Interval Valued Intuitionistic (S, T)-Fuzzy Hv-Submodules and their Characterizations

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Abstract: The notion of interval valued intuitionistic fuzzy Hv-submodules of an Hv-module with respect to a t-norm T and an s-norm S is given by J.M. Zhan. In this paper, we give some results on interval valued intuitionistic (S, T)-fuzzy Hv-submodules of an Hv-modules.

Keywords: Hv-module, interval valued intuitionistic (S, T)-fuzzy Hv-submodule, interval valued intuitinistic (S, T)-fuzzy relation.
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1. Introduction

The concept of hyperstructure was introduced in 1934 by Marty [1]. Hyperstructures have many applications to several branches of pure and applied sciences. Vougiouklis [2] introduced the notion of H_v-structures, and Davvaz [3] surveyed the theory of H_v-structures. After the introduction of fuzzy sets by Zadeh [4], there have been a number of generalizations of this fundamental concept. The notion of intuitionistic fuzzy sets introduced by Atanassov [5] is one among them. For more details on intuitionistic fuzzy sets, we refer the reader to [6, 7].

In [8] Biswas applied the concept of intuitoinistic fuzzy sets to the theory of groups and studied intuitionistic fuzzy subgroups of a group. In [9] Kim et al. introduced the notion of fuzzy subquasigroups of a quasigroup. In [10] Kim and Jun introduced the concept of fuzzy ideals of a semigroup. In [11] Zhan et al. introduced the notion of intuitionistic (S,T)-fuzzy H_v-submodule of an H_v-module. Basing on [11], in this paper, we apply the notion of interval valued intuitionistic (S,T)-fuzzy H_v-submodules of an H_v-module and describe the characteristic properties.The paper is organized as follows: in section 2 some fundamental definitions on H_v-structures and fuzzy sets are explored, in section 3 we establish some useful properties on interval valued intuitionistic (S,T)-fuzzy H_v-submodules and in section 4 interval valued intuitionistic (S,T)-fuzzy relations on an H_v -module are discussed.

2. Basic Definitions

We first give some basic definitions for proving the further results.

Definition 2.1 [12] Let X be a non-empty set. A mapping $\mu : X \to [0,1]$ is called a fuzzy set in X .The complement of μ, denoted by μ^c, is the fuzzy set in X given by

$$\mu^c(x) = 1 - \mu(x) \quad \forall x \in X .$$

Definition 2.2 [12] Let f be a mapping from a set X to a set Y. Let μ be a fuzzy set in X and λ be a fuzzy set in Y. Then the inverse image $f^{-1}(\lambda)$ of λ is a fuzzy set in X defined by

$$f^{-1}(\lambda)(x) = \lambda(f(x)) \quad \forall x \in X .$$

The image $f(\mu)$ of μ is the fuzzy set in Y defined by

$$f(\mu)(y) = \begin{cases} \sup\limits_{x \in f^{-1}(y)} \mu(x), & f^{-1}(y) \neq \phi \\ 0, & otherwise \end{cases}$$

For all $y \in Y$.

Definition 2.3 [12] An intuitionistic fuzzy set A in a non-empty set X is an object having the form $A = \{(x, \alpha_A(x), \beta_A(x)) : x \in X\}$, where the functions $\alpha_A : X \to [0,1]$ and $\beta_A : X \to [0,1]$ denote the degree of membership and degree of non membership of each element $x \in X$ to the set A respectively and $0 \le \alpha_A(x) + \beta_A(x) \le 1$ for all $x \in X$. We shall use the symbol $A = \{\alpha_A, \beta_A\}$ for the intuitionistic fuzzy set $A = \{(x, \alpha_A(x), \beta_A(x)) : x \in X\}$.

Definition 2.4 [12] Let $A = \{\alpha_A, \beta_A\}$ and $B = \{\alpha_B, \beta_B\}$ be intuitionistic fuzzy sets in X . Then

$(1)\ A \subseteq B \Leftrightarrow \alpha_A(x) \le \alpha_B(x)\ and\ \beta_A(x) \le \beta_B(x)$,

$(2)\ A^c = \{(x, \beta_A(x), \alpha_A(x)) : x \in X\},$

$(3)\ A \cap B = \left\{ \begin{matrix} (x, \min\{\alpha_A(x), \alpha_B(x)\}, \\ \max\{\beta_A(x), \beta_B(x)\}) : x \in X \end{matrix} \right\},$

$(4)\ A \cup B = \left\{ \begin{matrix} (x, \max\{\alpha_A(x), \alpha_B(x)\}, \\ \min\{\beta_A(x), \beta_B(x)\}) : x \in X \end{matrix} \right\},$

$(5)\ \Box A = \left\{(x, \alpha_A(x), \alpha_A^c(x)) : x \in X\right\},$

$(6)\ \Diamond A = \left\{(x, \beta_A^c(x), \beta_A(x)) : x \in X\right\}.$

Definition 2.5 [13] Let G be a non–empty set and $*: G \times G \to \wp^*(G)$ be a hyperoperation, where $\wp^*(G)$ is the set of all the non-empty subsets of G. Where $A * B = \bigcup_{a \in A, b \in B} a * b, \ \forall A, B \subseteq G$.

The $*$ is called weak commutative if $x * y \cap y * x \neq \phi, \quad \forall x, y \in G$.

The $*$ is called weak associative if $(x * y) * z \cap x * (y * z) \neq \phi, \quad \forall x, y, z \in G$.

A hyperstructure $(G, *)$ is called an H$_v$–group if

(i) $*$ is weak associative.

(ii) $a * G = G * a = G, \quad \forall a \in G$ (Reproduction axiom).

