POPULATION ESTIMATION UTILIZING KNOWN MEDIAN OF THE STUDY VARIABLE

[1]SURENDRA KUMAR, [2]S. K. YADAV

Assistant Professor
[1]Department of Mathematics,
[1]Govt. Degree College Pihani, Hardoi, U.P., India
[2] Department of Applied Statistics
[2]Babasaheb Bhimrao Ambedkar University, Lucknow, U.P., India

Abstract: **This paper deals with the estimation of population mean of the study variable by utilizing the known median of the study variable. A generalized ratio type estimator has been proposed for this purpose. The expressions for the bias and mean squared error of the proposed estimator have been derived up to the first order of approximation. The optimum value of the characterizing scalar has also been obtained. The minimum value of the proposed estimator for this optimum value of the characterizing scalar is obtained. A theoretical comparison of the proposed estimator has been made with the mean per unit estimator, usual ratio of Cochran (1940) and usual regression estimator of Watson (1937) and also with the Bahl and Tuteja (1991), Srivastava (1967), Reddy (1974), Kadilar (2016) and Subramani (2016) estimators. Through the numerical study, the theoretical findings are validated and it has been shown that proposed estimators perform better than the competing estimators.**

***Keywords*: Study variable, Bias, Ratio estimator, Mean squared error, Simple random sampling, Efficiency.**

2010 AMS Subject Classification: 62D05

I. INTRODUCTION

Some times in practice we come across the situations where population mean of the study variable is not known but the population median of the main variable under study is known. For example if we ask for the weight or basic salary of a person, it very hard to get the exact value but we get the information in terms of interval or the pay band. Here we can easily get the median of the study variable which can be utilized for improved estimation of population mean of study variable. The use of auxiliary variable which is highly correlated with study variable also improves the efficiency of the estimator but it is collected on additional cost of the survey. In this paper we have proposed an improved estimator of population mean of the study variable using median of the study variable.

Let us consider a the finite population consisting of *N* distinct and identifiable units and let $(x_i, y_i), i = 1, 2, ..., n$ be a bivariate sample of size *n* taken from (*X*, Y) using a simple random sampling without replacement (*SRSWOR)* scheme. Let $\bar{X}$ and $\bar{Y}$ respectively be the population means of the auxiliary and the study variables, and let $\bar{x}$ and $\bar{y}$ be the corresponding sample means. In SRSWOR, It is well established that sample means $\bar{x}$ and $\bar{y}$ are unbiased estimators of population means of $\bar{X}$ and $\bar{Y}$ respectively.

To demonstrate the problem in an effective manner, let us consider two interesting examples of mean estimation of study variable using median of study variable given by Subramani (2016). The tables have been used with permission of the author.

Example1. The estimation of body mass index (BMI) of the 350 patients in a Hospital based on a small simple random sample without replacement has been considered.

Table1: Body mass index of 350 patients in a hospital

Category	BMI range – kg/m2	Number of patients	Cumulative total
Very severely underweight	less than 15	15	15
Severely underweight	from 15.0 to 16.0	35	50
Underweight	from 16.0 to 18.5	67	117
Normal (healthy weight)	from 18.5 to 25	92	209
Overweight	from 25 to 30	47	256
Obese Class I (Moderately obese)	from 30 to 35	52	308
Obese Class II (Severely obese)	from 35 to 40	27	335
Obese Class III (Very severely obese)	over 40	15	350
Total		**350**	**350**

The median value will be between 18.5 and 25. So one can assume that the population median of the BMI is approximately 21.75

Example2. The problem is to estimate the average salary drawn by the faculty members (population mean) per month in an Indian university 800 faculty members are working in different categories and the basic salary drawn by different categories of the faculty members.

Table2: Salary of University faculty members

Category	**Basic Salary in Indian Rupees (IRs) Per month***	**Number of faculty members**	**Cumulative total**
Senior Professor	56000+10000**	20	20
Professor - Grade I	43000+10000	40	60
Professor - Grade II	37400+10000	60	120
Associate Professor - Grade I	37400+10000	80	200
Associate Professor - Grade II	37400+9000	100	300
Assistant Professor - Grade I	15100+8000	110	410
Assistant Professor - Grade II	15100+7000	140	550
Assistant Professor - Grade III	15100+6000	250	800
Total		**800**	**800**

*Actual salary depends on their experience in their designation and other allowances.

**The Basic salary is the sum of the basic (the first value) and the academic grade pay (the second value), which will differentiate people with same designation but different grades.

