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Effect of weak magnetic field and low thermal conductivity on thermal instability in a porous medium

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Abstract. The paper examines, within the framework of linear analysis, the thermal instability of compressible fluid layer in a porous medium in the presence of weak magnetic field and low thermal conductivity. The important results are obtained in this paper depend on $DT_0 + \ g/Cp$.

Keywords : Thermal instability, compressible fluid layer, porous medium, weak magnetic field, low thermal conductivity.

2000 Mathematics subject classification : 76S05

1. Introduction

Thermal instability of flows in a porous medium has been extensively studied by enormous research workers on the basis of Darcy's law[1] which completely neglects the viscous forces. An excellent review has been given by Scheidegger[2] and Yih[3] . Brinkman[4] proposed a plausible modification to Darcy's law that takes into account the viscous forces. Jaimala and Agrawal[5] , unlike Irmay[6] and Beck[7] , investigated a more physically realistic model in which the inertia and viscous forces are included in their usual forms to account for the flows with high flow rates, high permeability and high viscous forces.

Instability of compressible and incompressible flows has been studied extensively by a number of research workers in past few decades. In almost all such investigations, the Boussinesq's approximation is used to simplify the equations of motion. Jeffreys[8] tried to provide a justification of under certain conditions. If the vertical dimension of the fluid be much less than the scale height and the fluctuations in temperature, pressure and density, introduced due to the Boussinesq's approximation for steady, infinitesimal motions of compressible fluids and Spiegel and Veronis[9] simplified the set of equations governing the compressible flows motion, do not exceed their total static variations, then the flow equations are same as for incompressible fluids, the static temperature gradient being replaced by its excess over the adiabatic gradient and the specific heat at constant volume C_v by that at constant pressure C_p.

Using the model as suggested by Spiegel and Veronis Banerjee and Agrawal[10] investigated the thermal instability of parallel shear flows in the presence of both adverse and non-adverse temperature gradients. In an important paper, Bansal and Agrawal[11] investigated the thermal instability of a compressible shear flow with weak applied magnetic field in the presence of adverse or non-adverse temperature gradient.

In the present analysis, we have examined within the frame work of linear analysis, the thermal instability of a compressible shear layer in a porous medium. Though, some literature has been reported in which magnetic filed destabilizes a wave number range known to be stable in its absence [Kent[12] , Gilman[13] , Jain[14]], in most of the situations magnetic field has a stabilizing effect.

As explained by Chand and Agrawal[15] , the problem under investigation provides a reasonably good mathematical model to a narrow layer of atmosphere above earth's surface so that the curvature effects can be neglected and the boundaries be taken as horizontal, the lower boundary (the earth's surface) being rigid and the upper boundary being free. Temperature variations are due to the Sun and the pattern of temperature variations (increasing or decreasing in the vertical direction) depends upon the place/ time of investigation. The assumption of small thermal conductivity not only simplifies the mathematical analysis, thus leading to a number of interesting results, but also is important in the context of physical situations in which the transfer of heat is not instantaneous (case of very large thermal conductivity), rather it is slow. We consider the physical situation is which the wind speed is small so that our study is restricted to the case of low Mach number. The study is also restricted to the case of large wave numbers only.

2. Formulation of the problem

In this paper we consider an unidirectional flow of a compressible, viscous and heat conducting fluid in the presence of a horizontal magnetic field in a homogeneous and isotropic porous medium, confined between two infinite parallel plates situated at a distance d apart. In a cartesian frame of reference, the axis of x is in the main flow direction and the axis of z is against gravity. Boundaries are maintained at constant temperatures T_1 and T_2 respectively. The layer depth is small enough so that it is much less than the scale height as defined by Spiegel and Veronis. The basic state under investigation is, therefore, characterized by

$$\left.\begin{array}{lll}\mathbf{V}=[0,0,0], & T=T(z), & \mathbf{H}=(H,0,0) \\ \rho=\rho(z), & and & p=p(z),\end{array}\right\} \qquad ...(1)$$

where $\boldsymbol{V}$, T, $\boldsymbol{H}$ ρ and P are the fluid velocity, temperature, magnetic field, density and pressure respectively.

