A NOTE ON VAGUE IDEALS OF A NEAR ALGEBRA

L. Bhaskar[1], P. Narsimha Swamy[2] and T. Srinivas[3]
[1]Research Scholar, [2]Assistant Professor, [3]Professor
[1,3]Department of Mathematics, Kakatiya University, Telangana, India.
[2]Department of Mathematics, Gitam University, Hyderabad, India.

Abstract: This paper introduces the notion of vague ideal. Vague left ideal and vague right ideal of a near-algebra.
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1. Introduction

The notion of fuzzy subset was introduced by Zadeh [8] and many authors have considered several extension work in fuzzy sets. Gau and Buehrer[3] was the first to study the notion of vague set. Eswarlal and et al. [2] initiated the study of vague field and vague vector space etc. The concept of near-algebras was introduced by Brown [1], and fuzzy algebras over a fuzzy field is redefined by Gu and Lu [4]. Further, Narsimha Swamy [5] considered the notion of fuzzy ideals of a near-ring. Srinivas and Narasimha Swamy [7] developed the concept of fuzzy near-algebra over a fuzzy field and also discussed fuzzy ideals. Recently, Narsimhaswamy and et al.[6] introduced the notion of vague near-algebra over a vague field. Hence in present study, we proposed the new concept of vague ideals of a near-algebra.

2. Preliminaries

For the sake of continuity we recall some basic definitions.

Definition 1. [3] A vague set F in the universe of discourse X is a pair (t_F, f_F), where $t_F : X \to [0,1]$, $f_F : X \to [0,1]$ are mappings such that $t_F(x) + f_F(x) \leq 1$ for all $x \in X$. The functions t_F and f_F are called true membership function and false membership function in [0, 1] respectively. The interval $[t_F(x), 1 - f_F(x)]$ is called the vague value of x in F and it is denoted by $V_F(x)$, i.e. $V_F(x) = [t_F(x), 1 - f_F(x)]$.

Definition 2. [6] Let X be a field and Y be a near-algebra over X. Let F be a vague field of X and Ω be a vague set of Y. Then Ω is a vague near-algebra of Y over F if it satisfies the conditions:
(i) $V_\Omega(l_1 + l_2) \geq \min(V_\Omega(l_1), V_\Omega(l_2))$,
(ii) $V_\Omega(\lambda l_1) \geq \min(V_F(\lambda), V_\Omega(l_1))$,
(iii) $V_\Omega(l_1 l_2) \geq \min(V_\Omega(l_1), V_\Omega(l_2))$,
(iv) $V_F(1) \geq V_\Omega(l_1)$ $\forall l_1, l_2 \in Y$ and $\lambda \in X$.

3. Vague ideals

Definition 3. Let Ω be a vague near-algebra of Y over a vague field F of X. Then Ω is called an vague ideal of Y, if
(i) $V_\Omega(l_1 l_2) \geq V_\Omega(l_1)$ and(ii) $V_\Omega(l_2(l_1 + i) - l_2 l_1) \geq V_\Omega(i)$
i.e.,
(i) $t_\Omega(l_1 l_2) \geq t_\Omega(l_1)$ and $1 - f_\Omega(l_1 l_2) \geq 1 - f_\Omega(l_1)$
(ii) $t_\Omega(l_2(l_1 + i) - l_2 l_1) \geq t_\Omega(i)$ and $1 - f_\Omega(l_2(l_1 + i) - l_2 l_1) \geq 1 - f_\Omega(i)$ $\forall l_1, l_2, i \in Y$.
Ω is a vague right ideal of Y if $V_\Omega(l_1 l_2) \geq V_\Omega(l_1)$ $\forall l_1, l_2 \in Y$.
Ω is a vague left ideal of Y if $V_\Omega(l_2(l_1 + i) - l_2 l_1) \geq V_\Omega(i)$ $\forall l_1, l_2, i \in Y$.