The population median value will be assumed as IRs. 15100+8000 = IRs. 23100.

II. REVIEW OF EXISTING ESTIMATORS

The sample mean is the most suitable estimators of population mean of the study variable, given by,

$$t_o = \bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i \tag{1}$$

It is an unbiased estimator and its variance, up to the first order of approximation, is given by

$$V(t_0) = \frac{1-f}{n} S_y^2 = \frac{1-f}{n} \bar{Y}^2 C_y^2 \tag{2}$$

where, $C_y = \frac{S_y}{\bar{Y}}$, $S_y^2 = \frac{1}{N-1}\sum_{i-1}^{N}(Y_i - \bar{Y})^2 = \frac{1}{{}^N C_n}\sum_{i-1}^{{}^N C_n}(\bar{y}_i - \bar{Y})^2$, $f = \frac{n}{N}$.

Watson (1937) first utilized the highly correlated auxiliary variable and proposed the usual linear regression estimator of population mean as,

$$t_1 = \bar{y} + \beta_{yx}(\bar{X} - \bar{x}) \tag{3}$$

where β_{yx} is the regression coefficient of the line Y on X.

This estimator is also unbiased for population mean and its variance up to the first order of approximation, is given by,

$$V(t_1) = \frac{1-f}{n} \bar{Y}^2 C_y^2 (1-\rho_{yx}^2)\,. \tag{4}$$

Cochran (1940) also made use of highly positively correlated auxiliary variable and proposed the following usual ratio estimator as,

$$t_2 = \bar{y}\frac{\bar{X}}{\bar{x}} \tag{5}$$

He showed that it is a biased estimator of population mean and he gave the expressions for the bias and mean squared error for his estimator, up to the first order of approximation as,

$$B(t_2) = \frac{1-f}{n}\bar{Y}[C_x^2 - C_{yx}] \text{ and}$$

$$MSE(t_2) = \frac{1-f}{n}\bar{Y}^2[C_y^2 + C_x^2 - 2C_{yx}], \tag{6}$$

where, $C_x = \frac{S_x}{\bar{X}}$, $S_x^2 = \frac{1}{N-1}\sum_{i-1}^{N}(X_i - \bar{X})^2 = \frac{1}{{}^N C_n}\sum_{i-1}^{{}^N C_n}(\bar{x}_i - \bar{X})^2$, $\rho_{yx} = \frac{Cov(x,y)}{S_x S_y}$,

$$Cov(x,y) = \frac{1}{N-1}\sum_{i-1}^{N}(Y_i - \bar{Y})(X_i - \bar{X}), \text{ and } C_{yx} = \rho_{yx} C_y C_x\,.$$

Bahl and Tuteja (1991) proposed the following exponential ratio type estimator of population mean by making use of positively correlated auxiliary variable as,

$$t_3 = \bar{y}\exp\left[\frac{\bar{X}-\bar{x}}{\bar{X}+\bar{x}}\right] \tag{7}$$

The above estimator is biased and the bias and the mean squared error of this estimator, up to the first order of approximation, are given respectively by,

$$B(t_3) = \frac{1-f}{8n}\bar{Y}[3C_x^2 - 4C_{yx}] \text{ and}$$

$$MSE(t_3) = \frac{1-f}{n}\bar{Y}^2[C_y^2 + \frac{C_x^2}{4} - C_{yx}]. \tag{8}$$

Srivastava (1967) proposed the following generalized ratio type estimator of population mean using positively correlated auxiliary variable as,

$$t_4 = \bar{y}\left(\frac{\bar{x}}{\bar{X}}\right)^{\alpha} \tag{9}$$

where α is a suitably chosen constant.
It is also a biased estimator and its bias and the mean squared error up to the first order of approximation are given respectively by,

$$B(t_4) = \frac{1-f}{n}\bar{Y}[\frac{\alpha(\alpha-1)}{2}C_x^2 + \alpha C_{yx}] \text{ and}$$

$$MSE(t_4) = \frac{1-f}{n}\bar{Y}^2[C_y^2 + \alpha^2 C_x^2 + 2\alpha C_{yx}]$$

The optimum value of the constant α is, $\alpha = -C_{yx}/C_x^2$.

