# Theory Of Maximum Pressure And Temperature For Sonoluminescence As A Function Of Bubble Radius 

Dr. Anurag Saxena<br>Associate Professor, Department of Physics, D.A-V. College, C.S.J.M. University, Kanpur


#### Abstract

We have found that the maximum pressure and temperature give different trends of variation with initial radius, equilibrium radius and with their ratios are quite interesting. The optimum value of pressure and temperature found in this work as $2.0 \times 10^{9}$ atm and $5.5 \times 10^{6}{ }^{0} \mathrm{~K}$ for initial radius of $45 \mu \mathrm{~m}$ and equilibrium radius of $3.0 \mu \mathrm{~m}$. This work is fruitful and it may pave the way to propose a new theory and experiment for anomalous behavior of maximum pressure and temperature as a function of relative radius.


Index Terms - Acoustic cavitation, Sonoluminescence, Bubble Dynamics.

## I. INTRODUCTION

The sonoluminescence arises due to influence of strong ultrasonic field under the appropriate cavity conditions. This field has been reviewed by Walton and Raynolds ${ }^{(1)}$ Barber et al. ${ }^{(2)}$ and L.A. Crum ${ }^{(5)}$. In a standing wave sound field, the sonoluminescence originates from bubbles attracted to the pressure antinodes and has its maximum intensity when the bubble volume is minimum. Sonoluminescence is an amazingly efficient mechanism for conversion of sound into light. The mechanism of sonoluminescence has been discussed ${ }^{(3)}$ experimentally but it is not yet clearly understood. An identified mechanism is thermal bremsstrahlung. Our objective in this paper is to see the variations of the maximum pressure and temperature with radius of the bubble. We have chosen initial radius ( $R_{m}$ ). equilibrium radius ( $R_{0}$ ) and relative variation of $\left(\mathrm{R}_{\mathrm{m}} / \mathrm{R}_{0}\right)$ at different ambient pressures $\left(\mathrm{P}_{0}\right)$ Calculations have also been done to see the relationship between maximum temperature and the ambient pressure and maximum pressure.

## II. THEORETICAL DETAILS

When sound field is applied in the water during $99.5 \%$ of the acoustic cycle, one expects low Mach number hydrodynamics to describe accurately the bubble motion. The solution of the problem of energy absorption from the sound field (Railegh-Plessal- equation) can be described under the following situation
$\frac{\dot{\mathrm{R}}}{\mathrm{C}_{1}} \ll 12 \frac{\mathrm{R} \ddot{\mathrm{R}}}{\mathrm{C}_{1}^{2}} \ll 1_{2} \frac{\dot{\mathrm{R}}}{\mathrm{C}_{\mathrm{g}}} \ll 1_{2} \frac{\mathrm{R} \ddot{\mathrm{R}}}{\mathrm{C}_{\mathrm{g}}^{2}} \ll 1_{2}$,
and the wavelength of the sound field is large compared to bubble radius $\overrightarrow{\mathrm{k}} \mathrm{R} \ll 1$.

The hydrodynamic equation in its generalized form can be written as follows. The left hand

$$
\begin{equation*}
\mathrm{LHS}=\overline{\mathrm{R}} \ddot{\mathrm{R}}+\frac{3}{2}(\dot{\mathrm{R}})^{2}-\mathrm{M}\left[\mathrm{R} \ddot{\mathrm{R}}+\frac{1}{2}(\dot{\mathrm{R}})^{2}\right] \tag{1}
\end{equation*}
$$

