PERISTALTIC FLOW OF NON- NEWTONIAN FLUID IN AELASTIC TUBE

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ABSTRACT

A scientific model for the effect of slip speed on peristaltic transport of blood stream has been examined by using the Herschel-Bulkley model in an adaptable cylinder. The shut structure arrangements are gotten for speed, plug stream speed, and volume transition. It is seen that the effect of yield pressure, adequacy proportion, Darcy number, speed slip parameter, flexible parameters and liquid conduct record assumes a fundamental job in controlling the motion in a versatile cylinder. The results gained from the stream amounts uncover that, the volume transition in an adaptable cylinder diminishes with an expansion in the permeable parameter and it increments with an increment in the slip parameter. Further, the consequences of Newtonian, Bingham plastic and Power-law models have been displayed graphically and broke down.

Keywords :Amplitude ratio, Darcy number, Elastic tube, velocity slip parameter.

1.Introduction

Peristalsis is a component started by a dynamic inundation of district pressure and augmentation along the dividers of a distensible cylinder. Physiologically, peristaltic siphoning is a trademark neuromuscular property of a natural framework in which biofluids are moved along a cylinder by the propulsive improvements of the cylinder divider. The peristaltic wonder can be found in the development of the bolus through the throat, chyme transport in the stomach related tract, development of spermatozoa in the male regenerative tract, pee course through the ureter and stream of blood through little veins. As of late, the examination of peristaltic transport of non-Newtonian liquids in different geometric conditions and presumptions has gained the consideration of analysts because of their application in creating biomedical gadgets, for example, blood siphon and dialysis machine [1].

The instrument of peristalsis has been of consistent energy for certain analysts. Since the underlying examination by Latham [2], a couple of test and hypothetical examinations have been done to explore the peristaltic movement in different conditions by expecting little wave number, adequacy proportion and Reynolds number [3,4]. By inspecting the two-layered power law model, Usha and Ramachandra [5] presumed that the positive or negative mean stream was because of the rheology of the fringe layer. Likewise, similar examination was completed by Misra and Pandey [6] for the axisymmetric and channel stream. Blood comprises of plasma which is a suspension of cells and is in charge of the non-Newtonian nature at low shear rates. Along these lines, examines on blood stream, showing the non-Newtonian conduct, have pulled in a few analysts. In addition, at low shear rates, blood can be displayed either by Casson or Herschel-Bulkley model. Moreover, the utilization of Herschel-Bulkley model over Casson model is progressively suitable since it contains one additional parameter (fluctuating liquid conduct file) and is substantial for lower estimations of shear rates where the Casson model neglects to clarify the physiological conduct of blood. Notwithstanding these, the Herschel-Bulkley model can be utilized to infer Bingham-plastic, Power-law and Newtonian models for specific estimations of yield pressure and the liquid conduct list. The examinations on considering a Herschel-Bulkley liquid for various physiological circumstances has been accounted for by different creators [7-9]. As of late, a definite review with respect to peristaltic transport of physiological liquids was finished by Thanesh and Kavitha [10].

The Poiseuille's law demonstrates that for a liquid which is incompressible, the motion in the cylinder is a direct capacity of the weight contrast between the finishes of the inflexible cylinder through which it streams. Subsequently, the non-Newtonian liquids comply with Poiseuille's law in the vast majority of the hypothetical just as trial thinks about. The nonlinearity in vascular beds of warm blooded animals is doled out to the flexible idea of veins and their tremendous distensibility. This adaptable property of veins was first perceived by Youthful [11]. Afterward, Rubinow and Keller [12] showed that the scope of the cylinder can be constrained by the strain in the dividers and the transmural weight contrast by accepting that the Poiseuille law holds locally. Along these lines, there is a necessity for the emotional theory of blood course through cylinders which are versatile in nature. In this manner, Vajravelu et al. [13-15] considered the progression of blood through corridors and concentrated the diverse physiological practices either by Casson or Herschel-Bulkley model. The stream examples gotten by the models with inflexible cylinder can't clarify the conduct of blood coursing through tightened conduits altogether. Hereafter, it ends up urgent to consider the versatility in the present model.It is seen that, in numerous application issues that there exists a slip stream comparing to the stream design because of which there is lost bond at the dividers. In this manner, the liquid slides along the dividers of the cylinder. This slip stream of liquids is utilized in cleaning of the interior pits and counterfeit heart valves. As of late, a few scientists have explored the effect of slip and limit conditions on natural and old style liquids in various geometries and setups [16-28].