In other words, a vague set Ω of a near-algebra Y over a vague field F of X is said to be vague ideal if it satisfies the conditions:
(i) $V_\Omega(l_1 + l_2) \geq \min(V_\Omega(l_1), V_\Omega(l_2))$,((ii) $V_\Omega(\lambda l_1) \geq \min(V_F(\lambda), V_\Omega(l_1))$,
(iii) $V_F(1) \geq V_\Omega(l_1)$, (iv) $V_\Omega(l_1 l_2) \geq \min(V_\Omega(l_1), V_\Omega(l_2))$,
(v) $V_\Omega(l_2(l_1 + i) - l_2 l_1) \geq V_\Omega(i)$ (or equivalently $V_\Omega(l_2 l_3 - l_2 l_1) \geq V_\Omega(l_3 - l_1)$) for every $l_1, l_2, l_3, i \in Y$, where 1 is the unity in X.
If Ω satisfies (i), (ii), (iii) and (iv), then Ω is called a vague right ideal of Y
If Ω satisfies (i), (ii), (iii) and (v), then Ω is called a vague left ideal of Y.

Example 1. Let $X = Z_2 = \{0, 1\}_{\oplus 2, \otimes 2}$ be a field. A vague set $F = (t_F, f_F)$ of X defined as

$$t_F : X \to [0, 1] \text{ by } t_F(m) = \begin{cases} 0.9, if m = 0 \\ 0.8, if m = 1 \end{cases} \quad \text{and}$$

$$f_F : X \to [0, 1] \text{ by } f_F(m) = \begin{cases} 0.1, if m = 0 \\ 0.2, if m = 1 \end{cases}$$

For any m, n ∈ X, we get $V_F(m-n) \geq V_F(m) \wedge V_F(n)$ and
$n \neq 0, V_F(mn^{-1})\ V_F(m) \wedge V_F(n)$. Thus F is a vague field of X.

Let Y = {0, p, q, r} be a set with two binary operations " + " and " · " as follows:

+	0	p	q	r
0	0	p	q	r
p	p	0	r	q
q	q	r	0	p
r	r	q	p	0

·	0	p	q	r
0	0	0	0	0
p	0	q	0	q
q	0	0	0	0
r	0	q	0	q

Define a scalar multiplication on Y by $0 \cdot z = 0$, $1 \cdot z = z$ for each $z \in Y$, $0,1 \in X$.

Then it is clear that Y is a near-algebra over a field X.

A vague set $\Omega = (t_\Omega, f_\Omega)$ of Y defined as

$$t_\Omega : Y \to [0, 1] \text{ by } t_\Omega(z) = \begin{cases} 0.6, if z = 0 \\ 0.3, otherwise \end{cases} \quad \text{and}$$

$$f_\Omega : Y \to [0, 1] \text{ by } f_\Omega(z) = \begin{cases} 0.4, if m = 0 \\ 0.7, otherwise \end{cases}$$

For any $\lambda_1 \in X$ and $l_1, l_2, i \in Y$. Further (i) $V_\Omega(l_1+l_2) \geq \min(V_\Omega(l_1), V_\Omega(l_2))$, (ii) $V_\Omega(\lambda l_1) \geq \min(V_F(\lambda), V_\Omega(l_1))$, (iii)$V_F(1) \geq V_\Omega(l_1)$, (iv) $V_\Omega(l_1 l_2) \geq V_\Omega(l_1)$, (iv) $V_\Omega(l_1 l_2) \geq \min(V_\Omega(l_2), V_\Omega(l_1))$,
(v) $V_\Omega(l_2(l_1+i)-l_2 l_1) \geq V_\Omega(i)$(or equivalently $V_\Omega(l_2 l_3 - l_2 l_1) \geq V_\Omega(l_3 - l_1)$) Where 1 is the unity in X.
Hence Ω is called a vague ideal of Y.

Example 2. Consider X and Y as in above example1. A vague set $F = (t_F, f_F)$ of X defined as

$$t_F : X \to [0, 1] \text{ by } t_F(m) = \begin{cases} 0.8, if m = 0 \\ 0.7, if m = 1 \end{cases} \quad \text{and}$$

$$f_F : X \to [0, 1] \text{ by } f_F(m) = \begin{cases} 0.2, if m = 0 \\ 0.3, if m = 1 \end{cases}$$

For any $m, n \in X$, we get $V_F(m-n) \geq V_F(m) \wedge V_F(n)$ and $n \neq 0$, $V_F(mn^{-1}) \geq V_F(m) \wedge V_F(n)$.
Thus F is a vague field of X.
And also it is clear that Y is a near-algebra over a field X (∵ by example1).

