EOQ Models for non-Deteriorative items and Shortages – in third order Equation

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Abstract: The classical economic order quantity (EOQ) model is a cost-minimization inventory model with a constant demand rate. It is the most successful model of all inventory theories because it is simple to understand and easy to apply. However, the demand rate remains stable only in the maturity stage of a product life cycle. Therefore, using the EOQ formulation in stages other than the maturity stage will cause varying magnitudes of error. In the growth stage of a product life cycle, the demand rate can be well approximated by a linear form. This paper deals with an optimal inventory replenishment policy for a non-deteriorating items and shortages. Two models are developed (i) inventory model for non-deteriorating items, (ii) inventory model with shortages. A mathematical model is developed for each model and the optimal production lot size which minimizes the total cost is derived. The validation of result in this model was coded in Microsoft Visual Basic 6.0

Key words: Inventory, Deteriorating, shortages, optimality and optimal cycle time.

1. Introduction: The classical inventory models assume constant demand an infinite planning horizon. This assumption is valid during the maturity phase of the product life cycle and for a finite period of time. In other phases of a product life cycle, demand for the product may increase after is successful introduction into the market or decrease due to introduction of new competitor's products. Economic order quantity (EOQ) models have been studied since **Harries (1913)** presented the famous EOQ formulae. **Biswaranjan Mandal (2010)** considered in which it is depleted not only by demand but also by deterioration. The Weibull distribution, which is capable of representing constant, increasing and decreasing rate of deterioration, is used to represent the distribution of the time to deterioration. **Singh et al.(2011)** developed an inventory model for ecaying items with selling price dependent demand in inflationary environment. **Sana and De (2016)** developed an economic order quantity model for fuzzy variables with promotional effort and selling price dependent demand. They observed that demand rate decreases over time during shortage period. **Tripathi et al. (2017)** established inventory model with exponential time-dependent demand and time- dependent time-dependent deterioration. Shortages are allowed. In this paper, we developed an economic order quantity inventory models in which (i) inventory model for deteriorating items, (ii) the replenishment cycles and shortages interval are time varying. The remaining of the paper is organized as follows: Section 2 presents the assumptions and notations. Section 3 is for problem formulation and numerical examples. Finally, the paper summarizes and concludes in section 4.

2. Assumptions and Notations

2.1 Assumptions: The assumptions used to formulate the problem are 1) initially the inventory level is zero, 2) The planning horizon is finite, 3) Shortages are permitted, 4) Lead time is zero, 5)There is no repair or replacement of the deteriorated items.

2.2 Notations: The notations used in this analysis are 1) D – Demand rate in units per unit time, 2) Q^*- Optimal lot size , 3) θ-rate of deteriorative, 4) C_d - deterioration cost per unit, 5) B – number of shortages in unit, 6) C_S - shortage cost per unit, 7) C_0 – ordering cost/order, 8) C_h - holding cost per unit/time, 9) T – Cycle time and 10) TC - Total cost.

3. Mathematical Models

3. 1 EOQ Model for Non-Deteriorating items

The methodology adopted in this paper involves a number of steps. First, the differential inventory equation for the period is developed. Next, these differential equations are solved to formulate the cost model. The details of this methodology are discussed below. In order to develop the differential equation, we need to define the one stage of the inventory cycle shown in figure-1, a simplified representation of the cycle.

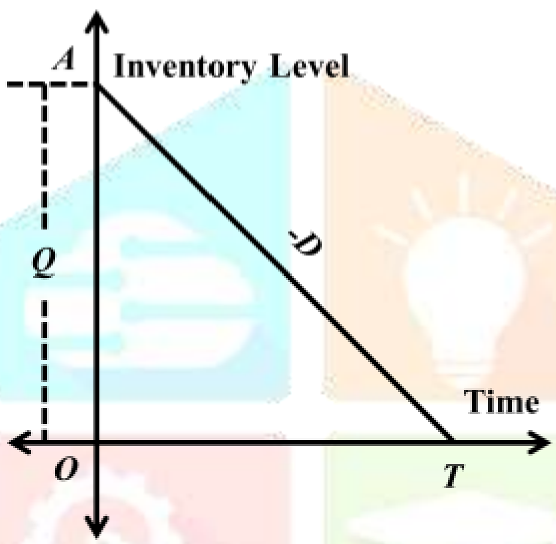

Figure -1 EOQ model with non - deteriorating items

$$\frac{dI(t)}{dt} = -D;\ 0 \le t \le T \qquad (1)$$

With the boundary conditions $I(0) = Q;\ I(T) = 0$ (2)

The solutions of above differential equation is

From (1); $I(t) = D(T - t)$ (3)

From the equation (2), and boundary conditions

$$I(0) = Q \Rightarrow Q = DT \qquad (4)$$

Total cost: The total cost comprises of the sum of the setup cost and holding cost. They are grouped together after evaluating the above cost individually. Therefore, total cost = Setup cost + holding cost.