To the best of the creator's learning, no endeavor has been made to consider the peristaltic transport of blood through a flexible cylinder under the impacts of slip speed. This particular examination is useful in filling the hole in this direction.The present paper expects to research the impacts of slip speed on the peristaltic transport of blood, displayed as a Herschel-Bulkley liquid, through a versatile cylinder. The physical amounts related with the numerical model are inspected in the non-dimensional structure, and careful arrangements are gotten for speed, plug stream speed, weight and transition. The impact of yield pressure, adequacy proportion, permeable parameter, slip parameter, versatile parameters and liquid conduct list on motion and weight are broke down and spoke to graphically. The results of the examination may be helpful to also grasp the peristaltic development of non-Newtonian progression of blood in tight supply routes.

2. Formulation of the problem

The flow of blood is modeled to be laminar, steady, incompressible, two-dimensional, fully-developed, axisymmetric and exhibiting peristaltic motion of Herschel-Bulkley fluid in an elastic tube of radius $a'(z)$ as shown in Fig. 1. The region between $r=0$ and $r=r_p$ is called as plug flow region where $|\tau_{rz}| \le \tau_0$. In the region between $r=r_p$ and $r=a(z,t)$, we have $|\tau_{rz}| \ge \tau_0$. The change in radius of the tube due to the elastic nature is given by $a''(z)$ and the change due to the peristaltic nature is given as,

$$a'(z,\, t) = a_0 + b\,\sin\left[\frac{2\pi}{\lambda}(z-ct)\right] \qquad (1)$$

where, a_0 is the radius of the tube in the absence of elasticity, b is the amplitude, λ is the wavelength, z is the axial direction, c is the wave speed and t is the time.

3. Mathematical model and closed form solutions

Considering the long wavelength approximation by neglecting the inertial terms and wall slope, the equations of motion in the wave frame of reference which is moving with speed c is given by

$$\frac{1}{r'}\frac{\partial}{\partial r'}\left(r'\tau_{r'z'}\right) = -\frac{\partial p'}{\partial z'}, \qquad (3)$$

$$\frac{\partial p'}{\partial r'} = 0. \qquad (4)$$

Where $\tau_{r'z'}$ for Herschel-Bulkley fluid given by [29]

$$-\frac{\partial u'}{\partial r'} = f(\tau) = \left[\frac{1}{\mu}\left(\tau'_{r'z'} - \tau'_0\right)\right]^{\frac{1}{n}}, \quad \tau'_{r'z'} \ge \tau'_0, \qquad (5)$$

$$-\frac{\partial u'}{\partial r'} = f(\tau) = 0, \qquad \tau'_{r'z'} \le \tau_0. \tag{6}$$

The variables are rendered dimensionless by the following transformations

$$p = \frac{a_0^{n+1} p'}{\lambda \mu c^n},\ r = \frac{r'}{a_0},\ z = \frac{z'}{\lambda},\ u = \frac{u'}{c},\ \varepsilon = \frac{b}{a_0},\ r_p = \frac{r_p'}{a_0},\ \tau_0 = \frac{a_0^n \tau_0'}{c^n \mu},\ \tau_{rz} = \frac{\tau'_{r'z'} a_0^n}{\mu c^n}. \tag{7}$$

Making use of the non-dimensional quantities in Eq. (7), the governing equations (3) and (4) (after dropping the primes) takes the form as,

$$\frac{1}{r}\frac{\partial}{\partial r} r\left[\left(-\frac{\partial u}{\partial r}\right)^n + \tau_0\right] = -\frac{\partial p}{\partial z}, \quad \tau_{rz} \ge \tau_0 \tag{8}$$

$$\frac{\partial u}{\partial r} = 0, \quad \tau_{rz} \le \tau_0 \tag{9}$$

where τ_{rz} and τ_0 are dimensionless shearing and yield stresses, respectively. The corresponding non-dimensional boundary conditions are [16]

$$a'\frac{\partial u}{\partial r} = \frac{-\alpha u}{\sqrt{Da}} \text{ at } r = a'(z,t) \tag{10}$$

$$\tau_{rz} \text{ is finite at } r = 0 \tag{11}$$

Solving Eqs. (8) and (9) under the boundary conditions (10) and (11), the expression for velocity so obtained is given by,

$$u = \frac{2}{P(K+1)}\left[\left(\frac{Pa'}{2} - \tau_0\right)^{K+1} - \left(\frac{P\mathrm{r}}{2} - \tau_0\right)^{K+1}\right] - \frac{a'\sqrt{Da}}{\alpha}\left(\frac{Pa'}{2} - \tau_0\right)^K \tag{12}$$

where $P = -\frac{\partial p}{\partial z},\ K = \frac{1}{n}$.