The minimum value of $MSE(t_4)$ for optimum value of α is given by,

$$MSE_{\min}(t_4) = \frac{1-f}{n}\bar{Y}^2 C_y^2(1-\rho_{yx}^2). \tag{10}$$

Reddy (1974) suggested the following class of ratio type estimators for population mean of study variable using positively correlated auxiliary variable as,

$$t_5 = \bar{y}\left[\frac{\bar{X}}{\bar{X}+\alpha(\bar{x}-\bar{X})}\right] \tag{11}$$

It is a biased estimator and the bias and the mean squared error of this estimator, up to the first order of approximation are given respectively by,

$$B(t_5) = \frac{1-f}{n}\bar{Y}[\alpha^2 C_x^2 - \alpha C_{yx}] \text{ and}$$

$$MSE(t_5) = \frac{1-f}{n}\bar{Y}^2[C_y^2 + \alpha^2 C_x^2 - 2\alpha C_{yx}]$$

The above MSE is minimum for optimum value of $\alpha = C_{yx}/C_x^2$ and the minimum MSE is given by,

$$MSE_{\min}(t_5) = \frac{1-f}{n}\bar{Y}^2 C_y^2(1-\rho_{yx}^2). \tag{12}$$

Kadilar (2016), using positively correlated auxiliary variable proposed the following exponential type estimator of population mean as,

$$t_6 = \bar{y}\left(\frac{\bar{x}}{\bar{X}}\right)^{\delta}\exp\left(\frac{\bar{X}-\bar{x}}{\bar{X}+\bar{x}}\right) \tag{13}$$

where δ is a characterizing scalar to be determined such the MSE of above estimator is minimum.
The bias and the mean squared error of the above estimator up to the first order of approximation respectively are,

$$B(t_6) = \frac{1-f}{n}\bar{Y}\left[\left\{\frac{\delta(\delta-1)}{2} + \frac{3}{8}\right\}C_x^2 + \left(\delta + \frac{1}{2}\right)C_{yx}\right]$$

$$MSE(t_6) = \frac{1-f}{n}\bar{Y}^2\left[C_y^2 + \left(\delta^2 + \delta + \frac{1}{4}\right)C_x^2 + (2\delta+1)C_{yx}\right] \tag{14}$$

The optimum value of the characterizing scalar δ which minimizes the mean squared error of t_6 is,

$$\delta_{opt} = \left(\frac{1}{2} - \rho_{yx} C_y / C_x \right)$$

The minimum value of the mean squared error of above estimator is,

$$MSE_{\min}(t_6) = \frac{1-f}{n} \bar{Y}^2 C_y^2 (1-\rho_{yx}^2) \tag{15}$$

which is equal to the variance of the usual regression estimator of Watson (1937).

Subramani (2016) used the population median of the study variable and proposed the following ratio estimator of population mean of the study variable,

$$t_7 = \bar{y}\left(\frac{M}{m}\right) \tag{16}$$

where M and m are the population and sample medians of study variable respectively.

It is a biased estimator and its bias and the mean squared error, up to the first order of approximation, are respectively given by,

$$B(t_7) = \frac{1-f}{n} \bar{Y}[C_m^2 - C_{ym} - \frac{Bias(m)}{M}] \text{ and}$$

$$MSE(t_7) = \frac{1-f}{n} \bar{Y}^2 [C_y^2 + R_7^2 C_m^2 - 2R_7 C_{ym}], \tag{17}$$

where, $R_7 = \frac{\bar{Y}}{M}$, $C_m = \frac{S_m}{M}$, $S_m^2 = \frac{1}{{}^N C_n} \sum_{i-1}^{{}^N C_n} (m_i - M)^2$, $S_{ym} = \frac{1}{{}^N C_n} \sum_{i-1}^{{}^N C_n} (\bar{y}_i - \bar{Y})(m_i - M)$ and $C_{ym} = \frac{S_{ym}}{\bar{Y}M}$.

Many authors have given various modified estimators of population mean using auxiliary variables. The latest references can be found in Subramani (2013), Subramani and Kumarapandiyan (2012, 2013), Tailor and Sharma (2009), Yan and Tian (2010), Yadav et al. (2014, 2015), Yadav et al. (2016), and Abid *et al.* (2016).

III. PROPOSED ESTIMATORS

Motivated by Srivastava (1967) and Jerajuddin & Kishun (2016), we propose the following ratio type estimators of population mean using known population median of study variable as,

$$t_p = \bar{y}\left(\frac{M+n}{m+n}\right)^{\alpha} \tag{18}$$

where α is a characterizing scalar to be determined such that the mean squared error of the proposed estimator t_p is minimum.

