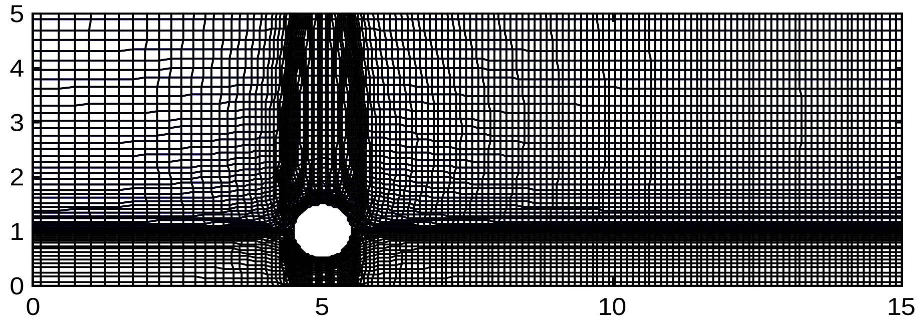


Fig. 2. Mesh of the partial flow domain around cylinder for a gap-ratio of G/D = 0.5.

A typical mesh arrangement (244 x 62) for a gap-ratio of G/D = 0.5 is shown in Fig. 2. Mesh size of (244 x 62) mesh, it is implied that there are 244 nodes in the longitudinal and 62 nodes in the transverse direction, respectively, with 72 nodes on cylinder surface In this study, calculations are performed with a non-dimensional time step of t = 0.002. Care was taken in refining the grid around the cylinder as well as near the wall so that the smaller size of grid is around 0.002 to satisfy the convective stability condition and the diffusive accuracy.

III. BOUNDARY CONDITIONS

At inlet, uniform longitudinal velocity is considered. No-slip boundary conditions are imposed on upper wall, lower wall and cylinder surfaces. Boundary conditions for vorticity and stream function are described as follows:

The no slip boundary conditions for vorticity at cylinder surface and wall can be written as per Thom’s formula, Thom (1933).

$$\omega_{i,0} = \frac{2(\psi_{i,1} - \psi_{i,0})}{(y_{i,1} - y_{i,0})^2} \quad \text{at bottom wall} \qquad (9)$$

$$\omega_{ic,jc} = \frac{2(\psi_{ic,jc} - \psi_{ic,jc-1})}{h_{ic,jc}^2} \quad \text{at cylinder surface} \qquad (10)$$

$$\omega_{i,j} = \frac{2(\psi_{i,j} - \psi_{i,j-1})}{(y_{i,j} - y_{i,j-1})^2} \quad \text{at top wall} \qquad (11)$$

where (ic, jc) denotes the nodes on the cylinder surface and $h_{ic,jc}$ is the radial distance between cylinder surface and first body fitted node around the cylinder.

The no penetration boundary condition for u indicates that at cylinder surface:

$$\psi = \text{constant} \qquad (12)$$

while no-slip boundary condition for u at cylinder surface shows that

$$\frac{\partial \psi}{\partial n} = 0 \qquad \text{and} \qquad \int_{cyl} \frac{\partial \omega}{\partial n} = 0 \qquad (13)$$

the value of ψ at cylinder is constant , which is updated for unsteady flow at every time step.

IV. RESULTS AND DISCUSSIONS

The fluid is considered to be air with a Prandtl number 0.705. The geometrical characteristic is taken into consideration through the introduction of non-dimensional gap-ratio, G/D base on nearest wall from cylinder. The flow past the cylinder in an unbounded domain to verificate the results on benchmark solutions, can be found from Singha et al (2008). The effect of the presence of parallel walls, along with the flow structures, will be considered for different gap ratios.