Using the condition $\tau_0 = \frac{Pr_p}{2}$ at $r = r_p$, the upper limit of plug flow region is obtained as $r_p = \frac{2\tau_0}{P}$. Also, by using the condition $\tau_{rz} = \tau_{a'}$ at $r = a'$ (Bird et al. [30]), we obtain

$$P = \frac{2\tau_{a'}}{a'} \text{ and } \frac{r_p}{a'} = \frac{\tau_0}{\tau_{a'}} = \tau. \tag{13}$$

Using relation (13) and by taking $r = r_p$ in Eq. (12), the plug flow velocity is obtained as,

$$u_p = a'^{K+1}\left(\frac{P}{2}\right)^K (1-\tau)^K \left[\frac{(1-\tau)}{K+1} - \frac{\sqrt{Da}}{\alpha}\right] \tag{14}$$

The instantaneous volume flux Q across any cross section of the artery is defined as,

$$Q = 2\left[\int_0^{r_p} u_p\, r\, dr + \int_{r_p}^{a'} u\, r\, dr\right] \tag{15}$$

$$Q = \frac{(1-\tau)^{K+1}}{2^K (K+1)}\left[1 - \frac{2(1-\tau)(\tau+K+2)}{(K+2)(K+3)} - \frac{\sqrt{Da}\,(K+1)}{\alpha(1-\tau)}\right] a'^{K+3} P^K \tag{16}$$

4. Theoretical determination of flux with an application to flow through an artery

A theoretical calculation of the flux Q is carried out for an incompressible Herschel-Bulkley fluid through an elastic tube of radius $a(t, z) = a'(t, z) + a''(z)$ where $a'(t, z)$ is the change in radius of the tube due to peristalsis and $a''(z)$ is the change in radius due to the elastic nature. The fluid is assumed to enter the tube with a pressure p_1 and leaves the tube with pressure p_2, while the pressure outside the tube is p_0. If z denotes the distance along the tube from the inlet end, then the pressure $p(z)$ in the fluid at z diminishes from $p(0) = p_1$ to $p(\lambda) = p_2$. The tube may contract or expand due to the difference in pressure of the fluid $p(z) - p_0$. Subsequently, the cross section of the tube may have a deformation due to the elastic property of the walls. Thus, the difference in pressure influences the conductivity σ_1 of the tube at z. We consider the conductivity $\sigma_1 = \sigma_1[p(z) - p_0]$ to be a known function of the pressure difference $(p(z) - p_0)$. This conductivity is assumed to be the same as that of a uniform tube having an identical cross section at z. It may be the conductivity for either laminar or turbulent flow, depending upon the type of the flow occurring at z in the non-uniform tube. The relation between Q and the pressure gradient is

$$Q = \sigma_1(p - p_0)P^K \tag{17}$$

This connection, which includes Poiseuille's law, is precisely right for a uniform tube. It is approximately right for a non-uniform one in which the cross-section changes bit by bit along the tube. It can be reasoned, together with correction terms, by an asymptotic analysis of stream in such tubes, in any case, we shall not present that analysis.

Under the considerations of peristaltic motion and the elastic property of the tube wall, we can rewrite Eq. (17) as

$$\sigma_1(p - p_0) = F(a' + a'')^{K+3} \tag{18}$$

where, a' is the change in radius due to the peristalsis and a'' is the change in radius due to the elastic nature of the tube. The pressure $(p - p_0)$ at each cross section due to the Poiseuille flow i.e. $[a''(p - p_0)]$. By taking the inlet condition $p(0) = p_1$ and integrating Eq. (17) gives

$$Q^n z = \int_{p(z)-p_0}^{p_1 - p_0} (\sigma_1(p'))^n\, dp' \tag{19}$$

where, $p' = p(z) - p_0$. This equation gives $p(z)$ in terms of z and Q. Setting $z = 1$ and $p(1) = p_2$ in Eq. (19), we get Q as,

$$Q = \left[\int_{p_2 - p_0}^{p_1 - p_0} (\sigma_1(p'))^n\, dp'\right]^{\frac{1}{n}} \tag{20}$$

