An Effect of Equalization Techniques on 2x2 MIMO System

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Abstract : **In wireless communication has been demonstrated that multiple antenna system provides very high gain without increasing the use of spectrum, reliability, throughput, power consumption and less sensitivity to fading, hence it leading to a data rate of wireless communication systems In this paper the MIMO system with optimally ordered successive interference cancellation (SIC) receiver in minimum mean square error(MMSE) equalizer and simulate this structure in Rayleigh fading channel.SIC receiver based on MMSE combined with optimal ordering improves the performance with lower complexity. In this paper we have investigated the bit error rate performance characteristics of MIMO system by different techniques consists of linear and non linear detectors or equalizers which aid in the elimination of Inter Symbol Interference (ISI) thus improving overall performance to analyze the BER of the designed system.BER analysis is done for BPSK modulation schemes using linear equalizer such as ZF, MMSE and non-linear equalizer such as ZF-SIC, MMSE-SIC.**

***Index Terms*—MIMO, MMSE,ZF, Optimally Ordered SIC.**

I. INTRODUCTION

In the past years, Multiple-Input Multiple-Output (MIMO) wireless communications has received much interest. Multiple antennas are employed at both the receiver and the transmitter in a MIMO communication system to enhance channel capacity.

The basic principle of MIMO is to take advantage of multi-path. MIMO uses multiple antennas to send multiple parallel signals (from transmitter). In an urban environment, these signals will bounce off trees, buildings, etc. and continue on their way to their destination (the receiver) but in different directions and at different arrival time. Multi-path occurs when the different signals arrive at the receiver at various times. With MIMO, the receiving end uses an algorithm or special signal processing to sort out the multiple signals to produce one signal that has the originally transmitted data. By transmitting multiple data streams in different channels at the same time and collecting multipath signals with multiple sensors, MIMO delivers simultaneous speed, coverage, and reliability improvements.

II. MIMO COMMUNICATION CHANNELS

Multi-path Spread

In conventional wireless communications, one antenna is used at the source, and another antenna is used at the destination as the receiver. This structure sometimes gives rise to problems of multipath effects. Because those multiple waves arrive at random delays (phases), angles and amplitudes, problems such as fading, cut-out, and intermittent reception occur [1,2]

Flat and Frequency Selective Fading

There are in general two fading effects, namely, flat fading and frequency selective fading signals are transmitted to receivers, if all the spectral components of the transmitted signals are affected by the same amplitude gains and phase shifts, the channel is called flat fading channel. On the other hand, if the spectral components of the transmitted signals are affected by different amplitude gains and phase shifts, the fading is said to be frequency selective.[3]

Rayleigh Fading

The Rayleigh distribution is used to model multipath fading with non-line-of- sight (NLOS). If there is a line of sight (LOS), Rician fading is more applicable.[3]

Doppler Fading

Another major concern in wireless communications is the Doppler effect (shift) this effect occurs due to the relative speed between the elements in the communication system. The Doppler effect is the change in frequency/wavelength of a wave as perceived by an observer moving relative to the source of the waves. The total Doppler effect may therefore result from either motion of the source or motion of the observer. The effect of the Doppler is directly proportional to the magnitude of the relative speed.

Slow And Fast Fading

In wireless communication, a channel can be time varying and those dynamic channels are characterized as slow or fast fading channels. Fast fading channel changes significantly during the duration of a symbol. On the other hand, slow fading occurs when the channel changes much slower than one symbol duration.[4].We define coherence time Tc of the channel as the period of time over which the fading process is correlated. Tc is closely related to Doppler spread fd as:

$$T_c \approx 1/f_d \qquad (1)$$

If the symbol time duration Ts is smaller than Tc, the fading is slow fading; otherwise, the channel fading is fast fading [4].

III. EQUALIZERS

An equalizer is a digital filter that is used to recover a signal that suffers from Inter symbol Interference (ISI) and the BER characteristics is improved and a good SNR can be obtained. It provides an approximate inverse of channel frequency response.