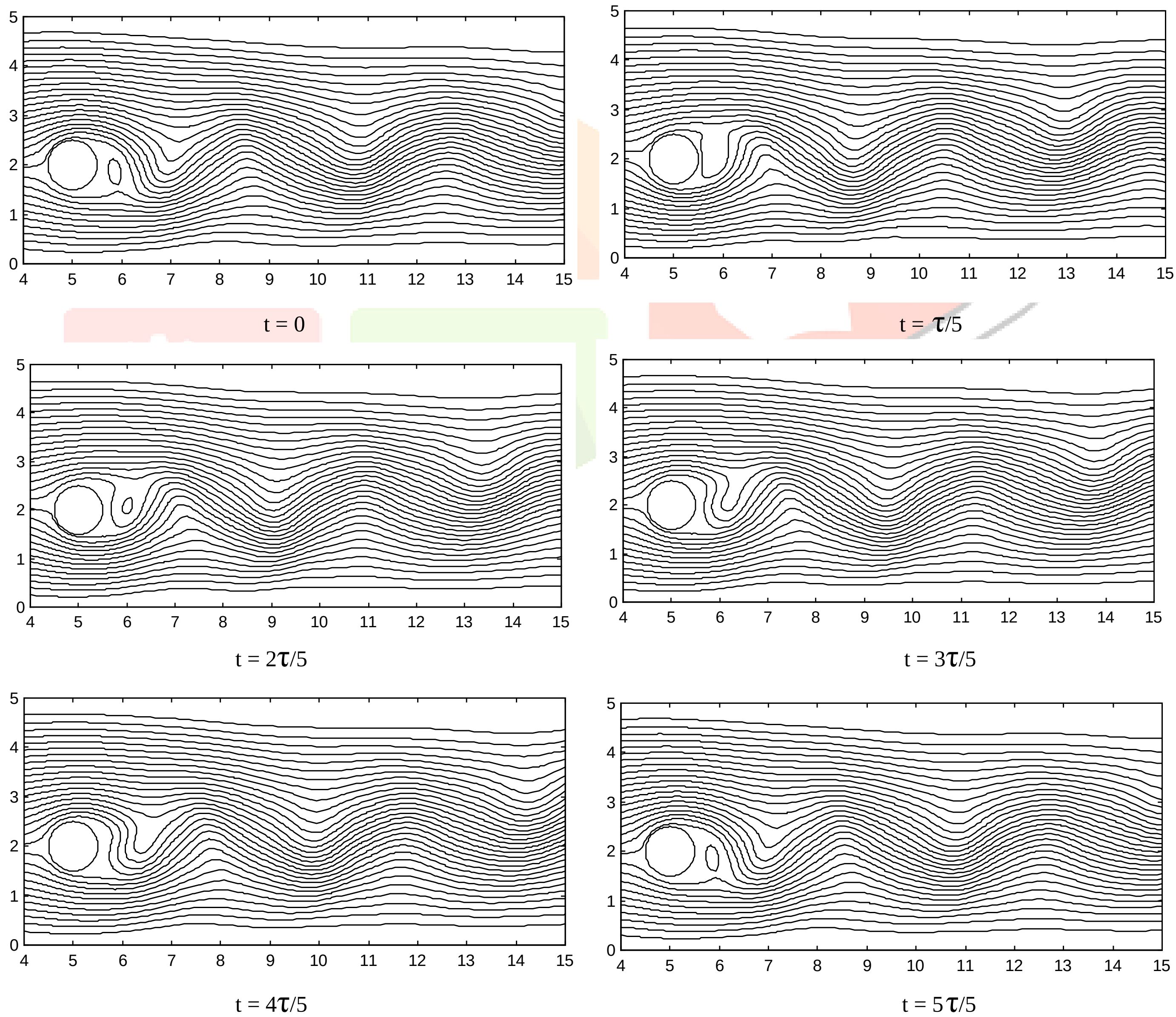
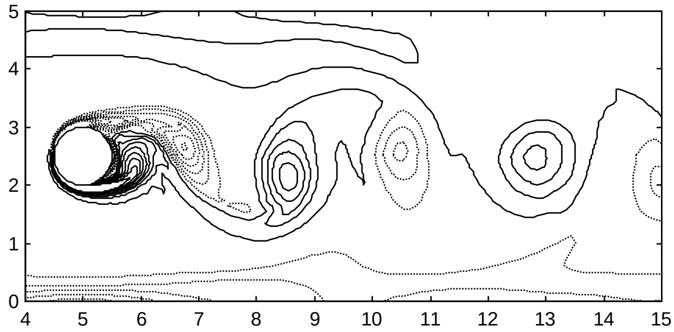
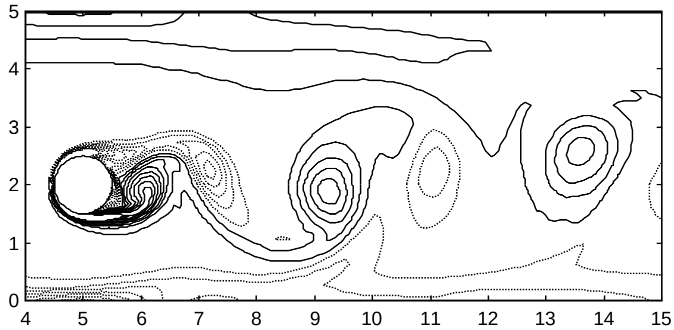