RHS $=\frac{1}{\rho}\left[\left(\mathrm{P}_{0}+\frac{2 \sigma}{\mathrm{R}_{0}}-\mathrm{P}_{\mathrm{v}}\right)^{3 r}\left(\frac{\mathrm{R}_{0}}{\mathrm{R}}\right)^{3 r}-\frac{2 \sigma}{\mathrm{R}}-\frac{4 \eta \mathrm{R}}{\mathrm{R}}-\mathrm{P}_{\alpha} \sin w t\right]$

$$
\begin{align*}
& +\frac{M}{\rho}\left[P_{g}\left(R_{s}, t\right)-P_{s}\left(t+t_{r}\right)-\frac{2 \sigma}{R}-\frac{4 \eta R}{R}-P_{0}-P_{\alpha} \sin w t\right] \\
& +\frac{t_{R}}{\rho} \frac{d}{d t}\left[P_{g}(R, t)-P_{s}(t)\right] \tag{2}
\end{align*}
$$

where $p_{0}$ is ambient pressure, $P_{a}$ is the amplitude of the sound field, $R$ is the radius, $\dot{\mathrm{R}}$ is the speed, $\ddot{\mathrm{R}}$ is the acceleration, $r$ is the polytropic exponent which includes the effects of heat flow between the bubble and the surrounding liquid $\sigma$ is surface tension, $\mu$ is the viscosity coefficient $\overline{\mathrm{M}}$ is the Mack number, $\rho$ is the density of liquid $w$ is the frequency of the sound field, $\mathrm{P}_{\mathrm{v}}$ is the vapour pressure and $c$ is the speed of sound. Also, $\mathrm{P}_{\mathrm{s}}(\mathrm{t})=$ $-\mathrm{P}_{\mathrm{a}} \sin \overline{\mathrm{w}} \overline{\mathrm{t}}, t_{R}=\frac{R}{C}$ and $M=\frac{\dot{R}}{C}$. The pressure on the gas side of the bubble is $\mathrm{P}_{\mathrm{g}}(\mathrm{R}, \mathrm{t})$ and it exceeds the pressure on the liquid side of the bubble wall by the effect of the surface tension and normal component of the viscous stress i.e.
$P_{g}(R, t)-\left(\tau_{\pi}\right)_{r-R}=P_{b}(R, t)+\frac{4 \eta \dot{R}}{R}+\frac{2 \sigma}{R}$

$$
\tau_{\pi}=-\frac{4 \eta \dot{R}}{\mathrm{R}}
$$

is negligible for the gas. One can take the assumption that $\mathrm{P}_{\mathrm{b}}(\mathrm{R}, \mathrm{t})$ is the pressure in the liquid just next to the bubble wall, then the time delayed during pressure $P_{s}\left(t+t_{R}\right)$ could be written as
$P_{s}\left(t+t_{R}\right) \approx P_{s}(t)+t_{R} \frac{d P_{s}}{d t}$ and $M \rightarrow 0$

The LHS represents the inertia of the accelerating bubble to the net force on it. The RHS is the pressure function. $\dot{\mathrm{R}}$ is the velocity of the bubble liquid inter surface.
$P_{\max }=\left(\frac{R_{m}}{R_{0}}\right)^{\frac{3}{r-1}}\left(P_{0}+\frac{2 \sigma}{R_{0}}\right)^{\frac{-1}{(r-1)}}\left[\left(P_{0}+P_{a}\right)(r-1)\right]^{\frac{r}{r-1}}$
$\mathrm{T}_{\text {max }}=\mathrm{T}_{0}\left(\frac{\mathrm{R}_{\mathrm{m}}}{\mathrm{R}_{0}}\right)^{3}\left(\mathrm{P}_{0}+\frac{2 \sigma}{\mathrm{R}_{0}}\right)^{-1}\left(\mathrm{P}_{0}+\mathrm{P}_{\mathrm{a}}\right)(\mathrm{r}-1)$

In equation (5) and (6) the maximum pressure temperature depends upon $\mathrm{R}_{\mathrm{m}}, \mathrm{R}_{0}, \mathrm{r}, \mathrm{P}_{0}$ and $\mathrm{P}_{\mathrm{a}}$ which play important role in the calculation and therefore we have chosen these parameters to see their effect on magnitudes of $\mathrm{P}_{\text {max }}$ and $\mathrm{T}_{\text {max }}$. These parameters are relevant even to the single bubble sonoluminescence (SBSL) ${ }^{(4)}$. We have used equation (5) and (6) which are appropriate for multibubble sonoluminescence (MBSL) ${ }^{(2)}$.