Definition 2.6 [14] Let G be a H$_v$-group and let μ be a fuzzy subset of G. Then μ is said to be a fuzzy H$_v$-subgroup of G if the following axioms hold:

$(i)\ \min\{\mu(x), \mu(y)\} \leq \inf_{z \in x*y} \{\mu(z)\}, \quad \forall x, y \in G$ *(ii)* For all $x, a \in G$ there exists $y \in G$ such that $x \in a * y$ and $\min\{\mu(a), \mu(x)\} \leq \{\mu(y)\}.$

Definition 2.7 [15] Let G be a H$_v$-group. An intuitionistic fuzzy set $A = \{\alpha_A, \beta_A\}$ of G is called intuitionistic fuzzy H$_v$-subgroup of G if the following axioms hold:

$(i)\ \min\{\alpha_A(x), \alpha_A(y)\} \leq \inf_{z \in x*y} \{\alpha_A(z)\}, \quad \forall x, y \in G.$

(ii) For all $x, a \in G$ there exists $y \in G$ such that $x \in a * y$ and $\min\{\alpha_A(a), \alpha_A(x)\} \leq \{\alpha_A(y)\}.$

$(iii)\ \sup_{z \in x*y} \{\beta_A(z)\} \leq \max\{\beta_A(x), \beta_A(y)\}, \quad \forall x, y \in G.$

(iv) For all $x, a \in G$ there exists $y \in G$ such that $x \in a * y$ and $\{\beta_A(y)\} \leq \max\{\beta_A(a), \beta_A(x)\}.$

Definition 2.8 [13] An H$_v$–ring is a system $(R, +, \cdot)$ with two hyperoperations satisfying the ring-like axioms:

(i) $(R, +)$ is an H$_v$-group, that is,

$$((x + y) + z) \cap (x + (y + z)) \neq \phi \quad \forall x, y \in R,$$
$$a + R = R + a = R \quad \forall a \in R;$$

(ii) $(R, \cdot)$ is an H$_v$-semigroup;

(iii) $(\cdot)$ is weak distributive with respect to $(+)$, that is, for all $x, y, z \in R$,

$$(x \cdot (y + z)) \cap (x \cdot y + x \cdot z) \neq \phi,$$
$$((x + y) \cdot z) \cap (x \cdot z + y \cdot z) \neq \phi.$$

Definition 2.9 [16] Let R be an H$_v$-ring. A nonempty subset I of R is called a left (resp., right) H$_v$-ideal if the following axioms hold:

(i) $(I, +)$ is an H$_v$-subgroup of $(R, +)$,

(ii) $R \cdot I \subseteq I$ (resp., $I \cdot R \subseteq I$).

Definition 2.10 [16] Let $(R,+,\cdot)$ be an H_v–ring and μ a fuzzy subset of R. Then μ is said to be a left (resp., right) fuzzy H_v-ideal of R if the following axioms hold:

(1) $\min\{\mu(x),\mu(y)\}\le\inf\{\mu(z): z\in x+y\}\forall x,y\in R,$

(2) For all $x,a\in R$ there exists $y\in R$ such that $x\in a+y$ and $\min\{\mu(a),\mu(x)\}\le\mu(y),$

(3) For all $x,a\in R$ there exists $z\in R$ such that $x\in z+a$ and $\min\{\mu(a),\mu(x)\}\le\mu(z),$

(4) $\mu(y)\le\inf\{\mu(z): z\in x\cdot y\}$ [respectively $\mu(x)\le\inf\{\mu(z): z\in x\cdot y\}$ $\forall x,y\in R$].

Definition 2.11 [16] An intuitionistic fuzzy set $A=\{\alpha_A,\beta_A\}$ in R is called a left (resp., right) intuitionistic fuzzy H_v-ideal of R if following axioms hold:

(1) $\min\{\alpha_A(x),\alpha_A(y)\}\le\inf\{\alpha_A(z): z\in x+y\}$ and $\max\{\beta_A(x),\beta_A(y)\}\ge\sup\{\beta_A(z): z\in x+y\}$ for all $x,y\in R,$

(2) For all $x,a\in R$ there exists $y\in R$ such that $x\in a+y$ and $\min\{\alpha_A(a),\alpha_A(x)\}\le\alpha_A(z)$ and $\max\{\beta_A(a),\beta_A(x)\}\ge\beta_A(y),$

(3) For all $x,a\in R$ there exists $z\in R$ such that $x\in z+a$ and $\min\{\mu_A(a),\mu_A(x)\}\le\mu_A(z)$ and $\max\{\beta_A(a),\beta_A(x)\}\ge\beta_A(z),$

(4) $\alpha_A(y)\le\inf\{\alpha_A(z): z\in x\cdot y\}$ [respectively $\alpha_A(x)\le\inf\{\alpha_A(z): z\in x\cdot y\}$ $\forall x,y\in R$] and $\beta_A(y)\ge\sup\{\beta_A(z): z\in x\cdot y\}$ [respectively $\beta_A(x)\ge\sup\{\beta_A(z): z\in x\cdot y\}$ $\forall x,y\in R$].

Definition 2.12 [16] A nonempty set M is called an H_v-module over an H_v -ring R if (M, +) is a weak commutative H_v -group and there exists a map

$.: R\times M\to\wp^*(M),(r,x)\to r.x$ Such that for all $a,b\in R$ and $x,y\in M$, we have

$$(a.(x+y))\cap(a.x+a.y)\ne\phi,$$
$$((x+y).a)\cap(x.a+y.a)\ne\phi,$$
$$(a.(b.x))\cap((a.b).x)\ne\phi.$$

Note that by using fuzzy sets, we can consider the structure of H_v -module on any ordinary module which is a generalization of a module.

Definition 2.13 [18] A fuzzy set μ in M is called a fuzzy H_v-submodule of M if

(1) $\min\{\mu(x),\mu(y)\}\le\inf\{\mu(z): z\in x+y\}\forall x,y\in M,$

(2) For all $x,a\in M$ there exists $y,z\in M$ such that $x\in(a+y)\cap(z+a)$ and $\min\{\mu(a),\mu(x)\}\le\inf\{\mu(y),\mu(z)\},$

(3) $\mu(y)\le\inf\{\mu(z): z\in x\cdot y\}$ for all $y\in M$ and $x\in R.$

Definition 2.14 [11] An intuitionistic fuzzy set $A=\{\alpha_A,\beta_A\}$ in an H_v –module M over an H_v –ring R is said to be an intuitionistic fuzzy H_v-submodule of M if the following axioms hold:

(1) $\min\{\alpha_A(x),\alpha_A(y)\}\le\inf\{\alpha_A(z): z\in x+y\}$ and $\max\{\beta_A(x),\beta_A(y)\}\ge\sup\{\beta_A(z): z\in x+y\}$ for all $x,y\in M,$

(2) For all $x,a\in M$ there exists $y\in M$ such that $x\in a+y$ and $\min\{\alpha_A(a),\alpha_A(x)\}\le\alpha_A(z)$ and $\max\{\beta_A(a),\beta_A(x)\}\ge\beta_A(y),$

(3) For all $x, a \in M$ there exists $z \in M$ such that $x \in z + a$ and $\min\{\mu_A(a), \mu_A(x)\} \leq \mu_A(z)$ and $\max\{\beta_A(a), \beta_A(x)\} \geq \beta_A(z)$,

(4) $\alpha_A(x) \leq \inf\{\alpha_A(z) : z \in r \cdot x\}$ and $\beta_A(x) \geq \sup\{\beta_A(z) : z \in r \cdot x\}$ for all $x \in M$ and $r \in R$.

Definition 2.15 [17] By a t-norm T, we mean a function $T : [0,1] \times [0,1] \to [0,1]$ satisfying the following conditions:

$(i) T(x,1) = x,$
$(ii) T(x,y) \leq T(x,z) \quad if \ y \leq z,$
$(iii) T(x,y) = T(y,x),$
$(iv) T(x, T(y,z)) = T(T(x,y), z)$
For all $x, y, z \in [0,1]$.

Definition 2.16 [17] By a s-norm S, we mean a function $S : [0,1] \times [0,1] \to [0,1]$ satisfying the following conditions:

$(i) S(x,0) = x,$
$(ii) S(x,y) \leq S(x,z) \quad if \ y \leq z,$
$(iii) S(x,y) = S(y,x),$
$(iv) S(x, S(y,z)) = S(S(x,y), z)$
For all $x, y, z \in [0,1]$.

It is clear that
$T(\alpha, \beta) \leq \min\{\alpha, \beta\} \leq \max\{\alpha, \beta\} \leq S(\alpha, \beta)$ For all $\alpha, \beta \in [0,1]$.

By an interval number $\tilde{a}$ we mean an interval $[a^-, a^+]$ where $0 \leq a^- \leq a^+ \leq 1$. The set of all interval numbers is denoted by $D[0,1]$. We also identify the interval $[a,a]$ by the number $a \in [0,1]$.

For the interval numbers $\tilde{a}_i = [a_i^-, a_i^+] \in D[0,1], i \in I,$ we define

$\max\{\tilde{a}_i, \tilde{b}_i\} = [\max(a_i^-, b_i^-), \max(a_i^+, b_i^+)],$

$\min\{\tilde{a}_i, \tilde{b}_i\} = [\min(a_i^-, b_i^-), \min(a_i^+, b_i^+)],$

$\inf \tilde{a}_i = \left[\wedge_{i \in I} a_i^-, \wedge_{i \in I} a_i^+\right], \sup \tilde{a}_i = \left[\vee_{i \in I} a_i^-, \vee_{i \in I} a_i^+\right]$

and put

$(1) \tilde{a}_1 \leq \tilde{a}_2 \Leftrightarrow a_1^- \leq a_2^-$ and $a_1^+ \leq a_2^+,$
$(2) \tilde{a}_1 = \tilde{a}_2 \Leftrightarrow a_1^- = a_2^-$ and $a_1^+ = a_2^+,$
$(3) \tilde{a}_1 < \tilde{a}_2 \Leftrightarrow \tilde{a}_1 \leq \tilde{a}_2$ and $\tilde{a}_1 \neq \tilde{a}_2,$
$(4) k\tilde{a} = [ka^-, ka^+]$, whenever $0 \leq k \leq 1$.

It is clear that $(D[0,1], \leq, \vee, \wedge)$ is a complete lattice with $0 = [0,0]$ as least element and $1 = [1,1]$ as greatest element.

By an interval valued fuzzy set F on X we mean the set $F = \{(x, [\alpha_F^-(x), \alpha_F^+(x)]) : x \in X\}$. Where α_F^- and α_F^+ are fuzzy subsets of X such that $\alpha_F^-(x) \leq \alpha_F^+(x)$ for all $x \in X$. Put $\tilde{\alpha}_F(x) = [\alpha_F^-(x), \alpha_F^+(x)]$. Then $F = \{(x, \tilde{\alpha}_F(x)) : x \in X\}$, where $\tilde{\alpha}_F : X \to D[0,1]$.

If A, B are two interval valued fuzzy subsets of X, then we define
$A \subseteq B$ if and only if for all $x \in X$, $\alpha_A^-(x) \leq \alpha_B^-(x)$ and $\alpha_A^+(x) \leq \alpha_B^+(x)$,

$A=B$ if and only if for all $x\in X$, $\alpha_A^-(x)=\alpha_B^-(x)$ and $\alpha_A^+(x)=\alpha_B^+(x)$,

Also, the union, intersection and complement are defined as follows: let A; B be two interval valued fuzzy subsets of X, then

$$A\cup B=\left\{\left(x,\left[\max\left\{\alpha_A^-(x),\alpha_B^-(x)\right\},\max\left\{\alpha_A^+(x),\alpha_B^+(x)\right\}\right]\right):x\in X\right\},$$

$$A\cap B=\left\{\left(x,\left[\min\left\{\alpha_A^-(x),\alpha_B^-(x)\right\},\min\left\{\alpha_A^+(x),\alpha_B^+(x)\right\}\right]\right):x\in X\right\},$$

$$A^c=\left\{\left(x,\left[\left\{1-\alpha_A^-(x),1-\alpha_A^+(x)\right\}\right]\right):x\in X\right\}.$$

According to Atanassov an interval valued intuitionistic fuzzy set on X is defined as an object of the form $A=\left\{\left(x,\tilde{\alpha}_A(x),\tilde{\beta}_A(x)\right):x\in X\right\}$, where $\tilde{\alpha}_A(x)$ and $\tilde{\beta}_A(x)$ are interval valued fuzzy sets on X such that $0\le\sup\tilde{\alpha}_A(x)+\sup\tilde{\beta}_A(x)\le 1$ for all $x\in X$.