Fig. 3. The vortex shedding from cylinder in a time cycle for Re =150, G/D=1.5.

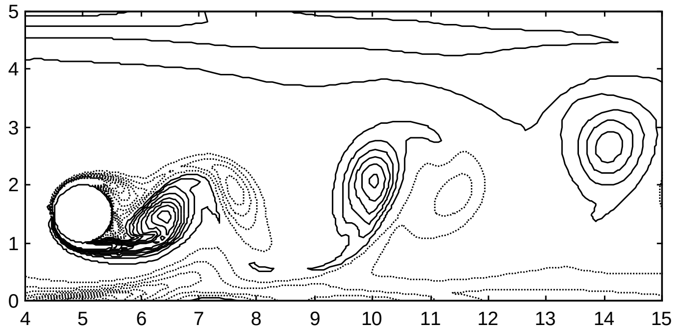
Due to instability of shear layers on the wake of the cylinder, two dimensional vortex shedding leading to the formation of von Karman vortex street, occurs; however, the flow is unsteady now. At a Reynolds number of 150 and G/D=1.5 typical streamline patterns of von Karman vortex street of the unsteady flow past an unbounded cylinder in a cross flow for a complete time cycle, of period τ is shown in Fig.3.



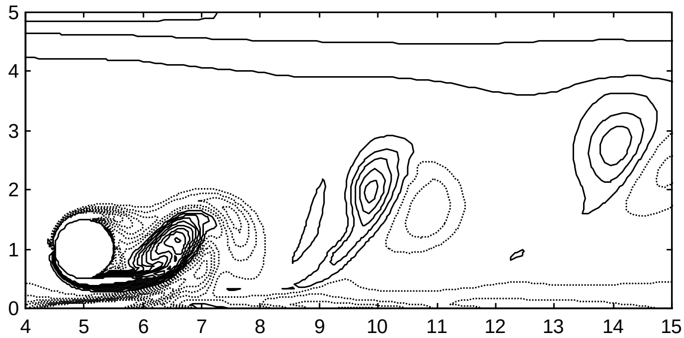
(a)



(b)



(c)



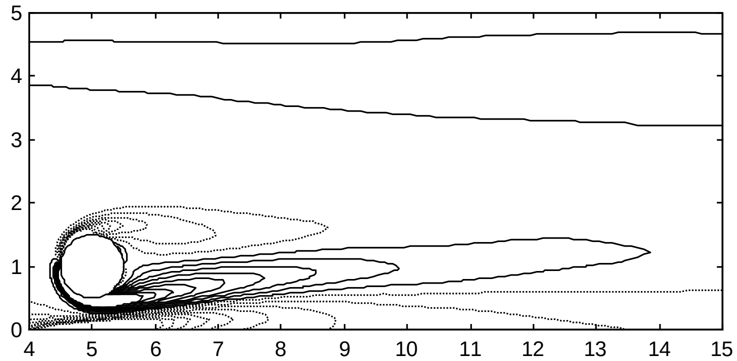
(d)