The following approximations have been made to study the properties of the proposed estimators as,

$\bar{y} = \bar{Y}(1+e_0)$ and $m = M(1+e_1)$ such that $E(e_0) = 0$, $E(e_1) = \frac{\bar{M} - M}{M} = \frac{Bias(m)}{M}$ and

$E(e_0^2) = \frac{1-f}{n} C_y^2$, $E(e_1^2) = \frac{1-f}{n} C_m^2$, $E(e_0 e_1) = \frac{1-f}{n} C_{ym}$,

where, $\bar{M} = \frac{1}{n} \sum_{i=1}^{n} m_i$

The proposed estimator t_p can be expressed in terms of e_i's ($i = 1, 2$) as,

$$t_{p_1} = \bar{Y}(1+e_0)\left(\frac{M+n}{M(1+e_1)+n}\right)^{\alpha} = \bar{Y}(1+e_0)(1+\theta e_1)^{-\alpha},$$

where $\theta = \frac{M}{M+n}$.

Expanding the right hand side of the above equation, simplifying and retaining the terms up to the first order of approximations, we get,

$$t_p = \bar{Y}[1 + e_0 - \alpha\theta e_1 - \alpha\theta e_0 e_1 + \frac{\alpha(\alpha+1)}{2} \theta^2 e_1^2]$$

Or

$$t_p - \bar{Y} = \bar{Y}[e_0 - \alpha\theta e_1 - \alpha\theta e_0 e_1 + \frac{\alpha(\alpha+1)}{2} \theta^2 e_1^2]. \tag{19}$$

Taking expectation on both sides and putting the values of various expectations, we get the bias of the proposed estimator t_p, up to the first order of approximation as,

$$B(t_{p_1})=\bar{Y}\left[\frac{1-f}{n}\frac{\alpha(\alpha+1)}{2}\theta^2C_m^2-\alpha\theta\frac{Bias(m)}{M}-\alpha\theta\frac{1-f}{n}C_{ym}\right]$$

From equation (19), up to the first order of approximation, we have,

$$t_p-\bar{Y}\cong\bar{Y}[e_0-\alpha\theta e_1]$$

Squaring both sides of above equation and taking expectations on both sides, we get the approximate mean squared error of the proposed estimator t_p as

$$MSE(t_p)=\bar{Y}^2E(e_0^2+\alpha^2\theta^2e_1^2-2\alpha\theta e_0e_1)$$
$$=\bar{Y}^2[E(e_0^2)+\alpha^2\theta^2E(e_1^2)-2\alpha\theta E(e_0e_1)]$$

Putting values of various expectations in above equation, we have,

$$MSE(t_p)=\frac{1-f}{n}\bar{Y}^2[C_y^2+\alpha^2\theta^2C_m^2-2\alpha\theta C_{ym}] \tag{20}$$

which is minimum for,

$$\alpha_{opt}=C_{ym}\big/\theta C_m^2.$$

and the minimum mean squared error of the proposed estimator t_p is,

$$MSE_{\min}(t_p)=\frac{1-f}{n}\bar{Y}^2\left[C_y^2-\frac{C_{ym}^2}{C_m^2}\right]. \tag{21}$$

IV. EFFICIENCY COMPARISON

Under this section, a theoretical comparison of the proposed estimator has been made with the competing estimators of population mean. The conditions under which the proposed estimator performs better than the competing estimators have also been given.

From equation (21) and equation (2), we have,

$$V(t_0)-MSE_{\min}(t_p)>0 \text{ if}$$

$$\frac{C_{ym}^2}{C_m^2}>0, \text{ or if } C_{ym}^2>0$$

Thus the proposed estimator is better than the usual mean per unit estimator of population mean.

From equation (21) and equation (4), we have,

$$MSE(t_1)-MSE_{\min}(t_p)>0 \text{ if}$$

$$\frac{C_{ym}^2}{C_m^2}-C_y^2\rho_{yx}^2>0$$

Under the above condition, the proposed estimator is better than the usual regression estimator of Watson (1937).

From equation (21) and equation (6), we have,

$$MSE(t_2)-MSE_{\min}(t_p)>0 \text{ if}$$

$$C_x^2-2C_{yx}+\frac{C_{ym}^2}{C_m^2}>0, \text{ if}$$

$$C_x^2+\frac{C_{ym}^2}{C_m^2}>2C_{yx}$$

Under the above condition, proposed estimators perform better than the usual ratio estimator given by Cochran (1940).