1. Setup cost = $\frac{C_0}{T}$ (5)
2. Holding cost = $\frac{C_h}{T}\int_0^T I(t)dt = \frac{C_h}{T}\int_0^T D(T-t)\,dt = \frac{DC_h}{T}\left[\frac{T^2}{2}\right] = \frac{DC_h T}{2}$ (6)

Therefore, Total cost (from the equations (5) to (6))

$$TC = \frac{C_0}{T} + \frac{DC_hT}{2} \qquad (7)$$

Optimality: It can be easily shown that TC (T) is a convex function in T. Hence, an optimal cycle time T can be calculated from

$\frac{d}{dT}TC(T) = 0$ and $\frac{d^2}{dT^2}TC(T) > 0$

Differentiate the equation (8) with respect to T,

$$-\frac{C_0}{T^2} + \frac{DC_hT}{2} = 0$$

$$and \quad \frac{d^2(TC)}{dT^2} > 0, \quad \text{Therefore,} \quad T = \sqrt{\frac{2C_0}{DC_h}}$$

For example, D = 4500, C_0 =100, C_h =10

Optimum Solution: The third order equation is
T = 0.0667, Q = 300, Setup cost = 1500,
Holding Cost = 1500, Total cost =3000

Sensitivity Analysis

The total cost functions are the real solution in which the model parameters are assumed to be static values. It is reasonable to study the sensitivity i.e. the effect of making chances in the model parameters over a given optimum solution. It is important to find the effects on different system performance measures, such as cost function, inventory system, etc. For this purpose, sensitivity analysis of various system parameters for the models of this research are required to observe whether the current solutions remain unchanged, the current solutions become infeasible, etc.

Table 1 Effect of Demand and cost parameters on optimal values

Parameters		Optimal Values				
		T	**Q**	**Setup cost**	**Holding cost**	**Total cost**
C_0	80	0.0596	268.33	1341.64	1341.64	2683.29
	90	0.0632	284.60	1423.02	1423.02	2846.05
	100	0.0667	300.00	1500.00	1500.00	3000.00
	110	0.0699	314.64	1573.21	1573.21	3146.43
	120	0.0699	314.64	1573.21	1573.21	3146.43
C_h	8	0.0745	335.41	1341.64	1341.64	2683.28
	9	0.0703	316.23	1423.02	1423.02	2846.05
	10	0.0667	300.00	1500.00	1500.00	3000.00
	11	0.0635	286.04	1573.21	1573.21	3146.43
	12	0.0608	273.86	1643.17	1643.17	3286.33

Observations: From the table 1, it is observed that 1) with the increase in setup cost per unit (C_0), optimum quantity (Q*), cycle time (T), setup cost, holding cost and total cost increase then there is positive relationship between them, 2) with the increase in holding cost per unit (C_h), optimum quantity (Q*), cycle time (T) and holding cost decreases then there is negative relationship them but setup cost and total cost increases then there is positive relationship between them.

3.2 Purchasing Inventory model for non - deteriorating items with shortages

The typical behavior of the inventory model is depicted in Figure 3. The inventory starts with zero stock at zero time. Shortage at T_1 to accumulate at the early stage of the inventory cycle. The instantaneous inventory level I(t) is given by the following differential equations. The instantaneously inventory level I(t) is given by the differential equation

$$\frac{dI(t)}{dt} = -D;\ 0 \le t \le T_1 \tag{8}$$

$$\frac{dI(t)}{dt} = -D; T_1 \le t \le T \tag{9}$$

with the boundary conditions $I(0) = Q_1;\ I(T_1) = 0;\ I(T) = B$ (10)

The solutions of above differential equations are

From (8); $I(t) = D(T_1 - t)$ (11)

From (9); $I(t) = D(t - T_1)$ (12)

From equations (10) and (11), we have, $I(0) = Q_1 \Rightarrow Q_1 = DT_1$ (13)

Total cost: The total cost comprises of the sum of the setup cost, holding cost, deteriorating cost and shortage cost. They are grouped together after evaluating the above cost individually.