Fig 4 Vorticity contours for Re = 150 and gap-ratios of (a) G/D = 2.0, (b) G/D =1.5, (c) G/D=1.0, (d) G/D=0.5 Continuous lines indicate positive contour levels and dotted line, negative contour levels.

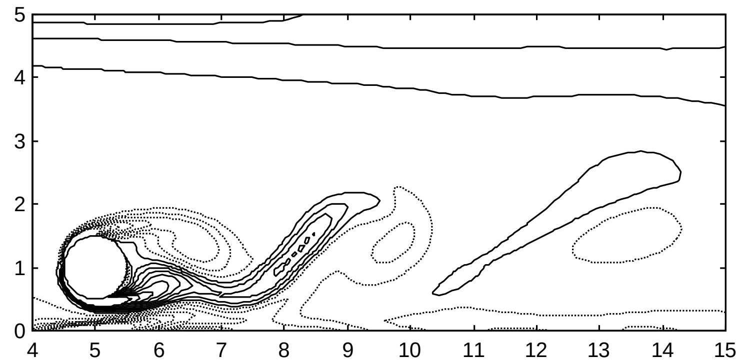
For four different gap ratios, the flow in the steady-state regimes is shown in Figure 4 for Re=150. The positive solid lines and negative dotted lines vorticity isolines are shown for these four gap ratios. When the cylinder is placed in the middle of the parallel walls, the wake is symmetric and free from interactions from wall vorticity as in the unbounded case. As the cylinder approaches one wall, the wake vorticity on the wall side is reduced in length. The interaction with the near wall vorticity is clearly seen.

Figure 5 shows the instantaneous vorticity contour for G/D =0.5 for three different Reynolds numbers of 50, 100 & 150. At low Reynolds no positive vorticity elongates with the interaction from neagative wall vorticity. The alternate shedding of vortices seen at Reynolds number at 100 and 150. As Reynolds number increase from 100 to 150 prominent alternate vortex shedding occurs with short positive vortices interacted with negative wall vortices.

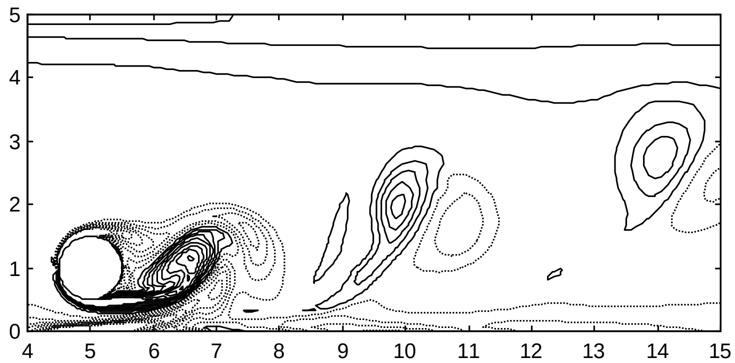
For flow past an unbounded cylinder the separation of boundary layer on the cylinder surface begins at Reynolds number equal to 5. A pair of steady symmetric vortices develops behind the cylinder between Reynolds numbers 10 to 40 and the length of re-circulation zone grows linearly with increase in Reynolds number. It is established that vortex shedding occurs for Reynolds number above 50. The vortex shedding flow remains laminar for Reynolds number up to around 150. Transition to three dimensional flow starts at Reynolds number of around 180-194 depending on experimental condition and ends at Reynolds number equal to about 260 at which fine scale three dimensional eddies appear. The vortex shedding is regular and the Strouhal number, which represents vortex shedding frequency, remains unchanged. Fig 5 shows vorticity isolines at Reynolds number 50 with three different gap ratio of 2.0, 1.5, 1.0. At gap ratio of 2.0 the cylinder is located at centre between the parallel walls. No periodic vortex