Now, using Eq. (18) in Eq. (20), we have

$$Q = F\left[\int_{p_2 - p_0}^{p_1 - p_0} (a' + a'')^{3n+1}\, dp'\right]^{\frac{1}{n}} \tag{21}$$

Eq. (21) can be solved if we explicitly know the function $a''(p-p_0)$. If a'' is known as a function of the tension $T(a'')$ or stress, then $a''(p')$ can be determined from the equilibrium condition given by [12]

$$\frac{T(a'')}{a''} = p - p_0. \quad (22)$$

Rubinow and Keller [12] carried out experimental investigations by controlling static pressure volume connection of a 4-cm long piece of a human iliac artery and gave an expression for tension in an elastic tube as:

$$T(a'') = t_1(a''-1) + t_2(a''-1)^5 \quad (23)$$

Using Eq. (23) with $t_1 = 13$ and $t_2 = 300$, Eq. (22) takes the following form:

$$dp' = \left[\frac{t_1}{a''^2} + t_2\left(4a''^3 - 15a''^2 + 20a'' - 10 + \frac{1}{a''^2}\right)\right]da'' \quad (24)$$

Eq. (21) can be written as

$$Q = F\left[\int_{p_2-p_0}^{p_1-p_0} (a'+a'')^{3n+1}\left[\frac{t_1}{a''^2} + t_2\left(4a''^3 - 15a''^2 + 20a'' - 10 + \frac{1}{a''^2}\right)\right]da''\right]^{\frac{1}{n}} \quad (25)$$

Letting $p = p_1$ and $p = p_2$ in Eq. (22) the solutions are obtained for a_1'' and a_2'' respectively. Eq. (25) can be rewritten as

$$Q = F[g(a_1'') - g(a_2'')]^{\frac{1}{n}} \quad (26)$$

Where

$$F = \frac{(1-\tau)^{K+1}}{2^K(K+1)}\left[1 - \frac{2(1-\tau)(\tau+K+2)}{(K+2)(K+3)} - \frac{\sqrt{Da}\,(K+1)}{\alpha(1-\tau)}\right] \text{ and}$$

$$g(a) = t_1\left(a'' + 2a'\log a'' - \frac{a'^2}{a''}\right) + t_2\left(\frac{2a''^6}{3} + \frac{a''^5}{5}(8a'-15) + \frac{a''^4}{4}(4a'^2 - 30a' + 20)\right.$$
$$\left. + \frac{a''^3}{3}(-15a'^2 + 40a' - 10) + \frac{a''^2}{2}(20a'^2 - 20a') + a''(-10a'^2 + 1) - \frac{a'^2}{a''} + 2a'\log a''\right).$$

It is worth noticing that from Eq. (26) one can obtain the results of Rubinow and Keller [12] as a special case of the present model by substituting $a' = 0,\ \tau = 0,\ Da = 0$ and $n = 1$. Also, in the absence of peristalsis and porous parameter $(a' = 0 \text{ and } Da = 0)$ the present results are in well agreement with the results of Vajravelu et al. [13, 14].

5. Results and Discussion

The present paper centers around the peristaltic transport of blood in an adaptable cylinder, displayed as a Herschel-Bulkley liquid. From the present examination, one can acquire the aftereffects of Rubinow and Keller [12], and Vajravelu et al. [14] as an exceptional case (without speed slip and permeable dividers). The results of the model are researched graphically by utilizing the fixed qualities for physiological parameters, for example, and

Fig. 2 demonstrates the variety yield weight on volume motion . It is seen from the assume that an expansion in the estimation of abatements the motion in a versatile cylinder. This conduct is relied upon because of the nearness of yield pressure present in the model which requires more measure of vitality to start the liquid stream and subsequently it diminishes the motion. Fig. 3 delineates the variety of motion along the hub of the cylinder for changing sufficiency proportion . It is seen that an expansion in builds the transition in a flexible cylinder. Since is the abundancy proportion, an increments in the estimation of results in an expansion in the wave stature which thus expands the transition. Fig. 4 shows the impact of shear thickening conduct of blood on the motion. It tends to be seen that the transition increments with the hub separation and diminishes with increment in the liquid conduct record . This abatement in transition is because of the shear thickening conduct of blood (thickness increments with expanding shear pressure).