## III. RESULTS AND DISCUSSIONS

The variation of maximum pressure ( $\mathrm{P}_{\max }$ ) and maximum temperature ( $\mathrm{T}_{\max }$ ) as a function of initial radius of the bubble ( $\mathrm{R}_{\mathrm{m}}$ ) and equilibrium radius ( $\mathrm{R}_{0}$ ) are given in Figs. 12,3 and 4 for different values of ambient pressure $\left(\mathrm{P}_{0}\right)$ and constant value of amplitude of sound field ( $\mathrm{P}_{\mathrm{a}}=1$ atmosphere) respectively. The range of $\mathrm{R}_{\mathrm{m}}$ values chosen is 20 to $60 \mu \mathrm{~m}$, while that for $\mathrm{R}_{0}$ is 2 to $11 \mu \mathrm{~m}$. The $\mathrm{P}_{\text {max }}$ and $\mathrm{T}_{\text {max }}$ have also been plotted as a function of $\mathrm{R}_{\mathrm{m}} / \mathrm{R}_{0}$ in Figs. 5 and 6 respectively. Figs. 7 and 8 variation of $T_{\text {max }}$ with $P_{\text {max }}$ and $\mathrm{P}_{\text {max }}$ with $\mathrm{P}_{0}$ respectively. The ambient pressure chosen for the calculations lie in the range 0.3 to 1.3 atm . The maximum pressure calculated is $2 \times 10^{9} \mathrm{~atm}$ while that the magnitude of $\mathrm{T}_{\text {max }}$ is about $5.5 \times 10^{6}{ }^{0} \mathrm{~K}$ for $\mathrm{R}_{\mathrm{m}}=45 \mu \mathrm{~m}$. An examination of Figs. 1 and 2 reveal that for each value of ambient pressure there are most significant peaks at the same value of $\mathrm{R}_{\mathrm{m}} \approx 45 \mu \mathrm{~m}$. The maximum pressure and temperature have the highest peak at $\mathrm{P}_{0}=0.3 \mathrm{~atm}$ and lowest one at $\mathrm{P}_{0}=1.3 \mathrm{~atm}$. On either side these peaks $\left(\mathrm{P}_{\max }\right)$ are just touching the abscissa indicating almost its zero found as $45 \mu \mathrm{~m}$. Figs. 3 and 4 illustrate variation of $P_{\text {max }}$ and $T_{\text {max }}$ as a function of equilibrium radius of the bubble. These calculations give $\mathrm{R}_{0}=4.5 \mu \mathrm{~m}$ for minimum values of $\mathrm{P}_{\text {max }}$ and $\mathrm{T}_{\text {max }}$. with a significant dip and they have maximum values for $\mathrm{P}_{0}=0.3 \mathrm{~atm}$ at about $3 \mu \mathrm{~m}$. These results characterize that when the bubble radius from its initial $\mathrm{R}_{\mathrm{m}}=45 \mu \mathrm{~m}$ oscillate and reaches to $\mathrm{R}_{0}=4.5 \mu \mathrm{~m}$ collapse to give sonoluminescence at $\mathrm{R}_{0}=3 \mu \mathrm{~m}$. The plots of maximum pressure and maximum temperature as a function of ratio of initial radius ( $\mathrm{R}_{\mathrm{m}}$ ) and equilibrium radius $\left(\mathrm{R}_{0}\right)$ at different pressure are given in Figs. 5 and 6 respectively. The ratio $R_{m} / R_{0}$ under these circumstances is $\mathrm{R}_{\mathrm{m}} / \mathrm{R}_{0}=10$ and $\mathrm{R}_{\mathrm{m}} / \mathrm{R}_{0}=15$. The maximum values of $\mathrm{P}_{\text {max }}$ are $2 \times 10^{9} \mathrm{~atm}$ and of $\mathrm{T}_{\text {max }}$ is about $5.5 \times 10^{6}{ }^{0} \mathrm{~K}$. This indicates that initial radius and equilibrium radius influence $\mathrm{P}_{\text {max }}$ and $\mathrm{T}_{\text {max }}$ in depend to each other and their combined effect represented by relative changes of $R_{m} / R_{0}$ does not corroborate their independent effects. This phenomenon is quite interesting and it may lead to formulation of a new theory and to improve existing experiments. The Figs. 7 and 8 indicate a non-linear relationship between $\mathrm{T}_{\text {max }}$ and $\mathrm{P}_{\max }$, and $\mathrm{P}_{\max }$ and $\mathrm{P}_{0}$.