A vague set $\Omega = (t_\Omega, f_\Omega)$ of Y defined as

$$t_\Omega : Y \to [0, 1] \text{ by } t_\Omega(z) = \begin{cases} 0.6, if z = 0 \\ 0.4, otherwise \end{cases} \quad \text{and}$$

$$f_\Omega : Y \to [0, 1] \text{ by } f_\Omega(z) = \begin{cases} 0.4, if z = 0 \\ 0.6, otherwise \end{cases}$$

For any $\lambda_1 \in X$ and $l_1, l_2, i \in Y$. Further (i) $V_\Omega(l_1+l_2) \geq \min(V_\Omega(l_1), V_\Omega(l_2))$, (ii) $V_\Omega(\lambda l_1) \geq \min(V_F(\lambda), V_\Omega(l_1))$, (iii)$V_F(1) \geq V_\Omega(l_1)$, (iv) $V_\Omega(l_1 l_2) \geq V_\Omega(l_1)$, (iv) $V_\Omega(l_1 l_2) \geq \min(V_\Omega(l_2), V_\Omega(l_1))$,
(v) $V_\Omega(l_2(l_1+i)-l_2 l_1) \geq V_\Omega(i)$(or equivalently $V_\Omega(l_2 l_3 - l_2 l_1) \geq V_\Omega(l_3 - l_1)$) Where 1 is the unity in X.
Hence Ω is called a vague ideal of Y.

4. Main Results

Throughout this section X stands for field and Y stands for near-algebra (right) over the field X otherwise it mentioned.

Theorem 1. Let Ω be a vague ideal of a near-algebra Y over a vague field F of X. Then each level subset $\Omega_t = \{l_1 \in Y : V_\Omega(l_1) \geq t, t \in [0, 1]\}$ is an ideal of Y , where $V_F(\lambda) \geq t$ for any $\lambda \in X$.

Proof. Let $l_1, l_2 \in \Omega_t$ and $\lambda \in X$. Then $l_1, l_2 \in Y$ and $V_\Omega(l_1) \geq t$, $V_\Omega(l_2) \geq t$.
Since Ω is a Vague ideal, we get $V_\Omega(l_1 - l_2) \geq \min\{V_\Omega(l_1), V_\Omega(l_2)\} \geq \min(t, t) = t$.
$\therefore l_1 - l_2 \in \Omega t$. Now $V_\Omega(\lambda l_1) \geq V_F(\lambda) \wedge V_\Omega(l_1) \geq t \wedge t = t$.
$\therefore \lambda l_1 \in \Omega t$. Thus Ωt is a linear subspace of Y.

Let $l_1, l_2 \in Y$ and $i \in \Omega_t$. Then $l_2(l_1 + i) - l_2l_1 \in Y$, and $V_\Omega(l_2(l_1 + i) - l_2l_1) \geq V_\Omega(i) \geq t$. $\therefore$ $l_2(l_1 + i) - l_2l_1 \in \Omega_t$.
Thus Ω_t is a left ideal of Y .
Let $l_1 \in Y$, $i \in \Omega_t$. Then $V_\Omega(il_1) \geq V_\Omega(i) \geq t$. $\therefore$ $il_1 \in \Omega_t$.
Thus Ω_t is a right ideal of Y. Hence Ω_t is an ideal of Y. |

Theorem 2. Intersection of a family of Vague ideals of a near-algebra Y is a Vague ideal of Y.

Proof. Let $\{\Omega_i\}_{i\in\Lambda}$ be a family of Vague ideals of near-algebra Y over a Vague field F of X.
Let $V_{\Omega_i}(l_1) = \bigcap_{i\in\Lambda} V_{\Omega_i}(l_1) = \inf_{i\in\Lambda} V_{\Omega_i}(l_1)$ For every $l_1, l_2 \in Y$ and $\lambda, \mu \in X$, we have

$$V_\Omega(\lambda l_1 + \upsilon l_2) = \inf_{i\in\Lambda} V_{\Omega_i}(\lambda l_1 + \upsilon l_2)$$
$$\geq \inf_{i\in\Lambda}\{\min(V_F(\lambda), V_{\Omega_i}(l_1)), \min(V_F(\upsilon), V_{\Omega_i}(l_2)))\}$$
$$\geq \min(\min(V_F(\lambda), \inf_{i\in\Lambda} V_{\Omega_i}(l_1)), \min(V_F(\upsilon), \inf_{i\in\Lambda} V_{\Omega_i}(l_2)))$$
$$= \min(\min(V_F(\lambda), V_\Omega(l_1)), \min(V_F(\upsilon), V_\Omega(l_2))))$$

Since each Ω_i is a ideal, we get $V_F(1) = V_{\Omega_i}(l_1) \geq \inf_{i\in\Lambda} V_{\Omega_i}(l_1) = V_\Omega(l_1)$ for every $l_1 \in Y$ and $i \in \Lambda$.