Further, $T(z) = D(T_0)\,z + T_1,$

$$p(z) = p_m - g\int_0^z (\rho_m + \rho_0)\,dz$$

and $\rho(z) = \rho_m\left[1-\alpha_m(T-T_m)+k_m(p-p_m)\right].$

The time independent solution of the governing equations does not change in the presence of a uniform magnetic field and therefore the basic velocity distribution remains the same as in Chand and Agrawal. DT_0 is the uniform static temperature gradient and α_m and k_m are given as

$$\alpha_m = \frac{1}{T_m} = -\left(\frac{1}{\rho}\,\frac{\partial \rho}{\partial T}\right)_m = \alpha\ (say) \quad \text{and} \quad k_m = \left(\frac{1}{\rho}\,\frac{\partial \rho}{\partial p}\right)_m . \qquad ...(2)$$

3. PERTURBED STATE SOLUTION AND THE LINEARIZED PERTURBATION EQUATIONS

We now suppose that the solution in the basic state is slightly perturbed so that every physical quantity is assumed to be the sum of a mean and a fluctuating component, later designated as primed quantity and assumed to be very small in comparison to its basic state value. The small disturbances are assumed to be the functions of the space as well as time variables

Following **Spiegel** and **Veronis,** the linearized perturbation equations of motion, continuity, heat conduction and magnetic field are

$$\frac{\rho_m}{\phi}\frac{\partial u'}{\partial t} = -\frac{\partial p'}{\partial x} + \mu\left(\frac{1}{\phi}\nabla^2 - \frac{1}{k_1}\right)u', \quad \text{... (3)}$$

$$\frac{\rho_m}{\phi}\frac{\partial v'}{\partial t} = -\frac{\partial p'}{\partial y} + \mu\left(\frac{1}{\phi}\nabla^2 - \frac{1}{k_1}\right)v' + \frac{H}{4\pi}\left(\frac{\partial h_y'}{\partial x} - \frac{\partial h_x'}{\partial y}\right), \quad \text{...(4)}$$

$$\frac{\rho_m}{\phi}\frac{\partial w'}{\partial t} = -\frac{\partial p'}{\partial z} + \mu\left(\frac{1}{\phi}\nabla^2 - \frac{1}{k_1}\right)w' + \frac{H}{4\pi}\left(\frac{\partial h_z'}{\partial x} - \frac{\partial h_x'}{\partial z}\right) + g\alpha\rho_m\theta', \quad \text{...(5)}$$

$$\frac{\partial u'}{\partial x} + \frac{\partial v'}{\partial y} + \frac{\partial w'}{\partial z} = 0, \quad \text{...(6)}$$

$$\frac{\partial h_x'}{\partial t} = -\frac{H}{\phi}\left[\frac{\partial v'}{\partial y} + \frac{\partial w'}{\partial z}\right], \quad \text{...(7)}$$

$$\frac{\partial h_y'}{\partial t} = \frac{H}{\phi}\frac{\partial v'}{\partial x}, \quad \text{...(8)}$$

$$\frac{\partial h_z'}{\partial t} = \frac{H}{\phi}\frac{\partial w'}{\partial x}, \quad \text{...(9)}$$

$$\frac{\partial h_x'}{\partial x} + \frac{\partial h_y'}{\partial y} + \frac{\partial h_z'}{\partial z} = 0, \quad \text{...(10)}$$

and

$$\frac{\partial \theta'}{\partial t} + \frac{w'}{\phi}\left(DT_0 + g/C_p\right) = \kappa\nabla^2\theta', \quad \text{...(11)}$$

where a dash denotes the perturbation quantity and $\kappa = \kappa'/\rho_m C_p .\kappa', \phi, k_1$ and μ are respectively the thermal conductivity, medium porosity, medium permeability and the coefficient of viscosity. $DT_0 - (-g/C_p)$ is the excess over the adiabatic gradient $(-g/C_p)$ and the other symbols have their same meaning as in **Spiegel** and **Veronis**.

We analyse the perturbations into normal modes, taking dependence of any perturbation quantity $f'(x,y,z,t)$ of the form.

$$f'(x,y,z,t) = f(z)\exp\left[i\left(k_x x + k_y y - \frac{k_x C}{\phi}t\right)\right],$$

where k_x and k_y are the real wave numbers in x and y directions, $k = \sqrt{k_x^{\,2} + k_y^{\,2}}$ and C, in general , is complex.

Now eliminating various physical quantities from the resulting equations and non-dimensionalysing the resulting equations, using

$(k^*, k_x^*, D^*) = d\,(k, k_x, D)$ and $(w^*, C^*) = \frac{1}{U_0}(w, C)$, where d is the characteristic length and U_0 is the characteristic velocity, we obtain after omitting the asterisks, the following non-dimensional equations

$$k^2\left(-C - ik_x^{-1}R_D^{-1}\right)w = D\left\{\left(-C - ik_x^{-1}R_D^{-1}\right)Dw\right\}$$

$$-S\left(D^2-k^2\right)\left\{-\frac{w}{C}\right\}+ik_x^{-1}R_e^{-1}\left(D^2-k^2\right)^2 w-\frac{iJ_0k^2\theta}{k_x} \qquad ...(12)$$

and
$$\kappa\left(D^2-k^2\right)\theta+\frac{ik_x dU_0}{\phi}C\theta=\frac{U_0d^2\left(DT_0+g/C_p\right)}{\phi}w, \qquad ...(13)$$

where
$$R_D^{-1}=\frac{\mu\phi^2 d}{k_1\rho_m U_0}, \qquad R_e^{-1}=\frac{\mu\phi}{\rho_m dU_0},$$

$$S=\frac{H^2\phi^2}{4\pi U_0^2\rho_m} \quad and \quad J_0=\frac{g\alpha\phi^2 d}{U_0^2}. \qquad ...(14)$$