## IV. CONCLUSION

It is concluded that maximum pressure and temperature generated under the condition of emission of light (sonoluminescence) are $5.0 \times 10^{9} \mathrm{~atm}$ and $5.5 \times 10^{6}{ }^{0} \mathrm{~K}$ for $\mathrm{R}_{\mathrm{m}}=45 \mu \mathrm{~m}$ and $\mathrm{R}_{0}=3 \mu \mathrm{~m}$. The rising trends of $\mathrm{P}_{\text {max }}$ and $\mathrm{T}_{\text {max }}$ with increasing values of $R_{m} / R_{0}$ is quite interesting to formulate a new model and to make improvement in existing experiment.

## REFERENCES

[1]. A.J. Walton and G.T. Reynolds: Adv. Phys. 33 (1984) 595.
[2]. B.P. Barber, R.A. Hiller, F. Lofstedt, S.J. Putterman and K.R. Weninger: Phys. Rep. (1997) 28165.
[3]. G.K. Johri, P. Ciuti, G. Iernetti and F. Tomasini: J. Acoust. Soc. Am. 84 (1988) 2170.
[4]. D.F. Gaitan, L.A. Crum, C.C. Church and R.A. Roy: J. Acoust. Soc. Am. 91 (1992) 3166.
[5]. Lawrence A. Crum.. The Journal of the Acoustical society of American 138, (2015), 2181.



Fig. 1 Variation of maximum temperature $T_{\max }\left(i n^{\circ} K\right)$ with equilibrium radius $R_{0}(i n \mu m)$


Fig. 2 Variation of maximum pressure $P_{\max }(i n$ atm) with equilibrium radius $R_{0}$ (in $\mu m$ )


Fig. 3 Variation of maximum temperature $T_{\text {max }}\left(i n^{\circ} \mathrm{K}\right)$ with ratio of initial radius with equilibrium radius $R_{m} / R_{o}$


Fig. 4 Variation of maximum pressure $P_{\text {max }}(i n a t m)$ with ratio of initial radius and equilibrium radius $\mathrm{Rm}_{\mathrm{m}} / \mathrm{Ro}_{0}$


Fig. 5 Variation of maximum temperature $T_{\max }\left(i n{ }^{\circ} \mathrm{K}\right.$ ) with maximum pressure $P_{\max }$ (in atm) as a function of ambient pressure $P_{0}$ (in atm)


Fig. 6 Variation of maximum pressure $P_{\max }(i n a t m$ ) with ambient pressure $P_{0}$ (in atm)


Fig. 7 Variation of maximum temperature $T_{\text {max }}$ (in $K$ ) with initial radius $\mathrm{R}_{\mathrm{m}}$ (in $\mu \mathrm{m}$ )


Fig. 8 Variation of maximum pressure $P_{\max }$ (in atm) with initial radius $\mathrm{Rm}_{\mathrm{m}}$ (in um)