For the sake of simplicity, in the following such interval valued intuitionistic fuzzy sets will be denoted by $A=\left(\tilde{\alpha}_A,\tilde{\beta}_A\right)$.

3. Interval Valued Intuitionistic (S, T)-Fuzzy Hv-Submodules

In what follows, let M denote an Hv-module over an Hv-ring R unless otherwise.

Definition 3.1. An interval valued intuitionistic fuzzy set $A=\left(\tilde{\alpha}_A,\tilde{\beta}_A\right)$ of M is called an intuitionistic fuzzy Hv-submodule of M with respect to t-norm T and s-norm S (briefly, intuitionistic (S, T)-fuzzy Hv-submodule of M) if it satisfies the following conditions:

(1) $T\left(\tilde{\alpha}_A(x),\tilde{\alpha}_A(y)\right)\le\inf_{z\in x+y}\tilde{\alpha}_A(z)$ and $S\left(\tilde{\beta}_A(x),\tilde{\beta}_A(y)\right)\ge\sup_{z\in x+y}\tilde{\beta}_A(z),\forall x,y\in M$,

(2) For all $x,a\in M$ there exists $y\in M$ such that $x\in a+y$ and $T\left(\tilde{\alpha}_A(a),\tilde{\alpha}_A(x)\right)\le\tilde{\alpha}_A(y)$ and $S\left(\tilde{\beta}_A(a),\tilde{\beta}_A(x)\right)\ge\tilde{\beta}_A(y)$,

(3) For all $x,a\in M$ there exists $z\in M$ such that $x\in z+a$ and $T\left(\tilde{\alpha}_A(a),\tilde{\alpha}_A(x)\right)\le\tilde{\alpha}_A(z)$ and $S\left(\tilde{\beta}_A(a),\tilde{\beta}_A(x)\right)\ge\tilde{\beta}_A(z)$,

(4) $\tilde{\alpha}_A(x)\le\inf_{z\in r\cdot x}\tilde{\alpha}_A(z)$ and $\tilde{\beta}_A(x)\ge\sup_{z\in r\cdot x}\tilde{\beta}_A(z)$, for all $x\in M$ and $r\in R$.

Definition 3.2. The norms T and S are called dual if for all $a,b\in[0,1],\overline{T(a,b)}=S\left(\bar{a},\bar{b}\right)$.

Lemma 3.3. Let T and S be dual norms. If $A=\left(\tilde{\alpha}_A,\tilde{\beta}_A\right)$ is an interval valued intuitionistic (S, T)-fuzzy Hv-submodule of M, then so is □A = $\left(\tilde{\alpha}_A,\overline{\tilde{\alpha}}_A\right)$.

Proof. It is sufficient to show that $\overline{\tilde{\alpha}}_A$ satisfies the conditions of Definition 3.1. For all $x,y\in M$, we have $T\left(\tilde{\alpha}_A(x),\tilde{\alpha}_A(y)\right)\le\inf_{z\in x+y}\tilde{\alpha}_A(z)$ and so $T\left(1-\overline{\tilde{\alpha}}_A(x),1-\overline{\tilde{\alpha}}_A(y)\right)\le\inf_{z\in x+y}\left(1-\overline{\tilde{\alpha}}_A(z)\right)$.

Hence $T\left(1-\overline{\tilde{\alpha}}_A(x),1-\overline{\tilde{\alpha}}_A(y)\right)\le\inf_{z\in x+y}\left(1-\overline{\tilde{\alpha}}_A(z)\right)$.

Which implies $T\left(1-\overline{\tilde{\alpha}}_A(x),1-\overline{\tilde{\alpha}}_A(y)\right)\le 1-\sup_{z\in x+y}\overline{\tilde{\alpha}}_A(z)$ since T and S are dual.

Now, let $a,x\in M$. Then there exists $y\in M$ such that $x\in a+y$ and $T\left(\tilde{\alpha}_A(a),\tilde{\alpha}_A(x)\right)\le\tilde{\alpha}_A(y)$. It follows that that $T\left(1-\overline{\tilde{\alpha}}_A(a),1-\overline{\tilde{\alpha}}_A(x)\right)\le 1-\overline{\tilde{\alpha}}_A(y)$,

so that $\overline{\tilde{\alpha}}_A(y)\le 1-T\left(1-\overline{\tilde{\alpha}}_A(a),1-\overline{\tilde{\alpha}}_A(x)\right)=S\left(\overline{\tilde{\alpha}}_A(a),\overline{\tilde{\alpha}}_A(x)\right)$.

Similarly, let $a,x\in M$. Then there exists $z\in M$ such that $x\in z+a$ and $\overline{\tilde{\alpha}}_A(z)\le S\left(\overline{\tilde{\alpha}}_A(a),\overline{\tilde{\alpha}}_A(x)\right)$.

Now, let $x\in M$ and $r\in R$, we have $\tilde{\alpha}_A(x)\le\inf_{z\in r\cdot x}\tilde{\alpha}_A(z)$ since α_A is a T -fuzzy Hv-submodule of M. Hence $1-\overline{\tilde{\alpha}}_A(x)\le\inf_{z\in r\cdot x}\left(1-\overline{\tilde{\alpha}}_A(z)\right)$ which implies $\sup_{z\in r\cdot x}\overline{\tilde{\alpha}}_A(z)\le\overline{\tilde{\alpha}}_A(x)$. Therefore □A = $\left(\tilde{\alpha}_A,\overline{\tilde{\alpha}}_A\right)$ is an intuitionistic (S, T)-fuzzy Hv-submodule of M.

Lemma 3.4. Let T and S be dual norms. If $A = \left(\tilde{\alpha}_A, \tilde{\beta}_A\right)$ is an interval valued intuitionistic (S, T)-fuzzy Hv-submodule of M, then so is $\Diamond A = \left(\overline{\tilde{\beta}_A}, \tilde{\beta}_A\right)$.

Proof. The proof is similar to the proof of Lemma 3.3.

Combining the above two lemmas it is not difficult to verify that the followingtheorem is valid.