Now $V_\Omega(l_1 l_2) = \inf_{i\in\Lambda}(V_{\Omega_i}(l_1 l_2)) \geq \inf_{i\in\Lambda}(V_{\Omega_i}(l_1)) = V_\Omega(l_1)$. Thus Ω is a vague right ideal of Y.

Let $l_1, l_2, j \in Y$. Then $V_\Omega(l_2(l_1 + j) - l_2 l_1)) = \inf_{i\in\Lambda} V_{\Omega_i}(l_2(l_1 + j) - l_2 l_1)) \geq \inf_{i\in\Lambda} V_{\Omega_i}(j) = V_\Omega(j)$. Thus Ω is a vague left ideal of Y. □

Hence Ω is a vague ideal of a near-algebra Y.

Theorem 3. Let Y and Y^1 be two near-algebras over a field X. Let Ω_1 and Ω_2 be two vague ideals of Y and Y^1 respectively over a Vague field F of X. Then $\Omega_1 \times \Omega_2$ is a vague ideal of a near-algebra $Y \times Y^1$.

Proof. Let Ω_1 and Ω_2 be two vague ideals of near-algebras Y and Y^1 respectively over a Vague field F of X. We have that $(V_{\Omega1} \times V_{\Omega2})(l_1, l_1^1) = \min(V_{\Omega1}(l_1), V_{\Omega2}(l_1^0))$ where $(l_1, l_1^1) \in Y \times Y^1$. Also know that $Y \times Y^1 = \{(l_2, l_2^1) : l_2 \in Y, l_2^1 \in Y^1\}$. Let $(l_1, l_1^1), (l_2, l_2^1) \in Y \times Y^1$ and $\lambda \in X$. Then

$$(V_{\Omega1} \times V_{\Omega_2})((l_1, l_1^1)(l_2, l_2^1)) = (V_{\Omega1} \times V_{\Omega_2})(l_1 + l_2, l_1^1 + l_2^1)$$
$$= \min(V_{\Omega1}(l_1 + l_2), V_{\Omega_2}(l_1^1 + l_2^1))$$
$$\geq \min\{\min(V_{\Omega1}(l_1), V_{\Omega1}(l_2)), \min(V_{\Omega_2}(l_1^1)V_{\Omega_2}(l_2^1)\}$$
$$= (V_{\Omega1} \times V_{\Omega_2})(l_1, l_1^1), (V_{\Omega1} \times V_{\Omega_2})(l_2, l_2^1)$$

$$(V_{\Omega1} \times V_{\Omega_2})(\lambda(l_1, l_1^1)) = (V_{\Omega1} \times V_{\Omega_2})(\lambda l_1, \lambda l_1^1)$$
$$= \min(V_{\Omega1}(\lambda l_1), V_{\Omega_2}(\lambda l_1^1))$$
$$\geq \min\{\min(V_F(\lambda), V_{\Omega1}(l_1)), \min(V_F(\lambda), V_{\Omega_2}(l_1^1)\}$$
$$= \min\{(V_F(\lambda), \min(V_{\Omega1}(l_1), V_{\Omega_2}(l_1^1)\}$$
$$= \min\{(V_F(\lambda), (V_{\Omega1} \times V_{\Omega_2})(l_1, l_1^1))$$

Since Ω_1 is a Vague ideal of Y, we get $V_F(1) \geq V_{\Omega1}(l_1)$ for every $l_1 \in Y$, 1 is the unity in X.
And Ω_2 is a vague ideal of Y^1, then $V_F(1) \geq V_{\Omega2}(l_1^1)$ for every Let $l_1^1 \in Y^1$
Then $V_F(1) \geq \min\{(V_{\Omega1}(l_1), V_{\Omega2}(l_1^1))\} = (V_{\Omega1} \times V_{\Omega2})(l_1, l_1^1))$

$$(V_{\Omega1} \times V_{\Omega_2})((l_1, l_1^1), (l_2, l_2^1)) = (V_{\Omega1} \times V_{\Omega_2})(l_1 l_2, l_1^1 l_2^1)$$
$$= \min\{V_{\Omega1}(l_1 l_2), V_{\Omega_2}(l_1^1 l_2^1)\}$$
$$\geq \min\{V_{\Omega1}(l_1), V_{\Omega_2}(l_1^1)\}$$
$$= (V_{\Omega1} \times V_{\Omega_2})(l_1, l_1^1).$$