Therefore, total cost = Setup cost + Holding cost + Shortage cost

1. Setup cost = $\frac{C_0}{T}$ (14)

2. Holding cost = $\frac{C_h}{T}\int_0^{T_1} I(t)dt = \frac{C_h}{T}\int_0^{T_1} D(T_1 - t)dt$

$$= \frac{DC_h}{T}\left(T_1^2 - \frac{T_1^2}{2}\right) = \frac{DC_h T_1^2}{2T} \tag{15}$$

3. Shortage cost = $\frac{C_S}{T}\int_{T_1}^{T} I(t)dt = \frac{DC_S}{T}\int_{T_1}^{T} D(t - T_1)dt = \frac{DC_S}{2T}[T - T_1]^2$ (16)

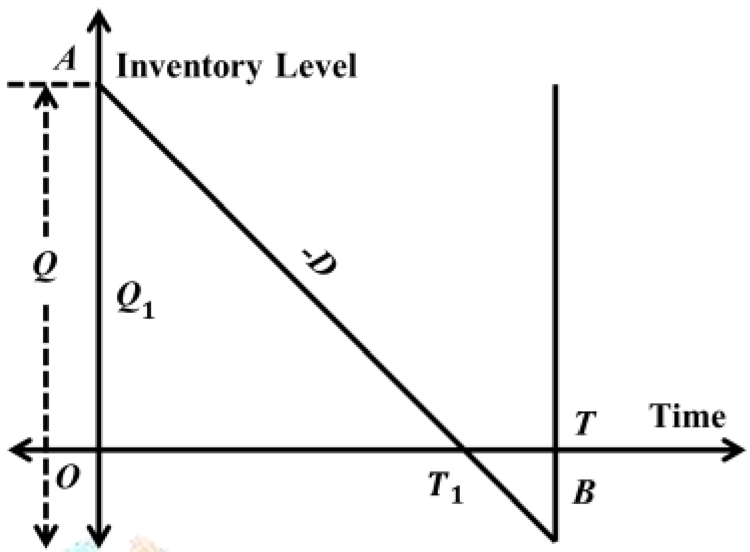

Figure 2 Purchasing inventory model with shortages

Total cost = Ordering cost + Holding cost + Deteriorating cost + Shortage cost

$$TC = \frac{C_0}{T} + \frac{DC_h T_1^2}{2T} + \frac{DC_S}{2T}\left(T - T_1\right)^2 \quad (17)$$

Optimality: It can be easily shown that $TC(T_1)$ and $TC(T)$ is a convex function in T_1 and T. Hence, an optimal cycle time T_1 and T can be calculated from

$$\frac{\partial}{\partial T_1} TC(T_1) = 0, \quad \frac{d^2}{dT_1^2} TC(T_1) > 0 \quad and \quad \frac{\partial}{\partial T} TC(T) = 0, \quad \frac{\partial^2}{\partial T^2} TC(T) > 0$$

Partially differentiate the equation (17) with respect to T_1,

$$\frac{\partial(TC)}{\partial T_1} = \frac{DC_h T_1}{T} - \frac{DC_S (T - T_1)}{T} = 0 \text{ and } \frac{\partial^2 (TC)}{\partial T_1^2} > 0$$

Therefore, $C_h T_1 = C_S (T - T_1)$, Therefore, $T_1 = \frac{C_S T}{C_h + C_S}$ (18)

Partially differential the equation (17) with respect to T,

$$\frac{\partial(TC)}{\partial T} = -\frac{C_0}{T^2} - \frac{DC_h T_1^2}{2T^2} + \frac{DC_S}{2T^2}\left(T^2 - T_1^2\right)$$

$$\frac{\partial^2 (TC)}{\partial T^2} = \frac{2C_0}{T^3} + \frac{DC_h T_1^2}{T^3} > 0$$

$$-2C_0 - DC_h T_1^2 + DC_S (T^2 - T_1^2) = 0$$

Substitute the value of T_1 and simplify

$$T^2\left(1 - \frac{C_S}{C_h + C_S}\right) = \frac{2C_0}{DC_S}$$

Therefore, $T = \sqrt{\dfrac{2C_0(C_h + C_S)}{DC_S C_h}}$ and $Q = \sqrt{\dfrac{2DC_0(C_h + C_S)}{C_S C_h}}$ (19)