Figs. 5 and 6 demonstrates the conduct of transition with hub areas for various estimations of flexible parameters. For a shear thickening liquid , the transition increments with an expansion in the versatile parameter (Fig. 5). A similar pattern holds useful for the other parameter, to be specific (Fig. 6). Figs. 7 and 8 separately investigate the impacts of Darcy number and slip parameter on the motion. From Fig. 7, it is seen that an expansion in the estimation of Darcy number reductions the motion. This is mostly a direct result of an expansion in the Darcy number, the porosity of the divider increments and along these lines, the transition diminishes. Further, an expansion in the slip parameter builds the transition in a versatile cylinder (Fig. 8). The transition profiles with bay and outlet flexible span varieties are appeared in Figs. 9 and 10. For a fixed estimation of outlet sweep, the impact of expanding estimations of channel versatile span makes the transition to increment and thus motion increments as the flexible range builds (Fig. 9). In any case, the contrary conduct is seen when we fix the delta flexible span and changing outlet versatile sweep (Fig. 10).

The effect of transition with pivotal areas for various liquids is plotted in Fig. 11. From the geometrical depiction, it is seen that the transition on account of a Newtonian liquid is more than that of the Bingham, Power-law, and Herschel-Bulkley liquid. Also, the transition in the Herschel-Bulkley model is less when diverged from substitute models (Newtonian, Power-law, and Bingham). This is because of the nearness of yield pressure, and liquid conduct file (shear thickening) present in the Herschel-Bulkley model lessens the transition.

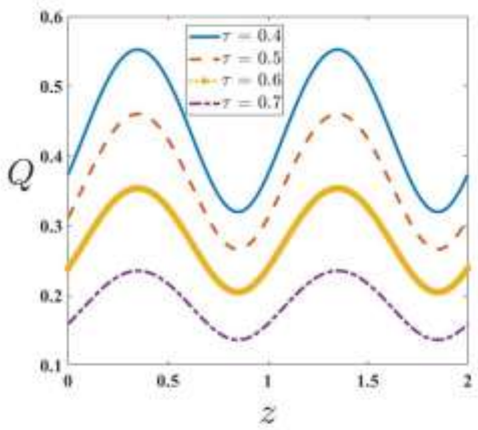

Fig. 2. Q **versus** z **for varying** τ **.**

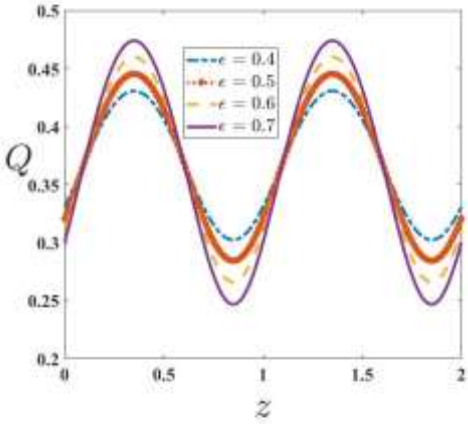

Fig. 3. Q versus z for varying ε .

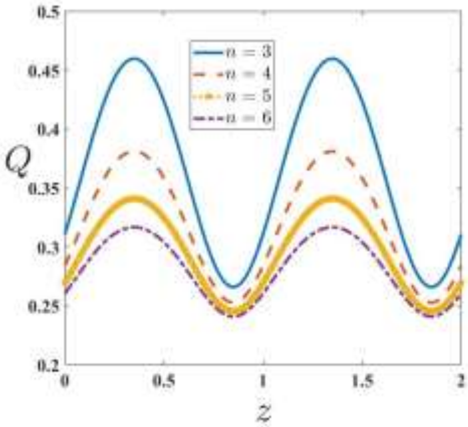

Fig. 4. Q versus z for varying n .

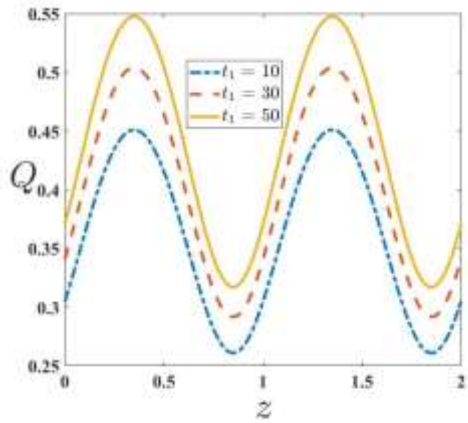

Fig. 5. Q versus z for varying t_1 .