(a)



(b)



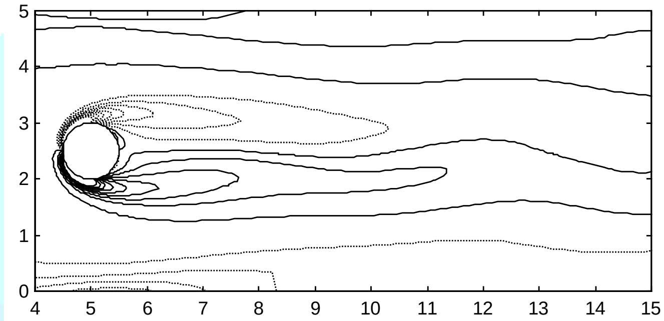
(c)

Fig 5 Vorticity contours for G/D = 0.5 (a) Re=50 (b) Re=100 and (c) Re=150.

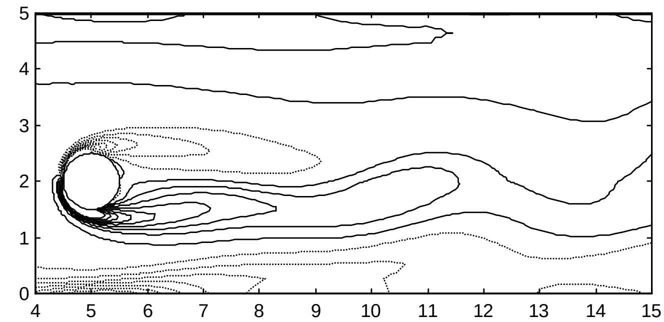
shedding is seen, The flow pattern resembles the pattern of an unbounded case. The lower wall induces with negative voticity (dotted lines) and cylinder generates positive vorticity (solid lines). The upper wall and top of cylinder induced with opposite lines i.e. positive vorticity at wall and negative vorticity at cylinder top surface. As the cylinder approaches near

Figure 5 shows the instantaneous vorticity contour for G/D =0.5 for three different Reynolds numbers of 50, 100 & 150. At low Reynolds no positive vorticity elongates with the interaction from neagative wall vorticity. The alternate shedding of vortices seen at Reynolds number at 100 and 150. As Reynolds number increase from 100 to 150 prominent alternate vortex shedding occurs with short positive vortices interacted with negative wall vortices.

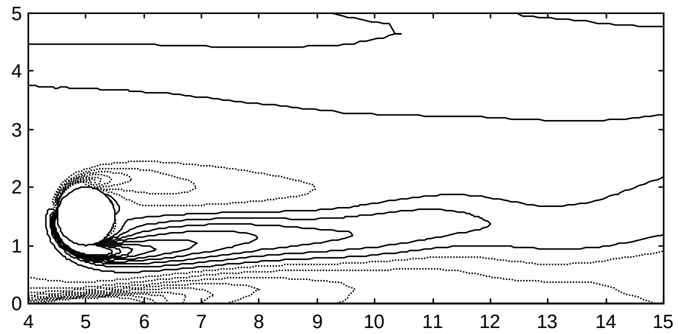
For flow past an unbounded cylinder the separation of boundary layer on the cylinder surface begins at Reynolds number equal to 5. A pair of steady symmetric vortices develops behind the cylinder between Reynolds numbers 10 to 40 and the length of re-circulation zone grows linearly with increase in Reynolds number. It is established that vortex shedding occurs for Reynolds number above 50. The vortex shedding flow remains laminar for Reynolds number up to around 150. Transition to three dimensional flow starts at Reynolds number of around 180-194 depending on experimental condition and ends at Reynolds number equal to about 260 at which fine scale three dimensional eddies appear. The vortex shedding is regular and the Strouhal number, which represents vortex shedding frequency, remains unchanged. Fig 6 shows vorticity isolines at Reynolds number 50 with three different gap ratio of 2.0, 1.5, 1.0. At gap ratio of 2.0 the cylinder is located at centre between the parallel walls. No periodic vortex shedding is seen, The flow pattern resembles the pattern of an unbounded case. The lower wall induces with negative voticity (dotted lines) and cylinder generates positive vorticity (solid lines). The upper wall and top of cylinder induced with opposite lines i.e. positive vorticity at wall and negative vorticity at cylinder top surface. As the cylinder approaches near lower wall vorticity isolines elongates with interaction through negative vorticity of wall. The elongation increases as the cylinder approaches the wall.