Theorem 3.5. Let T and S be dual norms. Then $A = \left(\tilde{\alpha}_A, \tilde{\beta}_A\right)$ is an interval valued intuitionistic (S, T)-fuzzy Hv-submodule of M if and only if $\square A$ and $\Diamond A$ are interval valued intuitionistic (S, T)-fuzzy Hv-submodules of M.

Corollary 3.6. Let T and S be dual norms. Then $A = \left(\tilde{\alpha}_A, \tilde{\beta}_A\right)$ is an interval valued intuitionistic (S, T)-fuzzy Hv-submodule of M if and only if $\tilde{\alpha}_A$ and $\overline{\tilde{\beta}_A}$ are T –fuzzy Hv-submodules of M.

Definition 3.7. An interval valued intuitionistic (S, T)-fuzzy Hv-submodule $A = \left(\tilde{\alpha}_A, \tilde{\beta}_A\right)$ of M is said to be imaginable if $\tilde{\alpha}_A$ and $\tilde{\beta}_A$ satisfy the imaginable property.

The following are obvious.

Lemma 3.8. Every imaginable interval valued intuitionistic (S, T)-fuzzy Hv-submodule of M is interval valued intuitionistic fuzzy Hv-submodule.

Lemma 3.9. [19] A fuzzy set μ in M is a fuzzy Hv-submodule of M if and only if the non-empty U (μ; α), $\alpha \in [0,1]$ is an Hv-submodule of M.

Lemma 3.10. [19] A fuzzy set μ in M is a fuzzy Hv-submodule of M if and only if the non-empty μ is an anti-fuzzy Hv-submodule of M.

By the above Lemmas, we can give the following results.

Theorem 3.11. If $A = \left(\tilde{\alpha}_A, \tilde{\beta}_A\right)$ is an imaginable interval valued intuitionistic fuzzy set in M. Then $A = \left(\tilde{\alpha}_A, \tilde{\beta}_A\right)$ is an interval valued intuitionistic (S, T)-fuzzy Hv-submodule of M if and only if the non-empty sets $U(\tilde{\alpha}_A;\alpha)$ and $L(\tilde{\beta}_A;\alpha)$ are Hv-submodules of M, for every $\alpha \in [0,1]$.

Theorem 3.12. Let $A = \left(\tilde{\alpha}_A, \tilde{\beta}_A\right)$ be an interval valued intuitionistic (S, T)-fuzzy Hv-submodule of M. Then $\tilde{\alpha}_A(x) = \sup\{\alpha \in [0,1] \mid x \in U(\tilde{\alpha}_A;\alpha)\}$ and $\tilde{\beta}_A(x) = \inf\{\alpha \in [0,1] \mid x \in L(\tilde{\beta}_A;\alpha)\}$, for all $x \in M$.

Definition 3.13. Let f: M → M′ be a strong epimorphism of Hv-modules. If $A = \left(\tilde{\alpha}_A, \tilde{\beta}_A\right)$ is an interval valued intuitionistic fuzzy set in M′, then the inverse image of A under f, denoted by $f^{-1}(A)$, is an interval valued intuitionistic fuzzy set in M, defined by $f^{-1}(A) = (f^{-1}(\tilde{\alpha}_A),\ f^{-1}(\tilde{\beta}_A))$.

By the above Definition, we can give the following result.

Theorem 3.14. Let f : M → M′ be a strong epimorphism of Hv-modules. If $A = \left(\tilde{\alpha}_A, \tilde{\beta}_A\right)$ is an interval valued intuitionistic (S, T)-fuzzy Hv-submodule of M′. Then the inverse image $f^{-1}(A) = (f^{-1}(\tilde{\alpha}_A),\ f^{-1}(\tilde{\beta}_A))$ of A under f is an interval valued intuitionistic (S, T)-fuzzy Hv-submodule of M.

4. Interval Valued Intuitionistic (S, T)-Fuzzy Relations

We first recall that a fuzzy relation on any set X is a fuzzy set μ: X×X → [0, 1]. We now give the following definitions and cite some known results.

Definition 4.1. An interval valued intuitionistic fuzzy set $A = \left(\tilde{\alpha}_A, \tilde{\beta}_A\right)$ is called an interval valued intuitionistic fuzzy relation on any set X if $\tilde{\alpha}_A$ and $\tilde{\beta}_A$ are fuzzy relations on X.

Definition 4.2. Let $A = \left(\tilde{\alpha}_A, \tilde{\beta}_A\right)$ and $B = \left(\tilde{\alpha}_B, \tilde{\beta}_B\right)$ be interval valued intuitionstic fuzzy sets on a set X. If $A = \left(\tilde{\alpha}_A, \tilde{\beta}_A\right)$ is an interval valued intuitionistic fuzzy relation on X, then $A = \left(\tilde{\alpha}_A, \tilde{\beta}_A\right)$ is called an interval valued intuitionistic (S, T)-fuzzy relation on $B = \left(\tilde{\alpha}_B, \tilde{\beta}_B\right)$ if and $\tilde{\beta}_A(x,\ y) \geq S(\tilde{\beta}_B(x),\ \tilde{\beta}_B(y))$, for all $x,\ y \in X$.

Definition 4.3. The interval valued intuitionistic (S, T)-Cartesian product of A and B, denoted by A × B, is an interval valued intuitionistic fuzzy set on X, which is defined by $A \times B = (\tilde{\alpha}_A, \tilde{\beta}_A) \times (\tilde{\alpha}_B, \tilde{\beta}_B) = (\tilde{\alpha}_A \times \tilde{\alpha}_B,\ \tilde{\beta}_A \times \tilde{\beta}_B)$, where $(\tilde{\alpha}_A \times \tilde{\alpha}_B)(x,\ y) = T(\tilde{\alpha}_A(x),\ \tilde{\alpha}_B(y))$ and $(\tilde{\beta}_A \times \tilde{\beta}_B)(x,\ y) = S(\tilde{\beta}_A(x),\ \tilde{\beta}_B(y))$ hold for all $x,\ y \in X$.