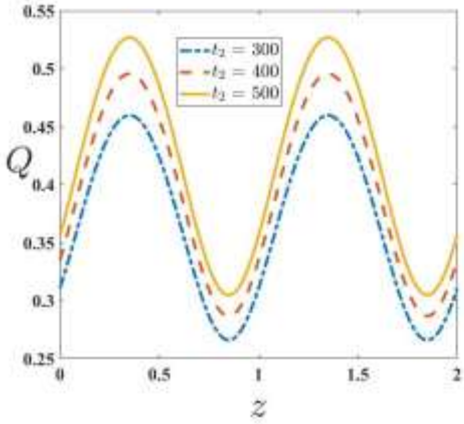

Fig. 6. Q **versus** z **for varying** t_2 **.**

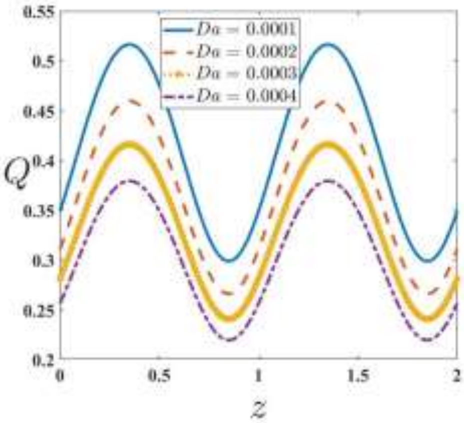

Fig. 7. Q **versus** z **for varying** Da **.**

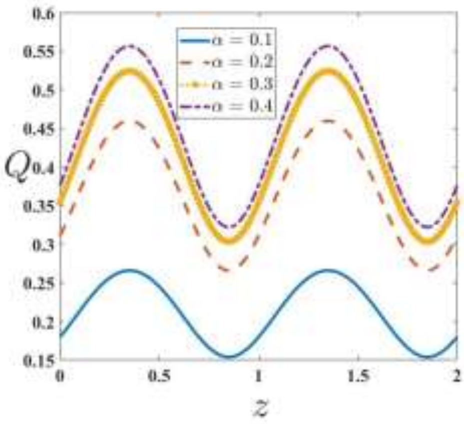

Fig. 8. Q **versus** z **for varying** α **.**

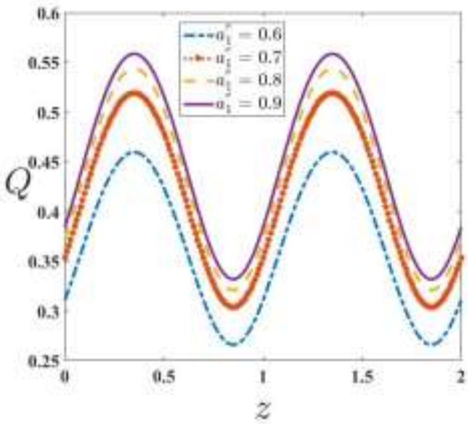

Fig. 9. Q **versus** z **for varying** a_1'' **.**

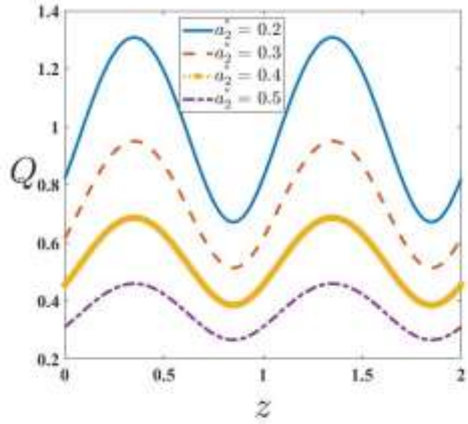

Fig. 10. Q **versus** z **for varying** a_2'' **.**

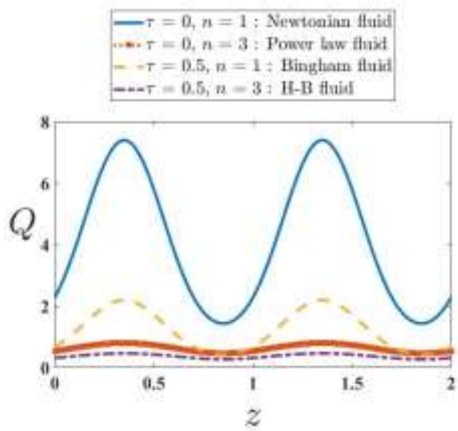

Fig. 11. Q **versus** z **for different types of fluids.**

The effects of τ, ε, Da, n and α on Pressure gradient along the axis are plotted in Figs. 12-16. From Figs. 12 and 13, the magnitude of pressure gradient increases with an increase in the values of τ and ε. Moreover, an increase in the Da (porosity of the walls) is accompanied with an increase in the magnitude of the pressure gradient (Fig. 14).Fig. 15 gives the variation of pressure gradient along the axis for different values of n. It is noticed that an increase in the value of fluid behavior index increases the magnitude of pressure gradient. Further, the effect of α on pressure gradient shows the opposite behavior as that of n (Fig. 16).

6. Conclusions

The present paper emphasizes on the peristaltic flow of blood in the human circulatory system by using Herschel-Bulkley Model in an elastic tube with porous walls. The study provides a satisfactory outcome that represents some of the natural phenomena, especially, the flow of blood in narrow arteries which can be handled and processed in case of dysfunction. The conclusions can be summarized as follows:

- The volume flux rate increases with an increase in slip parameter and decreases with an increase in the porous parameter.
- The presence of elastic parameters has a vital role in enhancing the flux.
- The flux in an elastic tube decreases with an increase in the values of yield stress, fluid behavior index and outlet elastic radius, and it increases with an increase in the values of inlet elastic radius and amplitude ratio.

References