MMSE Equalizer

A Minimum Mean Square Error (MMSE) estimator is a method which reduces the mean square error (MSE).[5]MMSE equalizer does not completely remove ISI but minimizes the total power of the noise and ISI components in the output

. In the first time slot, the received signal on the first receive antenna is,

$$y_1 = h_{11}x_1 + h_{12}x_2 + n_1 \qquad (2)$$

$$= \begin{bmatrix} h_{11} & h_{12} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + n_1$$

The received signal on second receive antenna is,

$$y_2 = h_{21}x_1 + h_{22}x_2 + n_2 \qquad (3)$$

$$= \begin{bmatrix} h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + n_1$$

Where,

- y1, y2 are the received symbol on the first and second antenna respectively,
- h1,1 is the channel from 1st transmit antenna to 1st receive antenna,
- h1,2 is the channel from 2nd transmit antenna to 1st receive antenna,
- h2,1 is the channel from 1st transmit antenna to 2nd receive antenna,
- h2,2 is the channel from 2nd transmit antenna to 2nd receive antenna,
- x1, x2 are the transmitted symbols and
- n1, n2 are the noise on 1st and 2nd receive antennas

.

The equation can be represented in matrix notation as follow,

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix} \qquad (4)$$

Equivalently,

$$y = Hx + n \qquad (5)$$

To solve for *x*,we need to find a matrix W which satisfies WH =I.MMSE detector for meeting this constraint is given by,

$$W=[(H^H H + N_oI)^{-1}H^H] \qquad (6)$$

Where,

W= Equalization Matrix

$$H^H H = \begin{bmatrix} h_{11}^* & h_{21}^* \\ h_{12}^* & h_{22}^* \end{bmatrix} \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix}$$

$$= \begin{bmatrix} |h_{11}|^2 + |h_{21}|^2 & h_{11}^* h_{12} + h_{21}^* h_{22} \\ h_{12}^* h_{11} + h_{22}^* h_{21} & |h_{12}|^2 + |h_{22}|^2 \end{bmatrix}$$

In fact, when the noise term is zero, the MMSE equalizer reduces to Zero Forcing equalizer [6]

MIMO with MMSE-SIC Equalizer

Simple Method

Using the MMSE equalization, the receiver can obtain an estimate of the two transmitted symbols x_1, x_2, i.e,

$$\begin{bmatrix} \hat{X}_1 \\ \hat{X}_2 \end{bmatrix} = (H^H H + N_0 I)^{-1} H^H \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} \qquad (7)$$

With Optimal Ordering Method

In this the receiver arbitrarily takes one of the estimated symbols, and subtract it effect from the received symbol y_1 and y_2. However, we can have choosing whether we should subtract the effect of $\hat{X}_1$ first or $\hat{X}_2$ first. To make that decision, find out the transmit symbol (after multiplication with the channel) which came at higher power at the receiver.

The received power at the both the antennas corresponding to the transmitted symbol x_1 is,

$$px_1 = |h_{11}|^2 + |h_{21}|^2 \qquad (8)$$

The received power at the both the antennas corresponding to the transmitted symbol x_2 is,

$$px_2 = |h_{12}|^2 + |h_{22}|^2 \qquad (9)$$

If $px_1 > px_2$ then the receiver decides to remove the effect of $\hat{X}_1$ from the received vector $\mathbf{y_1}$ and $\mathbf{y_2}$ and then re-estimate $\hat{X}_2$

$$\begin{bmatrix} r_1 \\ r_2 \end{bmatrix} = \begin{bmatrix} y_1 - h_{11}\hat{X}_1 \\ y_2 - h_{21}\hat{X}_1 \end{bmatrix} \qquad (10)$$

$$= \begin{bmatrix} h_{12}x_2 + n_1 \\ h_{22}x_2 + n_2 \end{bmatrix}$$

Expressing in matrix notation,

$$\begin{bmatrix} r_1 \\ r_2 \end{bmatrix} = \begin{bmatrix} h_{12} \\ h_{22} \end{bmatrix} x_2 + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix} \qquad (11)$$

$$r = hx_2 + n \qquad (12)$$

If $px_1 < px_2$ then the receiver decides to remove the effect of $\hat{X}_2$ from the received vector $\mathbf{y_1}$ and $\mathbf{y_2}$ and then re-estimate $\hat{X}_1$

$$\begin{bmatrix} r_1 \\ r_2 \end{bmatrix} = \begin{bmatrix} y_1 - h_{12}\hat{X}_2 \\ y_2 - h_{22}\hat{X}_2 \end{bmatrix} \qquad (13)$$