From equation (21) and equation (8), we have,

$$MSE(t_3)-MSE_{\min}(t_p)>0 \text{ if}$$

$$\frac{C_x^2}{4}-C_{yx}+\frac{C_{ym}^2}{C_m^2}>0, \text{ or}$$

$$\frac{C_x^2}{4}+\frac{C_{ym}^2}{C_m^2}>C_{yx}$$

Under the above condition, the proposed estimator performs better than Bahl and Tuteja (1991) ratio type estimator of population mean.

From equation (21) and equation (10), we have,

$$MSE(t_4) - MSE_{\min}(t_p) > 0 \text{, if}$$

$$\frac{C_{ym}^2}{C_m^2} - C_y^2 \rho_{yx}^2 > 0$$

Under the above condition, the proposed estimator is better than the Srivastava (1967) estimator.

The proposed estimator is better than Reddy (1974) and Kadilar (2016) estimators of population mean using auxiliary information under the same condition as for Srivastava (1967) estimator in above equation.

From equation (21) and equation (17), we have,

$$MSE(t_7) - MSE_{\min}(t_p) > 0 \text{, if}$$

$$R_7^2 C_m^2 - 2R_7 C_{ym} + \frac{C_{ym}^2}{C_m^2} > 0 \text{, or}$$

$$R_7^2 C_m^2 + \frac{C_{ym}^2}{C_m^2} > 2R_7 C_{ym}$$

Under the above condition, the proposed estimator is better than the Subramani (2016) estimator of population mean using information on median of the study variable.

V. NUMERICAL STUDY

To judge the theoretical findings, we have considered the natural populations given in Subramani (2016). He has used three natural populations. The population 1 and 2 have been taken from Singh and Chaudhary (1986, page no. 177) and the population 3 has been taken from Mukhopadhyay (2005, page no. 96). In populations 1 and 2, the study variable is the estimate the area of cultivation under wheat in the year 1974, whereas the auxiliary variables are the cultivated areas under wheat in 1971 and 1973 respectively. In population 3, the study variable is the quantity of raw materials in lakhs of bales and the number of labourers as the auxiliary variable, in thousand for 20 jute mills. Tables 3-6 represent the parameter values along with constants, biases of various estimators along with proposed estimator, variances and mean squared errors of existing and proposed estimator and percentage relative efficiencies of the proposed estimator over other existing estimators respectively.

Table 3: Parameter values and constants for three natural populations

Parameter	Population-1	Population-2	Population-3
N	34	34	20
n	5	5	5
${}^N C_n$	278256	278256	15504
$\bar{Y}$	856.4118	856.4118	41.5
$\bar{M}$	736.9811	736.9811	40.0552
M	767.5	767.5	40.5
$\bar{X}$	208.8824	199.4412	441.95
R_7	1.1158	1.1158	1.0247
C_y^2	0.125014	0.125014	0.008338
C_x^2	0.088563	0.096771	0.007845
C_m^2	0.100833	0.100833	0.006606
C_{ym}	0.07314	0.07314	0.005394
C_{yx}	0.047257	0.048981	0.005275
ρ_{yx}	0.4491	0.4453	0.6522

Table 4: Bias of various estimators

Estimator	Population-1	Population-2	Population-3
t_2	35.3748	40.9285	0.1067
t_3	1.39995	1.72380	0.0019
t_4	-1.60997	1.76775	0.0054
t_5	2.07309	1.85541	0.0167
t_6	27.4137	27.4137	0.3743
t_7	57.7705	57.7705	0.5061

Table 5: Mean squared error of various estimators

Estimator	Population-1	Population-2	Population-3
t_0	15640.97	15640.97	2.15
t_1	12486.75	12539.30	1.24
t_2	14895.27	15492.08	1.48
t_3	12498.01	12539.30	1.30
t_4	12486.75	12539.30	1.24
t_5	12486.75	12539.30	1.24
t_6	12486.75	12539.30	1.24
t_7	10926.53	10926.53	1.09
t_p	9002.22	9002.22	0.98

Table 6: PRE of the proposed estimator t_p with respect to existing estimators

Estimator	Population-1	Population-2	Population-3
t_0	173.7457	173.7457	219.3878
t_1	138.7075	139.2912	126.5306
t_2	165.4622	172.0918	151.0204
t_3	138.8325	139.2912	132.6531
t_4	138.7075	139.2912	126.5306
t_5	138.7075	139.2912	126.5306
t_6	138.7075	139.2912	126.5306
t_7	121.3759	121.3759	111.2245

