COMBINED CONVECTIVE HEAT TRANSFAR OF A CONDUCTING FLUID IN A VERTICAL CIRCULAR TUBE

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***ABSTRACT*: In this paper we study the combined free and forced Convection of a conducting fluid in a vertical tube of circular cross-section. The medium is assumed to be completely saturated and the boundary wall is impermeable. The expression for velocity, temperature mean velocity and mean temperature have been derived.**

***Keywords*: Magnetic permeability, imaginary flow, impermeable surface, magnetic field, conducting fluid.**

Introduction: The process of combined heat transfer is encountered very frequently, owing to its great practical importance; the problem under consideration has long been attracting the attention of investigators. A large number of theoretical and experimental studies have been conducted.

Convective heat transfer from a heated impermeable surface in a field is of growing interest because of its application in assessment of geothermal resources in design of underground energy storage system, nuclear power reactor engineering and so on.

Mathematical Formulation:

Let us consider steady fully developed linear flow of viscous in incompressible fluid in a long vertical circular tube of radius a. The wall of the tube is assumed to be impermeable and the fluid receives uniform heat input per unit length of tube. It is also assumed that there is a uniformly distributed constant heat source in the flow region, let (r,θ,z) be the cylindrical polar co-ordinate system with the axis of *z* coinciding with the axis of tube directed vertically upward. The fluid properties such as specific heat C_p, the co-efficient of viscosity μ as well as the medium properties such as the conductivity 6, thermal conductivity *k* etc are assumed to be constant.

The equation of state reduces to

$$\rho = \rho_0\left[1-\beta\left(t-t0\right)\right] \qquad (1)$$

Where ρ is the fluid density, ρ_0 the standard density, β the co-efficient of thermal expansion, *t* the fluid temperature and to the standard temperature.

We assume that the conducting fluid is acted on by uniform magnetic field H_0 along the radial direction. Due to the fully developed laminarity of flow, the wall and the fluid temperature gradients in the axial direction are equal and constant under these conditions, the steady flow is characterized by velocity $(0,0,w)$ and the temperature T.

The governing equations are.

$$0 = \frac{-\partial p}{\partial z} - \rho^* g + \mu\nabla^2 w - 6\beta\,\begin{matrix}2\\0\end{matrix} + pg\beta.T \qquad (2)$$

$$\rho C_p w\frac{\partial T}{\partial Z} = k\nabla^2 T + fi \qquad (3)$$

Equation of continuity is $$\frac{\partial w}{\partial z} = 0 \qquad (4)$$

Where $\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2} + \frac{\partial^2}{\partial z^2}$

ρ^* is the density of the fluid at the wall *p* the fluid pressure, *g* the acceleration due to gravity and f_i is the internal heat source.

$T = t - t^*, t^*$ being the temperature of the boundary wall and $\beta_0 = \mu_0 H_0, \mu$ being the magnetic permeability

The boundary conditions are

$$w = 0,\ T = 0 \text{ at } r = a \tag{5}$$

To convert the equation (2) and (3) in to non-dimensional form,

$$q = \frac{r}{a}, u = \frac{w}{wn}, \theta = \frac{KT}{\rho C_p w_m a^2 \frac{\partial T}{\partial z}}$$

$$S = \left(\frac{\partial p}{\partial z} + \rho^* g\right)\frac{a^2}{\mu w_m}, R = \frac{fi}{\rho C_p \frac{\partial T}{\partial z} w_m}$$

$$R^* = \frac{\rho^2 g C_p \frac{\partial T}{\partial z} \beta a^4}{k\mu}, m^2 = \frac{6\beta_0 a^2}{\mu} \tag{6}$$

Where w_m is the mean velocity

Then the equation (2) & (3) reduce to

$$\frac{d^2u}{dq^2} + \frac{1}{q}\frac{du}{dq} - m^2u + R^*\theta = S \tag{7}$$

And $$\frac{d^2\theta}{dq^2} + \frac{1}{q}\frac{d\theta}{dq} - u = -R \tag{8}$$

And the boundary conditions (5) is transformed into

$$u = 0 \qquad at\, q = 1$$
$$\theta = 0 \qquad at\, q = 1 \tag{9}$$

S and *R* in equation (7) & (8) characterize the pressure gradient and heat source respectively in the circular tube and are constant.

Solution of the Problem :

The sequence $< J_0(\lambda iq)$ where J_0 represents the zeroth order Bessel function of first kind and $\lambda i'S$ are successive roots of $J_0(\lambda) = 0$ vanishes on the tube boundary $q = 1$

We expand $f(q) = 1, 0 \le q \le 1$ in fourier-Bessel series in the form $1 = \sum_{i=1}^{\infty} C_i J_0(\lambda iq)$, where λi are roots of the equation $J_0(\lambda) = 0$ (10)

$$Ci = 2\frac{\int_0^1 J_0(\lambda iq)\,dq}{J_1^2(\lambda i)} \qquad (10) \quad [a]$$

$$\int_0^1 J_0(\lambda iq)\,dq = \frac{1}{\lambda i}\int_0^{\lambda i}\frac{d}{dt}\left[J_1(t)dt\right.$$

$$= \frac{1}{\lambda i}\left[J_1(t)\right]_0^{\lambda i}$$

$$= \frac{1}{\lambda i}\left[J_1(\lambda i) - J_1(0)\right] \qquad (11)$$

Using 10 [a] & (11) becomes

$$Ci = \frac{\frac{2}{\lambda i}}{J_1^2(\lambda i)} J_1(\lambda i) = \frac{2}{\lambda i J_1(\lambda i)}$$