Lemma 4.4. If $A = \left(\tilde{\alpha}_A, \tilde{\beta}_A\right)$ and $B = \left(\tilde{\alpha}_B, \tilde{\beta}_B\right)$ are interval valued intuitionistic fuzzy sets on a set X. Then we have

(i) A × B is an interval valued intuitionistic (S, T)-fuzzy relation on X;

(ii) $U(\tilde{\alpha}_A \times \tilde{\alpha}_B;\alpha) = U(\tilde{\alpha}_A;\alpha)\times U(\tilde{\alpha}_B;\alpha)$ and $U(\tilde{\beta}_A \times \tilde{\beta}_B;\alpha) = U(\tilde{\beta}_A;\alpha)\times U(\tilde{\beta}_B;\alpha)$ for all $\alpha \in [0,1]$.

Definition 4.5. If $A = (\tilde{\alpha}_A, \tilde{\beta}_A)$ and $B = (\tilde{\alpha}_B, \tilde{\beta}_B)$ are interval valued intuitionistic fuzzy sets on a set X, the strongest interval valued intuitionistic (S, T)-fuzzy relation on X is defined by $A_B = (\tilde{\alpha}_{A_{\alpha_B}}, \tilde{\beta}_{A_{\beta_B}})$, where $\tilde{\alpha}_{A_{\alpha_B}}(x, y) = T(\tilde{\alpha}_B(x), \tilde{\alpha}_B(y))$ and $\tilde{\beta}_{A_{\beta_B}}(x, y) = S(\tilde{\beta}_B(x), \tilde{\beta}_B(y))$ for all $x, y \in X$.

Lemma 4.6. For the interval valued intuitionistic fuzzy sets $A = (\tilde{\alpha}_A, \tilde{\beta}_A)$ and $B = (\tilde{\alpha}_B, \tilde{\beta}_B)$ on a set X, let A_B be the strongest interval valued intuitionistic (S, T)-fuzzy relation on X. Then for any $\alpha \in [0,1]$, we have $U(\tilde{\alpha}_{A_{\alpha_B}};\alpha) = U(\tilde{\alpha}_B;\alpha)\times U(\tilde{\alpha}_B;\alpha)$ and $L(\tilde{\beta}_{A_{\beta_B}};\alpha) = L(\tilde{\beta}_B;\alpha)\times L(\tilde{\beta}_B;\alpha)$.

Lemma 4.7. [20] For all $\alpha, \beta, \delta, \gamma \in [0,1]$, we have T (T (α, β), T (γ, δ)) = T (T (α, γ), (β, δ)); S(S(α, β), S(γ, δ)) = (S(α, γ), S(β, δ)).

By using the above lemmas, we have the following theorem.

Theorem 4.8. If $A = (\tilde{\alpha}_A, \tilde{\beta}_A)$ and $B = (\tilde{\alpha}_B, \tilde{\beta}_B)$ are interval valued intuitionistic (S, T)-fuzzy Hv-submodules of M. Then A×B is an interval valued intuitionistic (S, T)-fuzzy Hv-submodule of M ×M.

Corollary 4.9. If $A = (\tilde{\alpha}_A, \tilde{\beta}_A)$ and $B = (\tilde{\alpha}_B, \tilde{\beta}_B)$ are imaginable interval valued intuitionistic (S, T)-fuzzy Hv-submodules of M. Then A × B is an imaginable interval valued intuitionistic (S, T)-fuzzy Hv-submodule of M ×M.

The following theorem characterizes the imaginable interval valued intuitionistic (S, T)-fuzzy Hv-submodules on Hv-modules.

Theorem 4.10. If $A = (\tilde{\alpha}_A, \tilde{\beta}_A)$ and $B = (\tilde{\alpha}_B, \tilde{\beta}_B)$ are imaginable interval valued intuitionistic fuzzy sets of M and A_B is the strongest interval valued intuitionistic (S, T)-fuzzy relation on M. Then $B = (\tilde{\alpha}_B, \tilde{\beta}_B)$ is an imaginable interval valued intuitionistic (S, T)-fuzzy Hv-submodule of M if and only if A_B is an imaginable interval valued intuitionistic (S, T)-fuzzy Hv-submodule of M ×M.

Proof. Let $B = (\tilde{\alpha}_B, \tilde{\beta}_B)$ be an imaginable interval valued intuitionistic (S, T)-fuzzy Hv- submodule of M. Then we can verify the following conditions of definition 3.1.

(1) Let $x = (x_1, x_2), y = (y_1, y_2) \in M\times M$. For any $z = (z_1, z_2) \in x + y$, we have

$$\begin{aligned}
\inf_{z\in x+y} \tilde{\alpha}_{A_{\alpha_B}}(z) &= \inf_{(z_1,z_2)\in(x_1,x_2)+(y_1,y_2)} \tilde{\alpha}_{A_{\alpha_B}}(z_1, z_2)\\
&= \inf_{(z_1,z_2)\in(x_1+y_1,x_2+y_2)} \{T(\tilde{\alpha}_B(z_1), \tilde{\alpha}_B(z_2))\}\\
&= T(\inf_{z_1\in x_1+y_1} \tilde{\alpha}_B(z_1), \inf_{z_2\in x_2+y_2} \tilde{\alpha}_B(z_2))\\
&\geq T(T(\tilde{\alpha}_B(x_1), \tilde{\alpha}_B(y_1)), T(\tilde{\alpha}_B(x_2), \tilde{\alpha}_B(y_2)))\\
&= T(T(\tilde{\alpha}_B(x_1), \tilde{\alpha}_B(x_2)), T(\tilde{\alpha}_B(y_1), \tilde{\alpha}_B(y_2)))\\
&= T(\tilde{\alpha}_A\tilde{\alpha}_B(x_1, x_2), \tilde{\alpha}_A\tilde{\alpha}_B(y_1, y_2))\\
&= T(\tilde{\alpha}_A\tilde{\alpha}_B(x), \tilde{\alpha}_A\tilde{\alpha}_B(y)).
\end{aligned}$$

Similarly, we have $\sup_{z\in x+y}\tilde{\beta}_{A_{\alpha_B}}(z) \leq S(\tilde{\beta}_{A_{\alpha_B}}(x), \tilde{\beta}_{A_{\alpha_B}}(y))$.