VI. RESULTS AND CONCLUSION

In the present paper we have proposed an estimator of population mean of study variable using population median of study variable. The expressions for the bias and mean squared error for the proposed estimator have been obtained up to the first order of approximation. The optimum value of the characterizing scalar which minimizes the mean squared error of the proposed estimator has been obtained. For this optimum value of the characterizing scalar, the minimum value of the mean squared error of the proposed estimator has also been obtained. The proposed estimator is compared with the competing estimators of population mean under simple random sampling scheme. The theoretical conditions under which the proposed estimator performs better than the competing estimators have also been given. These theoretical conditions have also been verified through the different numerical examples from natural populations. From Table-5, it can be seen that the proposed estimator has minimum mean squared error among other competing estimators of population mean of study character. Thus proposed estimator is better than Watson (1937) usual regression estimator, Cochran (1940) usual ratio estimator, Bahl and Tuteja (1991) exponential ratio type estimator, Srivastava (1967) estimator, Reddy (1974) estimator, Kadilar (2016) estimator and Subramani (2016) estimator. Therefore it is recommended that the proposed estimator may be used for improved estimation of population mean under simple random sampling scheme.

REFERENCES

[1] M. Abid, N. Abbas, R.A.K. (2016). Sherwani and H. Z. Nazir, Improved Ratio Estimators for the population mean using non-conventional measure of dispersion, Pakistan Journal of Statistics and Operations Research, XII (2): 353-367.

[2] S. Bahl and R.K. Tuteja, (1991). Ratio and product type exponential estimator, Information and Optimization Sciences, XII (I): 159-163.

[3] W. G. Cochran, (1940). The Estimation of the Yields of the Cereal Experiments by Sampling for the Ratio of Grain to Total Produce, The Journal of Agric. Science, 30 : 262-275.

[4] M. Jerajuddin and J. Kishun, (2016). Modified Ratio Estimators for Population Mean Using Size of the Sample, Selected From Population, IJSRSET, 2, 2: 10-16.

[5] G.O. Kadilar, (2016). A New Exponential Type Estimator for the Population Mean in Simple Random Sampling, Journal of Modern Applied Statistical Methods, 15, 2: 207-214.

[6] V.N. Reddy, (1974). On a transformed ratio method of estimation, Sankhya, C, 36(1): 59-70.

[7] S.K. Srivastava, (1967). An estimator using auxiliary information in sample surveys, Cal. Statist. Assoc. Bull., 16: 62-63.

[8] J. Subramani, (2013). Generalized modified ratio estimator of finite population mean, Journal of Modern Applied Statistical Methods, 12 (2): 121–155.

[9] J. Subramani and G. Kumarapandiyan, (2012). Estimation of population mean using coefficient of variation and median of an auxiliary variable, International Journal of Probability and Statistics, 1 (4): 111–118.

[10] J. Subramani and G. Kumarapandiyan, (2013). A new modified ratio estimator of population mean when median of the auxiliary variable is known, Pakistan Journal of Statistics and Operation Research, Vol. 9 (2): 137–145.

[11] J. Subramani. (2016). A new median based ratio estimator for estimation of the finite population mean, Statistics in Transition New Series, 17, 4: 1-14.

[12] R. Tailor and B. Sharma, (2009). A modified ratio-cum-product estimator of finite population mean using known coefficient of variation and coefficient of kurtosis, Statistics in Transition-New Series, 10 (1): 15–24.

[13] D.J. Watson, (1937). The estimation of leaf area in field crops, The Journal of Agricultural Science, 27, 3: 474-483.

[14] S.K. Yadav, S.S. Mishra and A.K. Shukla, (2014). Improved Ratio Estimators for Population Mean Based on Median Using Linear Combination of Population Mean and Median of an Auxiliary Variable, American Journal of Operational Research, 4, 2: 21-27.

[15] S.K. Yadav, S.S. Mishra and A.K. Shukla, (2015). Estimation Approach to Ratio of Two Inventory Population Means in Stratified Random Sampling, American Journal of Operational Research, 5, 4: 96-101.

[16] S.K. Yadav, S.A.T. Gupta, S.S. Mishra and A.K. Shukla, (2016). Modified Ratio and Product Estimators for Estimating Population Mean in Two-Phase Sampling, American Journal of Operational Research, 6, 3: 61-68.