The necessary boundary conditions are

$w=o$ at $z=0$ and d, $Dw=0$ at $z=0$, $D^2w=0$ at $z=d$ and $\theta=0$ at $z=0$ and d.

Now multiplying the equation (12) by w* (complex conjugate of w) on both sides and taking the conjugate of (13), multiplying by θ, and then integrating over the range of z, we get

$$k^{-1}R_e^{-1}I_1+\left(k^{-1}R_D^{-1}-n\right)I_2=\frac{S}{n}I_2+J_0k\int\theta w^*dz. \qquad ...(15)$$

and
$$-I_3+\frac{kdU_0n^*}{\phi\kappa}I_4=\frac{U_0d^2\left(DT_0+g/C_p\right)}{\phi\kappa}\int\theta w^*\,dz, \qquad ...(16)$$

where
$$I_1=\int\left(\left|D^2w\right|^2+2k^2\left|Dw\right|^2+k^4\left|w\right|^2\right)dz$$

$$I_2=\int\left(\left|Dw\right|^2+k^2\left|w\right|^2\right)dz.$$

$$I_3=\int\left(\left|D\theta\right|^2+k^2\left|\theta\right|^2\right)dz$$

and
$$I_4=\int\left|\theta\right|^2dz.$$

Now to eliminate $\int\theta w^*$ from equations (15) and (16), we get

$$-J_0kI_3+\frac{k^2dU_0J_0n^*}{\phi\kappa}I_4=\frac{U_0d^2\left(DT_0+g/C_p\right)}{\phi\kappa}\left[k^{-1}R_e^{-1}I_1+\left(k^{-1}R_D^{-1}-n\right)I_2-\frac{Sn^*}{|n|^2}I_2\right]. \qquad ...(17)$$

Again substituting $n=n_r+in_i$ in the equation (17) and then equating real and imaginary parts, we get

$$-J_0kI_3+\frac{k^2dU_0J_0n_r}{\phi\kappa}I_4=\frac{U_0d^2\left(DT_0+g/C_p\right)}{\phi\kappa}\left[k^{-1}R_e^{-1}I_1+\left(k^{-1}R_D^{-1}-n_r\right)I_2-\frac{Sn_r}{|n|^2}I_2\right] \qquad ...(18)$$

and
$$n_i\left[-\frac{k^2dU_0J_0}{\phi\kappa}I_4+\frac{U_0d^2\left(DT_0+g/C_p\right)}{\phi\kappa}I_2\left\{1-\frac{S}{|n|^2}\right\}\right]=0 \qquad ...(19)$$

Now, multiplying equation (18) by n, and equating real and imaginary parts we get

$$J_0kI_3n_r-\frac{k^2dU_0J_0}{\phi\kappa}\left(n_r^2+n_i^2\right)I_4+\frac{U_0d^2\left(DT_0+g/C_p\right)}{\phi\kappa}\left[k^{-1}R_e^{-1}n_rI_1+\left\{k^{-1}R_D^{-1}n_r-n_r^2+n_i^2\right\}I_2-SI_2\right]=0 \qquad ...(20)$$

and $$n_i\left[\frac{U_0 d^2\left(DT_0+g/C_p\right)}{\phi\kappa}\left\{k^{-1}R_e^{-1}I_1+\left(k^{-1}R_D^{-1}-2n_r\right)I_2\right\}+J_0kI_3\right] = 0$$

...(21)

4. Results and Discussion

Now we have following result and theorems:

Result : Equation (19) is interesting in the sense that where as it does not allow oscillatory modes to exist in the absence of a magnetic field, there is a possibility of oscillatory modes to exist in the presence of magnetic field, this discussion holds for $DT_0+g/C_p<0$

Theorem1. If oscillatory modes whether stable or unstable under the condition $DT_0+g/C_p<0$, then n_r and n_i must lie inside the circle $|n|^2=S$.

Proof: For the existence of oscillatory modes equation (19) becomes

$$\frac{\kappa^2 dU_0J_0}{\phi\kappa}I_4-\frac{U_0d^2(DT_0+g/C_p)}{\phi\kappa}I_2\left(1-\frac{S}{|n|^2}\right)=0$$

If we impose the condition $DT_0+g/C_p<0$, then the validity of the above equation becomes

$$1-\frac{S}{|n|^2}<0$$

or

$$|n|^2<S$$

or

$$|n|^2=S$$

which proves the theorem.