(2) For all $x = (x_1, x_2), a = (a_1, a_2) \in M\times M$. Then $y_1, y_2 \in M$ such that $x_1 \in a_1 + y_1$ and $x_2 \in a_2 + y_2$, and thus $(x_1, x_2) \in (a_1 + y_1, a_2 + y_2) = (a_1, a_2) + (y_1, y_2)$. Moreover, we have

$$\begin{aligned}
\tilde{\alpha}_{A_{\alpha_B}}(y) = \tilde{\alpha}_{A_{\alpha_B}}(y_1, y_2) &= T(\tilde{\alpha}_B(y_1), \tilde{\alpha}_B(y_2))\\
&\geq T(T(\tilde{\alpha}_B(a_1), \tilde{\alpha}_B(x_1)), T(\tilde{\alpha}_B(a_2), \tilde{\alpha}_B(x_2)))\\
&= (T(\tilde{\alpha}_B(a_1), \tilde{\alpha}_B(a_2)), T(\tilde{\alpha}_B(x_1), \tilde{\alpha}_B(x_2)))\\
&= T(\tilde{\alpha}_{A_{\alpha_B}}(a_1, a_2), \tilde{\alpha}_{A_{\alpha_B}}(x_1, x_2))\\
&= T(\tilde{\alpha}_{A_{\alpha_B}}(a), \tilde{\alpha}_{A_{\alpha_B}}(x))
\end{aligned}$$

Similarly, $\tilde{\beta}_{A_{\beta_B}}(y) \leq S(\tilde{\beta}_{A_{\beta_B}}(a), \tilde{\beta}_{A_{\beta_B}}(x))$.

(3) is similar to (2).

(4) Let $x = (x_1, x_2) \in M \times M$ and $r = (r_1, r_2) \in R \times R$. For any $z = (z_1, z_2) \in (r_1, r_2) \cdot (x_1, x_2)$, we have

$$\begin{aligned}\inf_{z \in r \cdot x} \tilde{\alpha}_{A_{\alpha_B}}(z) &= \inf_{(z_1, z_2) \in (r_1, r_2) \cdot (x_1, x_2)} \tilde{\alpha}_{A_{\alpha_B}}(z_1, z_2) \\ &= \inf_{(z_1, z_2) \in (r_1 \cdot x_1, r_2 \cdot x_2)} T(\tilde{\alpha}_B(z_1), \tilde{\alpha}_B(z_2)) \\ &\geq T(\inf_{z_1 \in r_1 \cdot x_1} \tilde{\alpha}_B(z_1), \inf_{z_2 \in r_2 \cdot x_2} \tilde{\alpha}_B(z_2)) \\ &\geq T(\tilde{\alpha}_B(x_1), \tilde{\alpha}_B(x_2)) \\ &= \tilde{\alpha}_{A_{\alpha_B}}(x_1, x_2) = \tilde{\alpha}_{A_{\alpha_B}}(x)\end{aligned}$$

Similarly, $\sup_{z \in r \cdot x} \tilde{\beta}_{A_{\beta_B}}(z) \leq \tilde{\beta}_{A_{\beta_B}}(x)$.

This shows that A_B is an interval valued intuitionistic (S, T)-fuzzy Hv-submodule of M×M.

Now, for any $x = (x_1, x_2) \in M \times M$, we can easily show that $T(\tilde{\alpha}_{A_{\alpha_B}}(x), \tilde{\alpha}_{A_{\alpha_B}}(x)) = \tilde{\alpha}_{A_{\alpha_B}}(x)$ and $S(\tilde{\beta}_{A_{\beta_B}}(x), \tilde{\beta}_{A_{\beta_B}}(x)) = \tilde{\beta}_{A_{\beta_B}}(x)$.

Hence, A_B is an interval valued imaginable intuitionistic (S, T)-fuzzy Hv-submodule of M ×M.

To prove the converse of the theorem, we need prove the conditions (1)-(4) of definition 3.1 hold.

(1) Let $x, y \in M$. Then we have

$$\begin{aligned}\inf_{z \in x+y} \tilde{\alpha}_B(z) &= \inf_{z \in x+y} T(\tilde{\alpha}_B(z), \tilde{\alpha}_B(z)) \\ &= \inf_{z \in x+y} \tilde{\alpha}_{A_{\alpha_B}}(z, z) \\ &= \inf_{(z,z) \in (x,x)+(y,y)} \tilde{\alpha}_{A_{\alpha_B}}(z, z) \\ &\geq T(\tilde{\alpha}_{A_{\alpha_B}}(x, x), \tilde{\alpha}_{A_{\alpha_B}}(y, y)) \\ &\geq T(T(\tilde{\alpha}_B(x), \tilde{\alpha}_B(x)), T(\tilde{\alpha}_B(y), \tilde{\alpha}_B(y))) \\ &= T(\tilde{\alpha}_B(x), \tilde{\alpha}_B(y)).\end{aligned}$$

Similarly, we have $\sup_{z \in x+y} \tilde{\beta}_B(z) \leq S(\tilde{\beta}_B(x), \tilde{\beta}_B(y))$.

(2) For all $x, a \in M$, and thus $(x, x), (a, a) \in M \times M$. Then there exists $(y, y) \in M$ such that $(x, x) \in (a, a) + (y, y) = (a + y, a + y)$. That is, $x \in a + y$.

Moreover, we have

$$\begin{aligned}\tilde{\alpha}_B(y) &= T(\tilde{\alpha}_B(y), \tilde{\alpha}_B(y)) = \tilde{\alpha}_{A_{\alpha_B}}(y, y) \\ &\geq T(\tilde{\alpha}_{A_{\alpha_B}}(a, a), \tilde{\alpha}_{A_{\alpha_B}}(x, x)) \\ &= T(T(\tilde{\alpha}_B(a), \tilde{\alpha}_B(a)), T(\tilde{\alpha}_B(x), \tilde{\alpha}_B(x))) \\ &= T(\tilde{\alpha}_B(a), \tilde{\alpha}_B(x)).\end{aligned}$$

Similarly, $\tilde{\beta}_B(y) \leq S(\tilde{\beta}_B(a), \tilde{\beta}_B(x))$.

