E-Cordial Families Related To Cycle and Path

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***Abstract*: The two copies of graph G(p,q) are joined by t paths on n-points each. We represent the family by $G(tP_n)$.The paths are attached at the same fixed point on G. We discuss E-cordiality of C_4(Pn), $C_5(P_n)$, $w_4(P_n)$, $C_3P_nW_4$, $C_4P_nW_4$.We show that under certain conditions these graphs are E-cordial.**

***Keywords*: graph, E-cordial,shel graph,S_5,C_4.**

Subject Classification: 05C78

Introduction:

In 1997 Yilmaz and Cahit [4] introduced weaker version of edge graceful labeling E-cordial labeling. Let G be a (p,q) graph. f:E→{0,1}Define f on V by f(v) =$\sum\{f(vu)(vu)\in E(G)\}(mod\ 2)$.The function f is called as E-cordial labeling if $|v_f(0)-v_f(1)|\leq 1$ and $|e_f(0)-e_f(1)|\leq 1$ where $v_f(i)$ is the number of vertices labeled with i =0,1. And $e_f(i)$ is the number of edges labeled with i= 0,1,We follow the convention that $v_f(0,1)$ =(a,b) for $v_f(0)$=a and vf(1)=b further $e_f(0,1)$=(x,y) for $e_f(0)$=x and $e_f(1)$=y. A graph that admits E-cordial labeling is called as E-cordial graph. Yilmaz and Cahit prove that trees Tn are E-cordial iff for n not congruent to 2(mod 4), Kn are E-cordial if n not congruent to 2(mod 4), Fans Fn are E-cordial iff for n not congruent to 1(mod 4).Yilmaz and Cahit observe that A graph on n vertices cannot be E-cordial if n is congruent to 2 (mod 4). One should refer Dynamic survey of graph labeling by Joe Gallian [2] for more results on E-cordial graphs.

The graphs we consider are finite, undirected, simple and connected. For terminology and definitions we refer Harary [3] and Dynamic survey of graph labeling by Joe Gallian [2].The families we discuss are obtained by taking two copies of graph G and join them by t paths of equal length. The paths are attached at the same fixed point on G. We represent these families by G(tPn).We take t = 1 and choose G from C_3, C_4, C_5 and W_4.

3. Preliminaries:

3.1 $G_1(P_n)G_2$ is graph obtained by joining a vertex of G_1 with vertex of G_2.It has p1+p_2+n-2 vertices and q_1+q_2+n-1 edges where G_1 is (p_1q_1) and G_2 is (p_2,q_2) graph. When there are t paths from G1 to G2 starting at one vertex ans ending at one fixed vertex we denote this family of graphs as $G_1(tP_n)G_2$.

4. Main Results proved:

Theorem 4.1: G=$C_4(P_n)$ is e-cordial for n is not congruent to 0,2(mod 4)

Proof: We define G as $V_1 = \{v_1,v_2,..v_n\}$ .These are vertices on path Pn and end points are on respective cycle. $V_2 = \{u_1, u_2, u_3, u_4, u_5, u_6\}$, these are vertices on two copies of C_5. It does not includes the vertex common with path namely v_1 and v_n .Thus we have $V(G) = V_1UV_2$. E(G) =$\{e_i=(v_iv_{i+1})/i = 1,2,,\ldots,n-1\}$ U $\{c_1=(v_1u_1),c_2=(u_1u_2),c_3=(u_2u_3),c_4=(u_3v_1)$ and $c_5=(v_nu_4),c_6=(u_4u_5),c_7=(u_5u_6),c_8=(u_6v_n)\}$

Note that |E(V)|=q = n+7, |V(G)|=p=n+6

Define f:E(G) →{0,1} as follows :

$f(v_1u_1)=f(u_3v_1)$ =1;

$f(v_nu_4)$=1,

$f(u_4u_5)$=1;

$f(u_1u_2)$=0,

$f(u_2u_3)$=0,

$f(u_5u_6)$=0,

$f(u_6v_n)$=0.

f(ei) = 0 for i is odd number and i <2k where k =$[\frac{n}{4}]$

f(ei)=1 for i is even and i $\leq$2k

f(e_{2k+j}) =1 for j = 1 ,,,.,p-k where p = [$\frac{n}{2}$]

f(ei)=0 for i = k+p+1,,..,n-1.