Thus $V_{\Omega 1} \times V_{\Omega_2}$ is a vague right ideal of Y × Y [1]. Let $(l_1, l_1^1), (l_2, l_2^1), (i, i^1) \in Y \times Y^1$. Then

$$(V_{\Omega 1} \times V_{\Omega_2})((l_2, l_2^1)(l_1, l_1^1) + (i, i^1) - (l_2, l_2^1)(l_1, l_1^1)) = (V_{\Omega 1} \times V_{\Omega_2})((l_2, l_2^1)(l_1 + i, l_1^1 + i^1) - (l_2 l_1, l_2^1 l_1^1))$$
$$= (V_{\Omega 1} \times V_{\Omega_2})(l_2(l_1 + i), l_2^1(l_1^1 + i^1) - (l_2 l_1, l_2^1 l_1^1))$$
$$= (V_{\Omega 1} \times V_{\Omega_2})(l_2(l_1 + i) - l_2 l_1, l_2^1(l_1^1 + i^1 - l_2^1 l_1^1))$$
$$= \min\{V_{\Omega 1}(l_2(l_1 + i) - l_2 l_1), V_{\Omega_2}(l_2^1(l_1^1 + i^1 - l_2^1 l_1^1))\}$$
$$= \min(V_{\Omega 1}(i), V_{\Omega_2}(i^1) = (V_{\Omega 1} \times V_{\Omega_2})(i, i^1)$$

Thus $V_{\Omega 1} \times V_{\Omega_2}$ is a vague left ideal of $Y \times Y^1$. Hence $V_{\Omega 1} \times V_{\Omega_2}$ is a vague ideal of $Y \times Y^1$

Definition 4. Let Ω_1 and Ω_2 be two vague ideals of a zero symmetric near-algebra Y. Let $l_1 \in Y$ then their sum is denoted by $\Omega_1 + \Omega_2$ and is defined by $V_{\Omega 1+\Omega 2}(l_1) = \underset{l_1 = l_2 + l_3}{Sup}\{\min(V_{\Omega_1}(l_2), V_{\Omega 2}(l_3))\}$, where $l_1, l_2 \in Y$.

Note that, if $l^1 = -l_3 + l_2 + l_3$, and then $V_{\Omega_2}(-l_3 + l_2 + l_3) = V_{\Omega_2}(l_2)$. That is $V_{\Omega_2}(l_2^1) = V_{\Omega_2}(l_2)$.

From this it is clear that $V_{\Omega 1+\Omega 2}(l_1) = V_{\Omega 2+\Omega 1}(l_1)$. □

Theorem 4. Let Y and Z be two near-algebras over a field X. Let f: Y→ Z be an onto near-algebra homomorphism. If Ω is a vague ideal in Y , then f (Ω) is a Vague ideal in Z.

Proof. Let u, v ∈ Z and λ ∈ X. Now

(i) for all u, v ∈ Z and their exits $l_1, l_2 \in Y$ such that $u = f(l_1), v = f(l_2)$.

$$V_{f(\Omega)}(u+v) = Sup\{V_\Omega(l_3) : l_3 \in Y, l_3 \in f^{-1}(u+v)\}$$
$$= Sup\{(V_\Omega(l_1 + l_2) : l_1, l_2 \in Y, f(l_1) = u, f(l_2) = v\}$$
$$= Sup\{\min(V_\Omega(l_1), V_\Omega(l_2)) : l_1, l_2 \in Y, f(l_1) = u, f(l_2) = v\}$$
$$= \min\{Sup(V_\Omega(l_1) : l_1 \in Y, f(l_1) = u), Sup(V_\Omega(l_2) : l_2 \in Y, f(l_2) = v)\}$$
$$= \min(V_{f(\Omega)}(u), V_{f(\Omega)}(v)).$$