$$= \begin{bmatrix} h_{11}x_1 + n_1 \\ h_{21}x_1 + n_2 \end{bmatrix}$$

Expressing in matrix notation,

$$\begin{bmatrix} r_1 \\ r_2 \end{bmatrix} = \begin{bmatrix} h_{11} \\ h_{21} \end{bmatrix} x_1 + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix} \qquad (14)$$

$$r = hx_1 + n \quad (15)$$

ZF Equalizer

We will assume that the channel is a flat fading Rayleigh multipath channel and the modulation is BPSK.[5]Let us now try to understand the math for extracting the two symbols which interfered with each other.[7]

In the first time slot, the received signal on the first receive antenna is,

$$y_1 = h_{11}x_1 + h_{12}x_2 + n_1 \quad (16)$$

$$= \begin{bmatrix} h_{11} & h_{12} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + n_1$$

The received signal on second receive antenna is,

$$y_2 = h_{21}x_1 + h_{22}x_2 + n_2 \quad (17)$$

$$= \begin{bmatrix} h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + n_2$$

Where,

- y_1, y_2 are the received symbol on the first and second antenna respectively,
- $h_{1,1}$ is the channel from 1st transmit antenna to 1st receive antenna,
- $h_{1,2}$ is the channel from 2nd transmit antenna to 1st receive antenna,
- $h_{2,1}$ is the channel from 1st transmit antenna to 2nd receive antenna,
- $h_{2,2}$ is the channel from 2nd transmit antenna to 2nd receive antenna,
- x_1, x_2 are the transmitted symbols and
- n_1, n_2 are the noise on 1st and 2nd receive antennas.

The equation can be represented in matrix notation a follow,

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix} \quad (18)$$

Equivalently,

$$y = Hx + n \quad (19)$$

To solve for x,we need to find a matrix W which satisfies WH =I.ZF detector for meeting this constraint is given by,

$$W=[(H^H H)^{-1} H^H] \quad (20)$$

Where,
W= Equalization Matrix and H = Channel matrix

This matrix is Known as Pseudo inverse for a general mxn matrix where

$$H^H H = \begin{bmatrix} h_{11}^* & h_{21}^* \\ h_{12}^* & h_{22}^* \end{bmatrix} \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix}$$

$$= \begin{bmatrix} |h_{11}|^2 + |h_{21}|^2 & h_{11}^* h_{12} + h_{21}^* h_{22} \\ h_{12}^* h_{11} + h_{22}^* h_{21} & |h_{12}|^2 + |h_{22}|^2 \end{bmatrix}$$

The off diagonal elements in the matrix H^HH are not zero, because the off diagonal elements are non zero in values. Zero forcing equalizer tries to null out the interfering terms when performing the equalization, i.e. when solving for x_1 the interference from x_2 is tried to be nulled and vice versa. While doing so, there can be an amplification of noise. Hence the Zero forcing equalizer is not the best possible equalizer. However, it is simple and easy to implement. [8]

MIMO with ZF-SIC Equalizer

Simple Method

Using the ZF equalization, the receiver can obtain an estimate of the two transmitted symbols x_1, x_2, i.e,[9]

$$\begin{bmatrix} \hat{X}_1 \\ \hat{X}_2 \end{bmatrix} = (H^H H)^{-1} H^H \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} \quad (21)$$

With Optimal Ordering Method

In this the receiver arbitrarily takes one of the estimated symbols, and subtract it effect from the received symbol y_1 and y_2. However, we can have choosing whether we should subtract the effect of $\hat{X}_1$ first or $\hat{X}_2$ first. To make that decision, find out the transmit symbol (after multiplication with the channel) which came at higher power at the receiver.