Numerical example, D = 4500, C_0 =100, C_h =10, $\theta = 0.01$; C_d =100

Optimum Solution: T =0.0943, Q = 424.26, T_1 =0.0471, Q_1 = 212.13, Setup cost = 1060.66, Holding cost = 530.33, Shortage cost = 530.00, Total cost = 2121.32

Sensitivity Analysis:

Table 2 Effect of Demand and cost parameters on optimal values

Parameters		T	Q	T_1	Q_1	Setup cost	HC	SC	Total cost
C_o	80	0.0843	379.47	0.0422	189.74	948.68	474.34	474.34	1897.36
	90	0.0894	402.49	0.0447	201.25	1006.23	503.11	503.11	2012.46
	100	0.0943	424.26	0.0471	212.13	1060.66	530.33	530.33	2121.32
	110	0.0988	444.97	0.0494	222.48	1112.43	556.21	556.21	2224.86
	120	0.1033	464.76	0.0516	232.38	1161.89	580.94	580.94	2323.79
C_h	8	0.1000	450.00	0.0555	250.00	1000.00	555.55	444.44	2000.00
	9	0.0968	435.89	0.0509	229.41	1032.37	543.35	489.02	2064.74
	10	0.0943	424.26	0.0471	212.13	1060.66	530.33	530.33	2121.32
	11	0.0921	414.51	0.0438	197.38	1085.62	516.96	568.65	2171.24
	12	0.0903	406.20	0.0410	184.64	1107.82	503.55	604.27	2215.65
C_S	8	0.1000	450.00	0.0444	200.00	1000.00	444.44	555.55	2000.00
	9	0.0968	435.89	0.0458	206.47	1032.37	489.02	543.35	2064.74
	10	0.0943	424.26	0.0471	212.13	1060.66	530.33	530.33	2121.32
	11	0.0921	414.51	0.0482	217.12	1085.62	568.66	516.96	2171.24
	12	0.0903	406.20	0.0492	221.56	1107.82	604.27	503.56	2215.65

Sensitivity Analysis: A sensitivity analysis is performed to study the effects of change in the system parameters, ordering cost (C_0), holding cost (C_h), constant demand (a), varying demand (b) and (c) on optimal values that is optimal cycle time (T), optimal quantity (Q), replacement time (T_1), maximum inventory (Q_1), setup cost, holding cost, shortage cost and total cost. The sensitivity analysis is performed by changing (increasing or decreasing) the parameter taking at a time, keeping the remaining parameters at their original values. The following influences can be obtained from sensitivity analysis based on table 3. 1) with the increase in setup cost per unit (C_0) , optimum quantity (Q*), cycle time (T), setup cost, holding cost, shortage cost and total cost increases then there is positive relationship between them, 2) with the increase in holding cost per unit per unit time (C_h), the setup cost, holding cost and total cost increases then there is positive relationship between them but optimal cycle time (T) and optimal lot size (Q) decreases then there is negative relationship between, 3) with increase in the shortage cost (C_S), the optimal cycle time (T), optimal lot size (Q), and shortage cost decreases then there is negative relationship between them but the replacement time (T_1) , maximum inventory (Q_1), setup cost, holding cost and total cost increases then there is positive relationship between them. Similarly, other parameters "a", "b", and "c" can also be observed from the table -3.

4. Conclusion: In this paper, the inventory model for non-deteriorating items with quadratic demand, time value of money and shortages is considered. From the tables 1, and 2, the following points are observed. 1) with the increase in setup cost per unit (C_0) , optimum quantity (Q*), cycle time (T), setup cost, holding cost, shortage cost and total cost increases then there is positive relationship between them, 2) with the increase in holding cost per unit per unit time (C_h), the setup cost, holding cost and total cost increases then there is positive relationship between them but optimal cycle time (T) and optimal lot size (Q) decreases then there is negative relationship between, 3) with increase in the shortage cost (C_S), the optimal cycle time (T), optimal lot size (Q), and shortage cost decreases then there is negative relationship between them but the replacement time (T_1), maximum inventory (Q_1), setup cost, holding cost and total cost increases then there is positive relationship between them. Similarly, other parameters "a", "b", and "c" can also be observed from the table -3.

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