(a)



(b)



(c)

Fig 6 Vorticity contours for Re = 50 and gap-ratios of (a) G/D = 2.0, (b) G/D =1.5, (c) G/D=1.0

Typical Streamlines for a Reynolds number of 100, similar to patterns of von Karman vortex street of the unsteady flow is shown in Figure 7. Streamlines are shown with 30 values of equal interval through the distance of the parallel walls. The streamlines are going to be fewer as the cylinder approaches the nearest bottom wall.

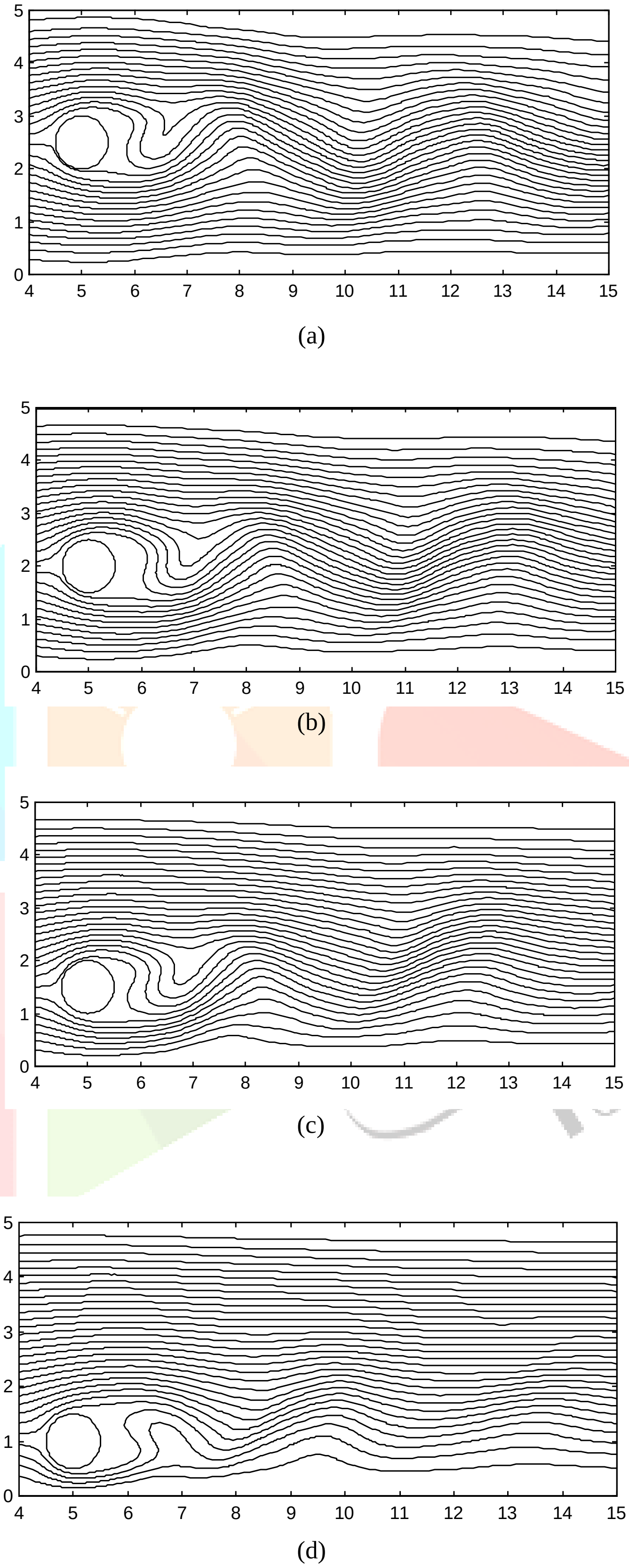


Fig 7 Streamlines for Re = 100 and gap-ratios of (a) G/D = 2.0, (b) G/D =1.5, (c) G/D=1.0, (d) G/D=0.5

V. Conclusion

The present study investigates the flow past a circular cylinder placed between parallel walls from vorticity stream function formulation using finite difference method with rectangular flow field. The cylinder is located at different location between the walls characterized by gap ratios G/D. The flow feature was discussed and results have been presented. The fluctuation of vorticity isolines noticed as the cylinder approaches near wall. The proximity of the cylinder to a wall has considerable influence on the flow and vortex formation at the wake of the cylinder. When the cylinder is in centre of two walls, it is natural to expect that the cylinder can be treated as an unbounded one. As the cylinder approaches nearest wall, cylinder vorticity elongates with the interactions from wall vorticity for lower Reylonds number. As Reynolds number increases suppression of shedding noticed as the cylinder vortices interacts with wall vorticity.

References

[1] Beaudan, P., Moin, P. 1994. Numerical Experiments on the flow past a circular cylinder at sub-critical Reynolds number. Report no TF-62, Department of Mechanical Engineering, Standford University, Standford, California, USA.