The received power at the both the antennas corresponding to the transmitted symbol x_1 is,

$$px_1 = |h_{11}|^2 + |h_{21}|^2 \quad (22)$$

The received power at the both the antennas corresponding to the transmitted symbol x_2 is,

$$px_2 = |h_{12}|^2 + |h_{22}|^2 \quad (23)$$

If $px_1 > px_2$ then the receiver decides to remove the effect of $\hat{X}_1$ from the received vector **y1** and **y2** and then re-estimate $\hat{X}_2$

$$\begin{bmatrix} r_1 \\ r_2 \end{bmatrix} = \begin{bmatrix} y_1 - h_{11}\hat{X}_1 \\ y_2 - h_{21}\hat{X}_1 \end{bmatrix} \quad (24)$$

$$= \begin{bmatrix} h_{12}x_2 + n_1 \\ h_{22}x_2 + n_2 \end{bmatrix}$$

Expressing in matrix notation,

$$\begin{bmatrix} r_1 \\ r_2 \end{bmatrix} = \begin{bmatrix} h_{12} \\ h_{22} \end{bmatrix} x_2 + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix} \quad (25)$$

$$r = hx_2 + n \quad (26)$$

If $px_1 < px_2$ then the receiver decides to remove the effect of $\hat{X}_2$ from the received vector **y1** and **y2** and then re-estimate $\hat{X}_1$

$$\begin{bmatrix} r_1 \\ r_2 \end{bmatrix} = \begin{bmatrix} y_1 - h_{12}\hat{X}_2 \\ y_2 - h_{22}\hat{X}_2 \end{bmatrix} \quad (27)$$

$$= \begin{bmatrix} h_{11}x_1 + n_1 \\ h_{21}x_1 + n_2 \end{bmatrix}$$

Expressing in matrix notation,

$$\begin{bmatrix} r_1 \\ r_2 \end{bmatrix} = \begin{bmatrix} h_{11} \\ h_{21} \end{bmatrix} x_1 + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix} \quad (28)$$

$$r = hx_1 + n \quad (29)$$

IV. RESULT ANALYSIS

A. Simulation Setup

In this section we analyzed BER performance for different modulation techniques in MIMO ZF and MMSE receiver. For analytic analysis we had taken the parameter as below:

TABLE I. SIMULATION PARAMETER

1	No of Bits or Symbols	10^6
2	Channel	Rayleigh Channel
3	Type of Modulation	BPSK
4	System	2x2 MIMO System

B. Simulation Results

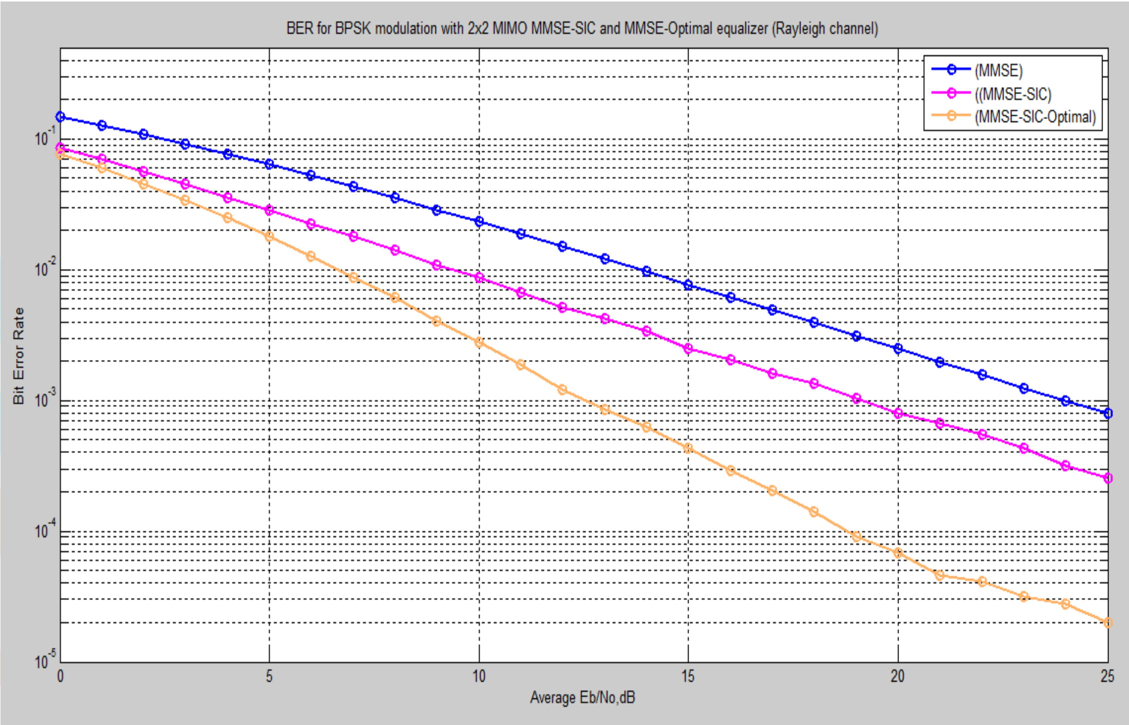

Fig. 1. BER plot for 2×2 MIMO channel with MMSE-SIC equalization with and without optimal ordering