(ii) For all u∈ Z and λ ∈X, consider

$$V_{f(\Omega)}(\lambda u) = Sup\{V_\Omega(l_3) : l_3 \in Y, l_3 \in f^{-1}(\lambda u)\}$$
$$\geq Sup\{(V_\Omega(\lambda l_3) : l_3 \in Y, f(l_3) = u\ \}$$
$$= Sup\{\min(V_F(\lambda), V_\Omega(l_3)) : l_3 \in Y, f(l_3) = u\}$$
$$= \min\{V_F(\lambda), Sup(V_\Omega(l_3) : l_3 \in Y, f(l_3) = u)\}$$
$$= \min(V_F(\lambda), V_{f(\Omega)}(u)).$$

(iii) We have $V_F(1) \geq V_\Omega(l_1)$ for every $l_1 \in Y$.

Then for all u∈ Z, $V_F(1) \geq Sup\{V_\Omega(l_3) : l_3 \in Y, l_3 \in f^{-1}(u)\} = V_{f(\Omega)}(u)$.

(iv) For all u,v∈ Z

$$V_{f(\Omega)}(uv) = Sup\{V_\Omega(l_3) : l_3 \in Y, l_3 \in f^{-1}(uv)\}$$
$$\geq Sup\{(V_\Omega(l_1 l_2) : l_1, l_2 \in Y, f(l_1) = u, f(l_2) = v\}$$
$$= Sup\{V_\Omega(l_1)) : l_1 \in Y, f(l_1) = u\}$$
$$= V_{f(\Omega)}(u).$$

(v). For all u,v,i∈ Z. Consider,

$$V_{f(\Omega)}(v(u+i)-vu) = Sup\{V_\Omega(l_3): l_3 \in Y, l_3 \in f^{-1}(v(u+i)-vu)\}$$
$$\geq Sup\{V_\Omega(l_2(l_1+j)-l_2l_1): l_1 \in f^{-1}(u), l_2 \in f^{-1}(v), j \in f^{-1}(i)\}$$
$$\geq Sup\{V_\Omega(j)): j \in f^{-1}(i)\}$$
$$= V_{f(\Omega)}(i).$$

Hence $f(\Omega)$ is a vague ideal in Z

Theorem 5. Let Y and Z be two near-algebras over a field X. Let f: Y→ Z be an onto near-algebra homomorphism. If Ω is a Vague ideal in Z, then $f^{-1}(\Omega)$ is a Vague ideal in Y.

Proof. For all l_1, l_2∈ Y and λ, μ ∈ X, consider

$$V_{f^{-1}(\Omega)}(\lambda l_1 + ul_2) = V_\Omega(f(\lambda l_1 + ul_2)$$
$$= V_\Omega(\lambda f(l_1) + uf(l_2))$$
$$\geq \min(V_\Omega(\lambda f(l_1) + V_\Omega(uf(l_2)))$$
$$= \min(\min(V_\Omega(\lambda), V_\Omega f(l_1)), \min(V_\Omega(u), V_\Omega f(l_2)))$$
$$= \min(\min(V_F(\lambda), V_{f^{-1}(\Omega)}(l_1)), \min(V_F(u), V_{f^{-1}(\Omega)}(l_2))).$$

We have $V_F(1) \geq V_\Omega(l_3)$ for each l_3∈ Z. This implies for every l_1∈ Y,
$V_F(1) \geq V_\Omega(f(l_1))$ (Since $f(l_1)$∈ Z)= $V_{f^{-1}(\Omega)}(l_1)$. Now

$$V_{f^{-1}(\Omega)}(l_1 l_2) = V_\Omega(f(l_1 l_2))$$
$$= V_\Omega(f(l_1) f(l_2))$$
$$\geq V_\Omega(l_1))$$
$$= V_{f^{-1}(\Omega)}(l_1).$$

$$V_{f^{-1}(\Omega)}(l_2(l_1+i)-l_2l_1) = V_\Omega(f(l_2(l_1+i)-l_2l_1))$$
$$= V_\Omega(f(l_2(l_1+i)) - f(l_2l_1))$$
$$= (V_\Omega(f(l_2)(f(l_1)+f(i)) - f(l_2)f(l_1))$$
$$\geq V_\Omega f(i))$$
$$= V_{f^{-1}(\Omega)}(i).$$

Hence $f^{-1}(\Omega)$ is a Vague ideal in Y . □

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