Therefore $1 = \sum_{i=1}^{\infty} C_i J_0(\lambda iq)$

$$\Rightarrow 1 = 2\sum_{i=1}^{\infty}\frac{J_0(\lambda iq)}{\lambda_i J_1(\lambda i)} \qquad (12)$$

Let us choose

$$u(q) = \sum_{i=1}^{\infty} r_i\, J_0(\lambda iq) \qquad (13)$$

$$\theta(q) = \sum_{i=1}^{\infty} S_i\, J_0(\lambda iq) \qquad (14)$$

Substituting (12), (13) and (14) in equation (7) and (8) we obtain

$$\sum_{i=1}^{\infty} r_i\, J_0''(\lambda iq) + \frac{1}{q}\sum_{i=1}^{\infty} r_i\, J_0^1(\lambda iq) - m^2\sum_{i=1}^{\infty} r_i\, J_0(\lambda iq) + R^*\sum_{i=1}^{\infty} S_i\, J_0(\lambda iq)$$

$$= 2S\sum_{i=1}^{\infty}\frac{J_0(\lambda iq)}{\lambda i\, J_1(\lambda i)} \qquad (15)$$

And $\sum_{i=1}^{\infty} S_i\, J_0''(\lambda iq) + \frac{1}{q}\sum_{i=1}^{\infty} S_i\, J_0^1(\lambda iq) - \sum_{i=1}^{\infty} r_i\, J_0(\lambda iq)$

$$= -2R\sum_{i=1}^{\infty}\frac{J_0(\lambda iq)}{\lambda i\, J_1(\lambda i)} \qquad (16)$$

Using $J_0'(\lambda iq) = J_1(\lambda iq)$

And $2J_0''(\lambda iq) = J_2(\lambda iq) - J_0(\lambda iq)$

In equation (15) & (16) and equation the co-efficient of $J_0(\lambda iq)$ on both side we get.

$$-\left(m^2+\lambda i^2\right)r_i+R^*S_j=\frac{2S}{\lambda i\,J_1(\lambda i)} \tag{17}$$

And $r_i+\lambda i^2 S_i=\dfrac{2R}{\lambda i\,J_1(\lambda i)}$ (18)

We thus have velocity and temperature fields :

$$u=2\sum_{i=1}^{\infty}\left[\frac{P\,R^*-S\lambda^2 i}{R^*+\lambda^2 i\left(m^2+\lambda^2 i\right)}\right]\frac{J_0(\lambda iq)}{\lambda i\,J_1(\lambda i)} \tag{19}$$

$$\theta=2\sum_{i=1}^{\infty}\left[\frac{R\left(m^2+\lambda^2 i\right)+S}{R^*+\lambda^2 i\left(m^2+\lambda^2 i\right)}\right]\frac{J_0(\lambda iq)}{\lambda i\,J_1(\lambda i)} \tag{20}$$

Mean velocity in non-dimensional form is

$$u_m=2\sum_{i=1}^{\infty}\frac{\left[R\,R^*-S\right]}{\lambda^2 i\left[R^*+\lambda^2 i\left(m^2+\lambda^2 i\right)\right]} \tag{21}$$

Mean temperature difference in non-dimensional form is

$$\theta_m=\frac{1}{\pi}\int_0^1 2\pi\, q\theta\, dq$$

$$=4\sum_{i=1}^{\infty}\frac{\left[R\left(m^2+\lambda^2 i\right)S\right]}{\lambda^2 i\left[R^*+\lambda^2 i\left(m^2+\lambda^2 i\right)\right]} \tag{22}$$

Vertical circular porous channel.

Mean-mixed-Temperature difference in non-dimensional form is

$$\theta_{my}=\frac{\int_{q=0}^{1}u\theta q\,dq}{\int_{q=0}^{1}uq\,dq}\qquad\frac{\dfrac{2\sum_{i=1}^{\infty}\left[R\,R^*-S\lambda^2 i\right]\left[R\left(m^2+\lambda^2 i\right)+S\right]}{\lambda^2 i\left[R^*+\lambda^2 i\left(m^2+\lambda^2 i\right)\right]^2}}{\dfrac{\left[R\,R^*-S\lambda^2 i\right]}{\sum_{i=1}^{\infty}\lambda^2 i\left[R^*+\lambda^2 i\left(m^2+\lambda^2 i\right)\right]}} \tag{23}$$

Numerical Results

The first few roots equation (10) are 2.4048, 5.5201, 8.6537, 11.7915

The expressions for velocity *u* and θ are plotted against *q* for difference values of *m*.

For numerical calculations it is assumed that $S=1,\ R=2,\ R^*=1,$

The magnetic field decreases the magnitude of the velocity and temperature.

References:

1. J. H. Merkin, (1978) : Int. J Heat Mass Transfer, 21

2. Reghavacharyulu N. C. (1985) : Combined free and forced connection in vertical circular chanel. Indian Journal and Applied Math's Vol. 16, p. 232-328

3. Soundalgekar, V. M : Reg. J. Energy heat transfer, Vol. 1, No. 2, P. P B_1-144 (1979)