The label number distribution is

$v_f(0,1)= (\frac{p+1}{2},\frac{p-1}{2})$, $e_f(0,1)=(\frac{q}{2},\frac{q}{2})$ for n ≡3(mod 4)

$v_f(0,1) =(\frac{p-1}{2},\frac{p+1}{2})$, $e_f(0,1)=(\frac{q}{2},\frac{q}{2})$ for n ≡1 (mod 4)

vf(0,1) = $(\frac{q}{2},\frac{q}{2})$ for n ≡ 2(mod 4), $e_f(0,1)=(\frac{q-1}{2},\frac{q+1}{2})$

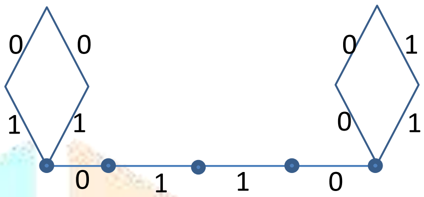

Fig 4.1:$C_4(P_5)$ labeled copy: edge labels are shown

for n is divisible by 4 the desired labeling does not exists.

Theorem 4.2. G=$C_5(P_n)$ is e-cordial for n is not congruent to 2(mod 4)

Proof: We define G as V(G) = {v_1,v_2,..v_n} U {u_1,u_2,u_3,u_4,u_5,u_6,u_7,u_8} and E(G) ={e_i=(v_iv_{i+1})/i = 1,2,, …,n-1} U {c_1=(v_1u_1),c_2=(u_1u_2),c_3=(u_2u_3),c_4=(u_3u_4) and c_5=(u_4v_1), and c_6=(v_nu_6),c_7=(u_6u_7),c_8=(u_7u_8),c_9=(u_8u_9),c_{10} = (u_9v_n)}

Define f:E(G) →{0,1} as follows :

f(c_1)=0;

f(c_2)=1;

f(c_3)=0;

f(c_4)=1;

f(c_5) = 0;

f(e_i)=0 for i = 2x-1,x = 1, 2, ,,k; where k=1+ $\frac{n-3}{2}$ if n-3 is divisible by 4 otherwise k is integer part of $\frac{n}{4}$;

f(e_i) = 1 for i = 2x, x= 1,2, k; where k=1+ $\frac{n-3}{2}$ if n-3 is divisible by 4 otherwise k is integer part of $\frac{n}{4}$;

f(e_{2k+j})=1 for j =1,2,.. (q_2-2-k), where f is e- cordial labeling we have $e_f(0,1)$ = (q_1,q_2);k as above.

f(ej)=0 for all others on C_5.

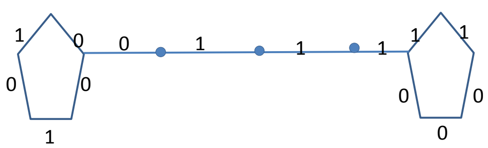

Fig 4.2: $C_4(P_5)$ labeled copy: edge labels are shown

Thus the graph is e-cordial. #

Theorem 4.3. $G=W_4(P_n)$ is e-cordial for n is not congruent to 2(mod 4).

Proof: We define G as follows: The vertices on two copies of w_4 are $V_1 = \{w_1, w_2, u_1, u_2, u_3, u_4, u_5, u_6, v_1, v_n\}$.The path vertices are $V_2 = \{v_1, v_2, .. v_{n-1}, v_n\}$.$v_1$and v_n are common vertices respectively with first and second copy of W_4. The edge set $E_1 = \{(x_1u_1), (u_1u_2), (u_2u_3), (u_4v_1)$, are cycle edges on first copy of W_4 and pokes on the same copy given by (w_1ui)/ i =1,2,3,4 where $u_4 = v_1\}$, $E_2 = \{\{en=(v_2u_4), e_{n+1}=(u_4u_5), e_{n+2}=(u_5u_6), e_{n+3}=(u_6v_4)$, these as cycle edges on second copy of W_4 and pokes given by (w_2ui)/ i =4,5, 6,7 where $u_7 = v_n\}$. E_3 are edges on path P_n given by $E_3 = \{ e_i=(v_iv_{i+1})/i=1,2 ,..(n-1)\}$. Thus we have $V(G) = V_1 \cup V_2$ and $E(G) = E_1 \cup E_2.\cup E_3$

Define $f:E(G) \rightarrow \{0,1\}$ as follows :$f(w_1u_i)=1$ for i = 1,2,3,4; $f(w_2u_i)=0$ for i = 5, 6, 7, 8; $f(u_iu_{i+1}) = 0$, i = 1, 2, 3 , 4 and i+1 taken (modulo 4);

$f(e_i)=0$ for i = 2x-1 where x = 1,2,..t, t=$integer\ part\ of\ \frac{n}{4}$ +1 for n-3 is divisible by 4 and t = integer part of $\frac{n}{4}$ otherwise. $f(e_i)=1$ for i= 2x/x=1,2,..2t.