The figure.1 shows the BER performance between MMSE equalization with simple SIC case and SIC with optimal ordering .The figure indicates that at BER of 10^{-3}, the MMSE equalizer shows SNR ~24 dB ,for MMSE-SIC case ~19dB. The MMSE-Optimal case SNR is ~13dB.Between the MMSE-SIC and optimal ordering there is a significant improvement in the SNR ~ 6dB.

The BER performance for different SNR values for a ZF and ZF-SIC detectors for a MIMO system is shown in figure.2 It can be seen from the figure the BER values decreases with the SNR in two cases. The figure indicates that for BER of 10^{-3}, ZF detectors indicate SNR ~24dB and MIMO ZF –SIC detectors SNR ~ 22dB.The SIC detectors indicate SNR improvement ~2 dB over that of ZF equalization detectors.

The MIMO- ZF equalization with SIC detection is compared with optimal ordering and shown in figure 3.It can be seen from the figure the performance of BER decreases with the SNR in all the cases. The figure indicates that for BER of 10^{-3} the SNR for optimal ordering is ~20 dB and for ZF-SIC the SNR ~22dB.also from fig. shown that ZF-SIC and optimal ordering there is an improvement ~2dB performance at BER of 10^{-3}

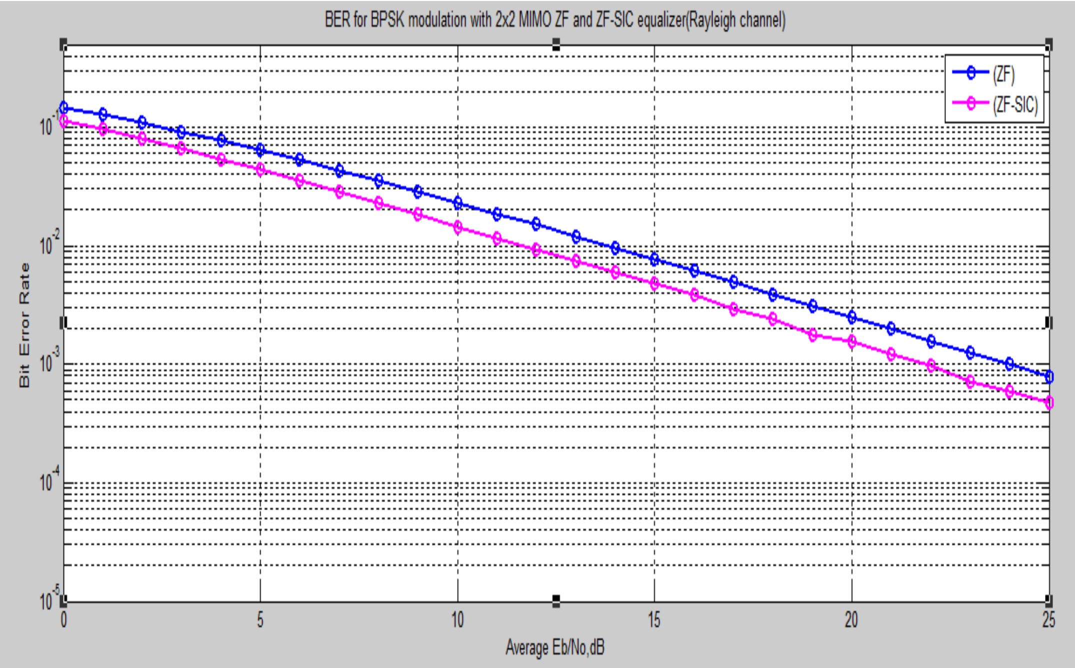

Fig.2 BER plot for BPSK in 2×2 MIMO channel with Zero Forcing Successive Interference Cancellation equalization