[2] Williamson, C. H. K. 1996. Vortex dynamics in the cylinder wake. Annual review of Fluid Mechanics, 28: 477-539.
[3] Goktun, S. 1975.The drag and lift characteristics of a cylinder placed near a plane Surface. M.Sc thesis, Naval Post Graduate School, Montery, California, USA.
[4] Bearman, P. W., Zdravkovich, M. M. 1978. Flow around a circular cylinder near a plane boundary, Journal of Fluid Mechanics, 89 (1): 33-47.
[5] Grass, A. J., Raven, P. W. J., Stuart, R. J., and Bray, J. A. 1984. The influence of boundary layer velocity gradients and bed proximity on vortex shedding from free spanning pipelines. Journal of Energy Resources Technology, Transaction of the ASME. 106: 70-78.
[6] Lei, C., Cheng, L., Armfield, S. W., and Kavanagh, K.. 2000. Vortex shedding suppression for flow over a circular cylinder near a plane Boundary. Ocean Engineering. 27: 1109-1127.
[7] Singha, A. K.., Sarkar, A., De, P. K.. 2008. Numerical study on heat transfer and fluid flow past a circular cylinder in the vicinity of a plane wall. Numerical Heat Transfer; Part A: Applications. 53:6: 641-666.
[8] Taneda, S., 1965. Experimental investigation of vortex street. Journal of the Physical Society of Japan, 20(9): 1714-1721.
[9] Zovatto, L., Pedrizzetti, G., 2001. Flow about a circular cylinder between parallel walls. Journal of Fluid Mechanics. 440: 1-25.
[10] Jayavel, S., Tiwari S., 2008. Numerical study of flow and heat transfer for flow past inline circular tubes built-in a rectangular channel in presence of vortex generators. Numerical. Heat Transfer - Part A. 54: 777-797.
[11] Singha, S., Sinhamahapatra, K.P., 2010. Flow past a circular cylinder between parallel walls at low Reynolds numbers. Ocean Engineering. 37: 757-769.
[12] Thom, A., 1933. The flow past circular cylinders at low speeds, Proc R Soc London Sect A, 141: 651-669.