$f(e_{2t+i})=1$ for i = 1, 2, .. , p where p =q_2-4-t.rest of $f(e_i)=0$ for all i >p

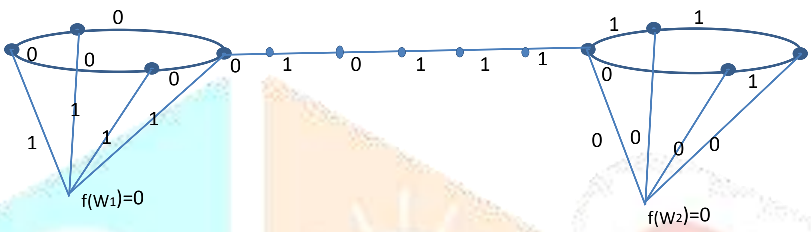

Fig 4.3 e-cordial copy of $w_4(p_7)$

Theorem 4.4. $G=C_3(P_n)W_4$ is e-cordial for n is not congruent to 0(mod 4).

Proof: We define G as follows: $V(G) = V_1 \cup V_2 \cup V_3$.Where the vertices of cycle C_3 are $V_1 = \{v_1,v_2,v_3\}$; The path vertices are $V_2 = \{ u_1, u_2, \ldots ,u_n\}$ here any v_i ;i= 1, 2, 3.will be same as u_1 and it depends on at which vertex we start path P_n. Further W_4 vertices are $V_3 = \{ u_n=v_1', v_2', v_3', v_4'$ and hub vertex v}.The edge set E(G) is defined as follows: $E_1 = \{c_i=(v_iv_{i+1})/i =1, 2, 3$, and i+1 taken (modulo 3).}, these are cycle edges on C_3., $E_2=\{e_i= (u_iu_{i+1})/ i = 1,2, ..,(n-1)\}$, These edges are on path P_n. $E_3 =\{(vv_i')$ / i= 1,2,3,4},These are pokes of W_4 , $E_4 = \{e_n=(u_nv_2'), e_{n+1}=(v_2'v_3'), e_{n+2}=(v_3'v_4'), e_{n+3}=(v_4'u_n)$; un is same as v_1'and are four edges on C_4 of W_4. We get $E(G) = E_1 \cup E_2 \cup E_3 \cup E_4$.

Define $f:E(G) \rightarrow \{0,1\}$ as follows :

$f(c_1)=1$;

$f(c_2)=1$;

$f(c_3)= 0$;

$f(e_i)=0$ for i = 2x-1, x= 1, 2, ..,t. and t = k+1..

f(ei) =1 for i =2x; x= 1, 2, ..k. Where k = ($integer\ part\ of\ \frac{n}{4}$).

If f we defined is e-cordial labeling then have got say, $e_f(0,1) = (q_1,q_2)$. i.e. number of edges with label 1 are say, q_2.

Let s = q_2-2-k.Then $f(e_{2k+1+i})= 1$ for I =1, 2, ..s. For i>s.$f(e_{2k+1+i})$=0.That completes e-cordial labeling 0f $C_3(p_n)W_4$. We showcase f in following diagram taking n = 5.

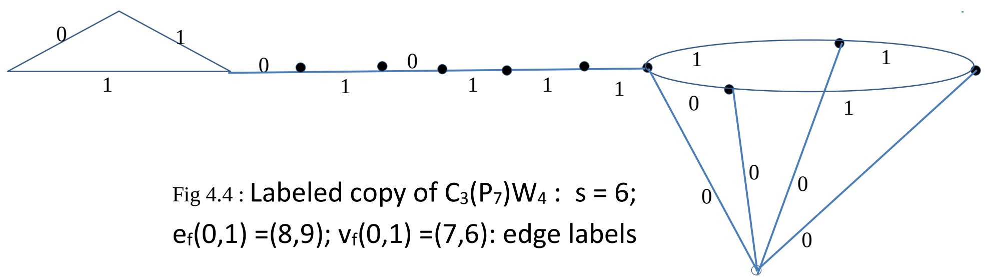

Fig 4.4 : Labeled copy of $C_3(P_7)W_4$: s = 6; $e_f(0,1) =(8,9)$; $v_f(0,1) =(7,6)$: edge labels