[1] Jaggy, C., Lachat, M., Leskosek, B., Znd, G. and Turina, M.“Affinity pump system: a new peristaltic blood pump for cardiopulmonary bypass”.*Perfusion* 15, (2000): 77-83.

[2] Latham, W. (1966). *Fluid motions in the peristaltic pump*. M.S. thesis, Massachusetts Institute of Technology, US.

[3] Burns, J. C. and Parkes, T.“Peristaltic motion.”*Journal of Fluid Mechanics*29, (1967): 731-743.

[4] Shapiro, A. H., Jaffrin, M. Y. and Weinberg, S. L. “Peristaltic pumping with long wavelengths at low Reynolds number.” *Journal of Fluid Mechanics* 37, (1969): 799-825.

[5] Usha, S. and Ramachandra, R. A. “Peristaltic transport of two layered power law fluids.“*Journal of Biomechanical Engineering* 119, (1997): 483-488.

[6] Misra, J. C. and Pandey, S. K. “Peristaltic Transport of Blood in Small Vessels: Study of Mathematical Model.“*Computers and Mathematics with applications* 43, (2002): 1183-1193.

[7] Vajravelu, K., Sreendh, S. and Ramesh, B. V. “Peristaltic Transport of a Herschel-Bulkley fluid in contact with Newtonian fluid.“*Journal of Applied Mathematics* 64, (2006): 593-604.

[8] Manjunatha, G., Basavarajappa, K. S., Thippeswamy, G. and Hanumesh, V. “Peristaltic Transport of Three Layered Viscous Incompressible Fluid.“*Global Journal of Pure and Applied Mathematics* 09, (2013): 93-107.

[9] Rajashekhar, C, Manjunatha, G, Prasad, K. V., Divya, B. B. and Hanumesh Vaidya. Peristaltic transport of two-layered Herschel Bulkley fluid.” *Cogent Engineering* 5, (2018): 1495592.

[10] Thanesh, K. K. and Kavitha, A. “A review report of recent developments in peristaltic transport of physiological fluids.“*International Journal of Pharmacy and Technology* 08, (2016): 4105-4120.

[11] Young, DF. “Effects of a time-dependent stenosis on flow through a tube.“*J. Engg. Ind. Trans. ASME* 90, (1968): 248-254.

[12] Rubinow, SI, and Keller, JB. “Flow of a viscous fluid through an elastic tube with application to blood flow.“*J. Theor. Bio.* 35, (1972): 299-313.

[13] Vajravelu, K., Sreenadh, S., Devaki, P. and Prasad, K. V. “Mathematical model for a Herschel-Bulkley fluid in an elastic tube.“*Cent. Eur. J. Phys.*09, (2011): 1357-1365.