Theorem 2. The modes are stable under the condition

$$DT_0+g/C_p>0$$

Proof : If stable modes exist in the problem then equation (18) can be written as

$$n_r\left[\frac{\kappa^2 dU_0J_0}{\phi\kappa}I_4+\frac{U_0d^2(DT_0+g/C_p)}{\phi\kappa}\left(I_2+\frac{SI_2}{|n|^2}\right)\right]$$
$$=J_0\kappa I_3+\frac{U_0d^2(DT_0+g/C_p)}{\phi\kappa}\left[k^{-1}R_e{}^{-1}I_1+k^{-1}R_D{}^{-1}I_2\right]$$

…(22)

For stable modes we must necessarily have $n_r>0$ and obviously if $DT_0+g/C_p>0$ then definitely $n_r>0$ and hence the modes are stable under the given condition. Hence the theorem.

Theorem 3. If $DT_0+g/C_p<0$ then the modes are unstable under the condition

$$|DT_0+g/C_p|<\frac{\kappa^2|n|^2J_0I_4}{(S+|n|^2)dI_2}$$

and

$$J_0\kappa I_3<\frac{U_0d^2(DT_0+g/C_p)}{\phi\kappa}\left[k^{-1}R_e{}^{-1}I_1+k^{-1}R_D{}^{-1}I_2\right]$$

Proof: If $DT_0 + g/C_p < 0$, then the equation (22) can be written as

$$n_r\left[\frac{\kappa^2 dU_0 J_0}{\phi\kappa} I_4 - \frac{U_0 d^2 |DT_0 + g/C_p|}{\phi\kappa}\left(I_2 + \frac{SI_2}{|n|^2}\right)\right] = J_0\kappa I_3 + \frac{U_0 d^2}{\phi\kappa}|DT_0 + g/C_p|\left[k^{-1}R_e{}^{-1}I_1 + k^{-1}R_D{}^{-1}I_2\right] \quad \ldots (23)$$

If we impose the conditions

$$|DT_0 + g/C_p| < \frac{\kappa^2 |n|^2 J_0 I_4}{(S + |n|^2) d I_2}$$

and

$$J_0\kappa I_3 < \frac{U_0 d^2 (DT_0 + g/C_p)}{\phi\kappa}\left[k^{-1}R_e{}^{-1}I_1 + k^{-1}R_D{}^{-1}I_2\right]$$

Then in equation (23) the quantity in bracket in LHS is positive definite and the whole quantity in RHS is negative definite, so n_r must be negative and hence the modes are unstable.

Theorem 4: If unstable $(DT_0 + g/C_p < 0)$ oscillatory modes $(n_i \neq 0)$ exist in the problem, then necessary condition is given by

$$S > \left(U_0 d^2 + |DT_0 + g/C_p| I_2 - \phi\kappa\right)(n_r{}^2 + n_i{}^2)$$

Proof : For oscillatory modes $(n_i \neq 0)$ equation (19) becomes

$$\frac{\kappa^2 dU_0 J_0}{\phi\kappa} I_4 - \frac{U_0 d^2 (DT_0 + g/C_p)}{\phi\kappa} I_2\left(1 - \frac{S}{|n|^2}\right) = 0$$

or

$$\frac{\kappa^2 dU_0 J_0}{\phi\kappa} I_4 + \frac{U_0 d^2}{\phi\kappa}|DT_0 + g/C_p| I_2\left(1 - \frac{S}{|n|^2}\right) = 0$$

For the validity of the above equation we must necessarily have

$$\frac{U_0 d^2}{\phi\kappa}|DT_0 + g/C_p| I_2\left(1 - \frac{S}{|n|^2}\right) < 0$$

or

$$\left(1 - \frac{S}{|n|^2}\right) < \frac{\phi\kappa}{U_0 d^2 |DT_0 + g/C_p| I_2}$$

or

$$S > \left(U_0 d^2 + |DT_0 + g/C_p| I_2 - \phi\kappa\right)(n_r{}^2 + n_i{}^2)$$

which proves the theorem.

Conclusion: In this paper we examined the framework of linear stability analysis of compressible fluid layer in a porous medium in the presence of weak magnetic field and low thermal conductivity. The assumption of small thermal conductivity not only simplifies the mathematical analysis, thus leading to a number of interesting results, but also is important in the context of physical situations in which the transfer of heat is not instantaneous (case of very large thermal conductivity), rather it is slow.The important and different results are obtained in this paper depend on $DT_0 + g/C_p$.

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