Theorem 4.5. $G=C_3(P_n)C_4$ is e-cordial for n is not congruent to 1 (mod 4)

Proof: We define G as follows: $V_1 = \{v_1,v_2,.. v_n\}$ .These are vertices on path P_n and end points are on respective cycle. $V_2 = \{u_1, u_2, u_3, u_4, u_5, u_6, u_7\}$, these are vertices on two cycles C_3 and C_4. The end point on path namely $v_1=u_1$ on C_3 and $v_n=u_4$ on C_4. Thus we have $V(G) = V_1UV_2$. $E(G) =\{e_i=(v_iv_{i+1})/i = 1,2,,\ldots,n-1\}$ U $\{c_1=(u_1u_2),c_2=(u_2u_3),c_3=(u_3u_1)$ and $e_n=(u_4u_5)$, $e_{n+1}=(u_5u_6)$, $e_{n+2}=(u_6u_7)$, $e_{n+3}=(u_7u_4)\}$

Note that $|E(V)|=q = n+6$, $|V(G)|=p=n+5$

Define $f:E(G) \rightarrow\{0,1\}$ as follows :
$f(u_1u_2)=f(u_3u_1) =1$;
$f(u_2u_3)=0$;for n is divisible by 4 for all n,n-1,n-2 we have $k = \frac{n}{4}-1$ and $t = n-2-k$.

$f(e_i)=0$ for i= 2x-1 ,x=0, 1, 2, .., k.

$f(e_i)=1$for i = 2x, x= 1,2, ..k.
$f(e_{2k+1+i})=1$ for i = 1,2,t.

$f(e_i)= 0$ for all i>2k+1+t

The observed label numbers are $e_f(0,1) =(x,x)$; $v_f(0,1) =(x-1,x)$ for n is even number and $x =\frac{n+6}{2}$.

If n is odd number $x' = \frac{n+5}{2}$ we have $e_f(0,1) =(x'+1,x')$; $v_f(0,1) =(x',x')$.

For n≡1 (mod 4) E- cordial labeling does not exists. #

Theorem 4.6. $G=C_4(P_n)w_4$ is e-cordial for n is not congruent to 0,2 (mod 4).

Proof: We define G as follows: The vertices of cycle C_4 are $V_1 = \{v_1,v_2,v_3,v_4\}$; The path vertices are $V_2 = \{ u_1, u_2,\ldots ,u_n\}$ here any v_i ;i= 1, 2, 3,4.will be same as u_1 and it depends on at which vertex of C_4 we start path P_n. Further W_4 vertices are $V_3 = \{ u_n=v_1', v_2', v_3',v_4'$ and hub vertex v}. We have $V(G) = V_1$ U V_2 U V_3. The edge set E(G) is defined as follows: $E_1= \{c_i=(v_iv_{i+1})/i =1, 2, 3,4$. and i+1 taken (modulo 4).}, these are cycle edges on C_4., $E_2=\{e_i= (u_iu_{i+1})/ i = 1,2, ..,(n-1)\}$, These edges are on path P_n. $E_3 =\{(vv_i') / i= 1,2,3,4\}$, are pokes of W_4 , $E_4 = \{e_n=(u_nv_2'), e_{n+1}=(v_2'v_3'), e_{n+2}=(v_3'v_4'), e_{n+3}=(v_4'u_n)$; u_n is same as v_1'and are four edges on C_4 of W_4. We get $E(G) = E_1$ U E_2 U E_3 U E_4.

Define $f:E(G) \rightarrow\{0,1\}$ as follows :

$f(c_1)=1$;

$f(c_2)=1$;

$f(c_3)= 0$; $f(c_3)= 0$;

$f(e_i)=0$ for i = 2x-1, x= 1, 2, ..,t. and t = k+1..

f(ei) =1 for i =2x; x= 1, 2, ..k. Where k = ($integer\ part\ of\ \frac{n}{4}$).

If f we defined is e-cordial labeling then have got say, $e_f(0,1) = (q_1,q_2)$. i.e. number of edges with label 1 are say q_2.

Let $s = q_2-2-k$.Then $f(e_{2k+1+i})= 1$ for I =1, 2, ..s. For i>s. $f(e_{2k+1+i})=0$.That completes e-cordial labeling of $C_3(p_n)W_4$. We showcase f in following diagram taking n = 5.

Conclusion: In this paper we discuss the families of graphs obtained by joining two graphs by a path Pn. We denote these graph families by $G_1(P_n)G_2$.We have taken G_1 and G_2 from C_3, C_4, C_5 and W_4.

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