[14] Vajravelu, K., Sreenadh, S., Devaki, P. and Prasad, K. V. “Peristaltic transport of a Herschel-Bulkley fluid in an elastic tube.“*Heat Transfer-Asian Research* 44, (2015): 585-598.

[15] Vajravelu, K., Sreenadh, S., Devaki, P. and Prasad, K. V. "Peristaltic pumping of a Casson fluid in an elastic tube.“*Journal of Applied Fluid Mechanics* 09, (2016): 1897-1905.

[16] Santosh, N., Radhakrishnamacharya, G. and Chamka, A. J. “Effect of slip on Herschel-Bulkley fluid flow through narrow tubes.“*Alexandria Engineering Journal* 54, (2015): 889-896.

[17] MebarekOudina, F. and Bessaih, R. “Oscillatory magnetohydrodynamic natural convection of liquid metal between vertical coaxial cylinders.” *Journal of Applied Fluid Mechanics* 9, (2016): 1655-1665.

[18] MebarekOudina, F. “Numerical modelling of the hydrodynamic stability in vertical annulus with heat source of different lengths.” *Engineering Science and Technology, an International Journal* 20, (2017): 1324-1333.

[19] Vajravelu, K., Prasad, K. V., Chiu-On Ng and Vaidya, H. “MHD squeeze flow and heat transfer of a nanofluid between parallel disks with variable fluid properties and transpiration.” *International Journal of Mechanical and Materials Engineering* 12, (2017): 9.

[20] Prasad, K. V., Vajravelu, K., Vaidya, H., Basha, N. Z. and Umesh, V. “Thermal and species concentration of MHD Casson fluid at a vertical sheet in the presence of variable fluid properties.” *Ain Shams Engineering Journal* (2017).

[21] Prasad, K. V., Vaidya, H. and Vajravelu, K. “MHD mixed convection heat transfer over a non-linear slender elastic sheet with variable fluid properties.” *Applied Mathematics and Nonlinear Sciences* 2, (2017): 351-366.

[22] Wakif, A.,Boulahia, Z. and Sehaqui, R. "A semianalytical analysis of electro-thermo-hydrodynamic stability in dielectric nanofluids using Buongiorno's mathematical model together with more realistic boundary conditions." *Results in Physics* 9, (2018): 1438-1454.

[23] Wakif, A., Boulahia, Z., Ali, F., Eid, M. R. and Sehaqui, R. "Numerical Analysis of the Unsteady Natural Convection MHD CouetteNanofluid Flow in the Presence of Thermal Radiation Using Single and Two-Phase Nanofluid Models for Cu-Water Nanofluids."*International Journal of Applied and Computational Mathematics* 4, (2018): 81.

[24] Wakif, A., Boulahia, Z., Mishra, S. R., Rashidi, M. M. and Sehaqui, R. "Influence of a uniform transverse magnetic field on the thermo-hydrodynamic stability in water-based nanofluids with metallic nanoparticles using the generalized Buongiorno's mathematical model."*The European Journal Plus* 133, (2018): 181.

[25] Manjunatha, G. and Rajashekhar, C. "Slip effects on peristaltic transport of Casson fluid in an inclined elastic tube with orous walls." *Journa of advanced research in fluid mechanics and thermal sciences* 43, (2018): 67-80.

[26] Rajashekhar, C., Manjunatha, G. and Naveen, C. "Analytical Solutions on the Flow of blood with the Effects of Hematocrit, Slip and TPMA in a porous tube." *Journal of Advanced Research in Fluid Mechanics and Thermal Sciences* 47, (2018): 201-208.

[27] Ashwini, B. and Nagaraj, N. K. "Analysis of Stagnation Point flow of an Incompressible Viscous Fluid between Porous Plates with Velocity Slip." *Journal of Advanced Research in Fluid Mechanics and Thermal Sciences* 48, (2018): 40-52.

[28] Sampath Kumar, V. S. and Pai, N. P. "Analysis of Porous Eliptical Slider through Semi-Analytical Technique." *Journal of Advanced Research in Fluid Mechanics and Thermal Sciences* 48, (2018): 80-90.

[29] Chaturani, P. and Narasimhan, S. "Theory for flow of Casson and Herschel-Bulkley fluids in cone-plate viscometers." *Biorheology* 25, (1988): 199-207.

[30] Bird, R. B., Stewart, W. E. and Lightfoot, E. N. (1976).*Transport Phenomena,* New York: Wiley.