(3) is similar to (2).

(4) Let $x \in M$ and $r \in R$, we have

$$\begin{aligned}\inf_{z \in r \cdot x} \tilde{\alpha}_B(z) &= \inf_{z \in r \cdot x} T(\tilde{\alpha}_B(z), \tilde{\alpha}_B(z)) \\ &= \inf_{(z,z) \in (r,r) \cdot (x,x)} \tilde{\alpha}_{A_{\alpha_B}}(z, z) \\ &\geq \tilde{\alpha}_{A_{\alpha_B}}(x, x) \\ &= T(\tilde{\alpha}_B(x), \tilde{\alpha}_B(x)) \\ &= \tilde{\alpha}_B(x).\end{aligned}$$

Similarly, $\sup_{z \in r \cdot x} \tilde{\beta}_B(z) \leq \tilde{\beta}_{A_{\beta_B}}(x)$.

This shows that conditions (1)-(4) hold and hence $B = \left(\tilde{\alpha}_B, \tilde{\beta}_B\right)$ is an imaginable interval valued intuitionistic (S, T)-fuzzy Hv-submodule of M.

Definition 4.11. If $A = \left(\tilde{\alpha}_A, \tilde{\beta}_A\right)$ and $B = \left(\tilde{\alpha}_B, \tilde{\beta}_B\right)$ are imaginable intuitionistic fuzzy sets on any set X, then the intuitionistic (S, T)-product of A and B, denoted by $[A\cdot\ B]_{(S,T)}$, is defined by

$$[A\cdot\ B]_{(S,T)} = [(\tilde{\alpha}_A,\ \tilde{\beta}_A)\cdot(\tilde{\alpha}_B,\ \tilde{\beta}_B)]_{(S,T)}$$

$$= ([\tilde{\alpha}_A\cdot\ \tilde{\alpha}_B],[\tilde{\beta}_A\cdot\ \tilde{\beta}_B])_{(S,T)}$$

$$= ([\tilde{\alpha}_A\cdot\ \tilde{\alpha}_B]_T,[\tilde{\beta}_A\cdot\ \tilde{\beta}_B]_S),$$

Where $[\tilde{\alpha}_A\cdot\ \tilde{\alpha}_B]_T(x) = T(\tilde{\alpha}_A(x),\ \tilde{\alpha}_B(x))$ and $[\tilde{\beta}_A\cdot\ \tilde{\beta}_B]_S(x) = S(\tilde{\beta}_A(x),\ \tilde{\beta}_B(x)),$ for all $x \in X.$

Theorem 4.12. If $A = \left(\tilde{\alpha}_A, \tilde{\beta}_A\right)$ and $B = \left(\tilde{\alpha}_B, \tilde{\beta}_B\right)$ are imaginable interval valued intuitionistic (S, T)-fuzzy Hv-submodules of M. If T^* (resp. S^*) is a t-norm (resp. s-norm) which dominates T (resp. S), that is, $T^*(T(\alpha,\ \beta),\ T(\gamma,\ \delta)) \geq T(T^*(\alpha,\ \gamma),\ T^*(\beta,\ \delta))$ and $S^*(S(\alpha,\ \beta),\ S(\gamma,\ \delta)) \leq S(S^*(\alpha,\ \gamma),\ S^*(\beta,\ \delta))$ for all $\alpha, \beta,\ \gamma,\ \delta \in [0,1].$ Then for the intuitionistic $(S^*,\ T^*)$-product of A and B, $[A\cdot\ B]_{(S^*,T^*)}$ is an intuitionistic (S, T)-fuzzy Hv-submodule of M.

Proof. In proving this theorem, we only need verify that the conditions (1)-(4) hold. The verification is montioned and we omit the details.

Let f: M → M′ be a strong epimorphism of Hv-modules. Let T (resp. S) and T^* (resp. S^*) be the t-norms (resp. s-norms) such that T^* (resp. S^*) dominates T (resp. S). If $A = \left(\tilde{\alpha}_A, \tilde{\beta}_A\right)$ and $B = \left(\tilde{\alpha}_B, \tilde{\beta}_B\right)$ are imaginable interval valued intuitionistic fuzzy Hv-submodules of M′, then the intuitionistic $(S^*,\ T^*)$-product of A and B, we have $[A\cdot\ B]_{(S^*,T^*)}$ is an intuitionistic (S, T)-fuzzy Hv-submodule of M′. Since every strong epimorphic inverse image of an intuitionistic (S, T)-fuzzy Hv-submodule is an intuitionistic (S, T)-fuzzy Hv-submodule, the inverse images $f^{-1}(A),\ f^{-1}(B),$ and $f^{-1}([A\cdot\ B]_{(S^*,T^*)})$ are also intuitionistic (S, T)-fuzzy Hv-submodules of M. In the next theorem, we described that the relation between $f^{-1}([A\cdot\ B]_{(S^*,T^*)})$ and intuitionistic $(S^*,\ T^*)$-product $[f^{-1}(A)\cdot\ f^{-1}(B)]_{(S^*,T^*)}$ of $f^{-1}(A)$ and $f^{-1}(B).$

Based on the above discussion, we have:

Theorem 4.13. Let f: M → M′ be a strong epimorphism of Hv-modules. Let T^* (resp. S^*) be a t-norm (resp. s-norm) such that T^* (resp. S^*) dominates T (resp. S). If $A = \left(\tilde{\alpha}_A, \tilde{\beta}_A\right)$ and $B = \left(\tilde{\alpha}_B, \tilde{\beta}_B\right)$ are intuitionistic (S, T)-fuzzy Hv-submodule of M′. Then for the intuitionistic $(S^*,\ T^*)$ -product $[A\cdot\ B]_{(S^*,T^*)}$ of A and B and the intuitionistic $(S^*,\ T^*)$-product $[f^{-1}(A)\cdot\ f^{-1}(B)]_{(S^*,T^*)}$ of $f^{-1}(A)$ and $f^{-1}(B)$ we have $f^{-1}([A\cdot\ B]_{(S^*,T^*)}) = [f^{-1}(A)\cdot\ f^{-1}(B)]_{(S^*,T^*)}$.

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