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Fractional Roman Domination

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Abstract: A subset $\boldsymbol{S}$ of the vertex set $\boldsymbol{V}$ of the graph $\boldsymbol{G} = (\boldsymbol{V}, \boldsymbol{E})$ is called a dominating set, if every vertex in $\boldsymbol{V} - \boldsymbol{S}$ is adjacent to at least one vertex in $\boldsymbol{S}$. The characteristic function of the dominating set is a function $\boldsymbol{f}: \boldsymbol{V} \to [\boldsymbol{0}, \boldsymbol{1}]$ such that $\boldsymbol{f}(\boldsymbol{v}) = \boldsymbol{1}$ if $\boldsymbol{v} \in \boldsymbol{S}$ and $\boldsymbol{f}(\boldsymbol{v}) = \boldsymbol{0}$ otherwise. An important version of the dominating set is called the Roman dominating set. The Roman dominating set can be defined in the form of a function $\boldsymbol{f}: \boldsymbol{V} \to \{\boldsymbol{0}, \boldsymbol{1}, \boldsymbol{2}\}$ such that every vertex $\boldsymbol{v}$ with $\boldsymbol{f}(\boldsymbol{v}) = \boldsymbol{0}$ has a neighbor $\boldsymbol{u}$ with $\boldsymbol{f}(\boldsymbol{u}) = \boldsymbol{2}$. This function is called a Roman dominating function (RDF) of the graph $\boldsymbol{G}$. In this paper we initiate a study the fractional version of the Roman dominating function.

***Index Terms* – Dominating set, Dominating function, Roman domination, Fractional domination, fractional Roman domination.**

I. Introduction

A subset S of the vertex set V of the graph $G = (V, E)$ is called a dominating set, if every vertex in $V - S$ is adjacent to at least one vertex in S. The characteristic function of the dominating set is a function $f: V \to [0,1]$ such that $f(v) = 1$ if $v \in S$ and $f(v) = 0$ otherwise. A fractional version of the dominating function is obtained when the function f is allowed to take all real values from the interval $[0,1]$. A dominating set S is minimal if no proper subset of S is a dominating set. Equivalently, the dominating function f representing S is minimal if we cannot obtain another dominating function g by reducing the function values of f. The fractional version of dominating function and related concepts were studied by Grinstead et al. [5], Cockayne et al. [1, 2, 3], and subsequently by Reji Kumar et al. [6, 7, 8, 9, 10, 11, 12, 13].

An important variation of the dominating set is called the Roman dominating set. The Roman dominating set can be defined in the form of a function $f: V \to \{0,1,2\}$ such that every vertex v with $f(v) = 0$ has a neighbor u with $f(u) = 2$. This function is called a Roman dominating function (RDF) of the graph G [4].

An RDF $f = (V_0, V_1, V_2)$ is reducible if it is possible to reduce the value of $f(v)$ for some vertex v and still the resulting function be an RDF. An RDF is irreducible if it is not reducible. But the minimality of Roman dominating function is something more than irreducibility. Minimality has four versions, which are described in [4]. An RDF is a Type - I minimal if it is irreducible.

The second type of minimality has been suggested by Renu Laskar [4]. An RDF $f = (V_0, V_1, V_2)$ is Type II minimal if it is irreducible and V_0 is not empty. The third definition of minimality was developed by Steve Hedetniemi [4], The third definition of minimality was developed by Steve Hedetniemi [4], which uses the notion of sliding tokens between vertices in the graph. For a given RDF $f = (V_0, V_1, V_2)$, $f(v) = i$ means that there are i tokens on v. A slide of a token from $u \in V_1 \cup V_2$ to $v \in V_0 \cup V_1$ is called a valid slide if the new function f', defined by $f'(u) = f(u) - 1$, $f'(v) = f(v) + 1$, $f'(w) = f(w)$, $w \neq u, v$ is also an RDF. If f is an irreducible RDF, there are only two types of possible valid slides: sliding a token from a vertex in V_1 to another vertex in V_1, or sliding a token from a vertex in $v \in V_2$ to a vertex in $w \in V_0$, provided that w is the only external $V_2 - pn$ of v. An RDF f is Type III minimal if

(1) f is irreducible and

(2) no sequence of valid slides changes f into a reducible RDF f'.

The last version of minimality is due to Alice McRae [4]. An RDF $f = (V_0, V_1, V_2)$ is Type IV minimal if (1) f is irreducible and

(2) the set V_1 is an independent set.

In Graph Theory, many concepts are dealt with treating it as the presence or absence of elements in a specified set. The presence of a vertex is denoted by the number 1 and absence by 0. For example, a dominating set D of G is the whole vertex set V of the graph together with a function f defined on it such that $f(v) = 1$ if $v \in D$ and $f(v) = 0$ if v is not an element of D. This function is integer-valued.

Fractional graph theory deals with the generalization of integer-valued graph theoretic concepts such that they take fractional values. One of the standard methods for converting a graph-theoretic concept from integer version to fractional version is to formulate the concept as an integer program and then to consider the linear programming relaxation. In the following section, we discuss the fractional version of Roman Domination.