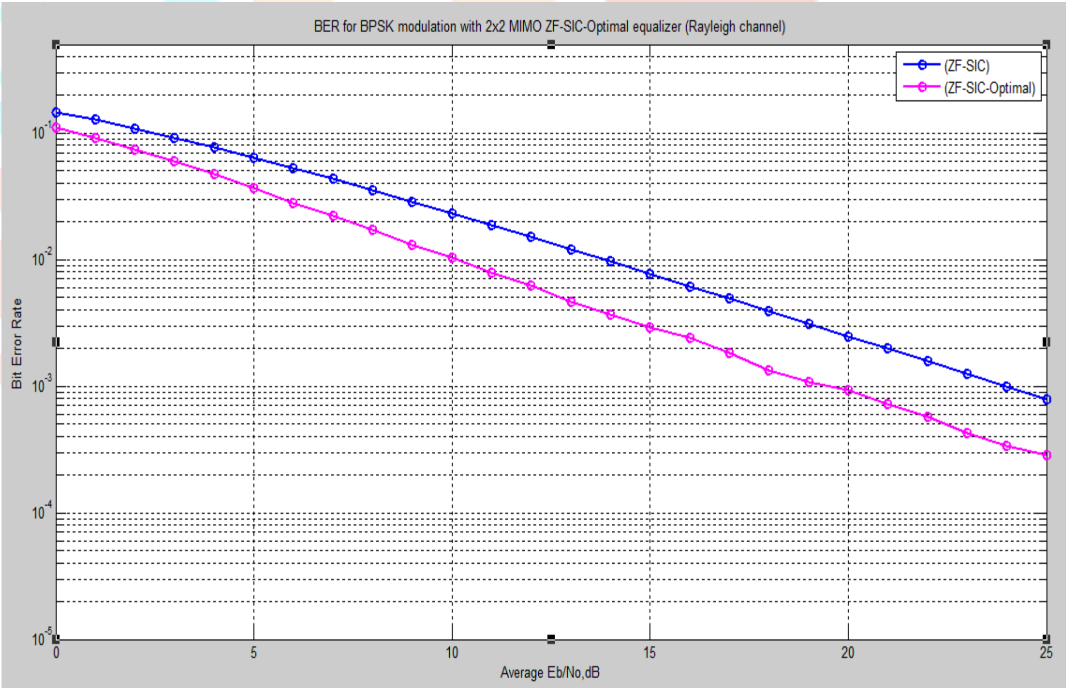

Fig.3 BER plot for BPSK in 2×2 MIMO equalized by ZF-SIC with optimal Ordering

COMPARISON OF AN DIFFERENT EQUALIZER

TABLE-2 BER FOR DIFFERENT EQUALIZER AT SNR ~20DB

SNR at ~20 dB		
Modulation System	***Equalizer Used***	***BER***
BPSK 2x2 MIMO System	ZF	$2.5\text{x}10^{-3}$
	ZF-SIC	$1.5\text{x}10^{-3}$
	ZF-SIC-Optimal	$9\text{x}10^{-4}$
	MMSE	$2.5\text{x}10^{-3}$
	MMSE-SIC	$7\text{x}10^{-4}$
	MMSE-SIC-Optimal	$7\text{x}10^{-5}$

V. CONCLUSIONS

In this performance of different equalization techniques has been analyzed to find out suitable equalizer for 2x2 MIMO channel in Rayleigh multipath fading channel. Zero Forcing equalizer performs well only in theoretical assumptions that are when noise is zero. The MMSE equalizer results is improvement when compared to zero forcing equalizer. Zero forcing with Successive interference cancellation improves the performance of equalizer. Compared to Zero Forcing equalization case, addition of successive interference cancellation results in around 2dB of improvement for BER. Zero forcing with Successive interference cancellation with optimal ordering that the reliability of the symbol which is decoded first is guaranteed to have a lower error probability than the other symbol. Compared to Zero Forcing equalization with successive interference cancellation case, addition of optimal ordering results in around 2dB of improvement for BER. Minimum Mean Square Equalization with simple successive interference cancellation case, addition of optimal ordering results in around 6.0dB of improvement for BER. So, by observing the simulation results we have conclude that by using MMSE with SIC optimal ordering, interference can be cancelled.

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