II. Fractional Roman Domination

We can define different varieties of fractional Roman domination taking into account different aspects of Roman domination. First, we consider the fact that a Roman dominating function $f = (V_0, V_1, V_2)$ has the property that every vertex in V_0 is adjacent to at least one vertex in V_2. We can fractionalize the Roman dominating function by allowing the function values to vary freely in the closed interval $[0, 2]$. The vertices having function values 2 can supply one item to one of the vertices having zero as function value and adjacent to it. The vertex can do this only if it has one item for its use. We can fractionalize the situation as follows. A function $f: V \to [0, 2]$ such that every vertex having function value zero is adjacent to at least one vertex having function value greater than one is a kind of fractional Roman dominating function. To make the discussion more clear, we need the following definitions.

For the function $f: V \to [0,2]$,

$V_0^f = v \in V | f(v) = 0\}$, $V_1^f = v \in V | f(v) = 1\}$, $V_2^f = v \in V | f(v) = 2\}$.

$V_{(0,1]}^f = v \in V |\ 1 \geq f(v) > 0\}$ and $V_{[2,1)}^f = v \in V |\ 1 < f(v) \leq 2\}$.

A function $f: V \to [0,2]$ is a fractional Roman dominating function if $V_{[2,1)}^f$ dominates $V_0{}^f$.

A fractional Roman dominating function f is reducible if it is possible to reduce the value of $f(v)$ for some vertex v and obtain a new fractional Roman dominating function of the same graph. Otherwise, it is irreducible. We can reduce the function value of a fractional Roman dominating function f freely if the function value $f(v)>1$. This reduction will not change the vertex sets $V_{[2,1)}^f$, V_0^f, and $V_2{}^f$. But the function value can be reduced to one (then the vertex is removed from $V_{[2,1)}^f$ and included in $V_1{}^f$) only if all vertices in V_0, which are adjacent to $v \in V_{[2,1)}^f$. Thus if the new function obtained is g, then $V_{[2,1)}^g$ is a subset of $V_{[2,1)}^f$, V_1^g is a superset of $V_1{}^f$, and $V_0{}^g = V_0{}^f$. In addition to this if a vertex $u \in V_1^g$ is adjacent to some vertices in $V_{[2,1)}^g$, we can reduce the function values at the vertices to zero and obtain a new fractional Roman dominating function h. Then $V_{[2,1)}^g = V_{[2,1)}^f$, V_0^h is a superset of V_0^g, and $V_1{}^h$ is a subset of V_1^g.

III. Conclusion

Study on the fractional version of various domination parameters is a very active area of research. In many situations, the fractional version allows us to express the values of the parameters as fractions, which gives us more insight into the characteristics of the parameter specifically and that of the underlying graph generally. In fractional versions, we choose function values for the vertices or edges from intervals, instead of a set of integers. The study on the fractional version of Roman domination must get more attention.

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REFERENCES

[1] E. J, Cokayne, Gary MacGillivray, and C. M Mynhardt, Convexity of Minimal dominating functions and universal graphs, Bull, Inst. Combin. Appl., 5 (1992) 37-48.

[2] E. J. Cockayne, G. MacGillivray, and C.M Mynhardt, Convexity of minimal dominating functions of Trees-II, Discrete Mathematics.,125 (1994) 137-146.

[3] E. J. Cockayne, G. Fricke, S. T. Hedetniemi, and C.M Mynhardt, Properties of minimal dominating functions of graphs.ARS Combinatoria.,41 (1995}, 107-115.

[4] P. A. Dreyer Jr., Applications and Variations of Dominations in graphs, A dissertation submitted to the Graduate School for the award for the degree of Doctor of Philosophy, New Brunswick, New Jersy October 2000

[5] D. I. Grinstead and P.J. Slater, Fractional domination and fractional packing graphs, Congr. Numer, 71 (1990), 153-172.

[6] K Reji Kumar, Topological properties of the set of all minimal dominating of graphs, to appear in the proceedings of ICDM 2006 published by RMS lecture notes series (2008).

[7] K. Reji Kumar, Gary MacGillivray, and R. B. Bapat, Topological properties of the set of all minimal total dominating functions of graphs, J. Combin. Math. Combin.Comput.76 (2011), 213-223.

[8] K Reji Kumar, Gary MacGillivray, Structure of a set of all minimal total dominating functions of some classes of graphs, Discuss. Math Graph Theory 30(2010), No.3, 407-423.

[9] K Reji Kumar, Convexity of minimal dominating functions and minimal edge dominating functions, Complexity in the discrete structures,97104, Ramanujan Math. Soc. Lect. Notes Ser., 5, Ramanujan Math. Soc. Mysore 2008.

[10] K. Reji Kumar, S Arumugam, Basic minimal dominating functions Util.Math.77 (2008), 235-247.

[11] K Reji Kumar, S Arumugam, G. MacGillvray, On maximal fractional independent sets in graphs. Ars Combin. 87 (2008), 205-212.

[12] K Reji Kumar, Convexity graph of minimal dominating functions. Bull. Kerala Math. Assoc.4 (2007), No.2, 27-36.

[13] K. Reji Kumar, Studies in Graph Theory-Dominating functions, Ph.D. Thesis, Manonmaniam Sundaranar University, Thirunelveli, India, 2004