EOQ MODEL WITH SELLING PRICE & TIME DEPENDENT DEMAND AND CONSTANT DETERIORATION RATE WITH INVENTORY DEPENDENT HOLDING COST

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Abstract: In this paper we consider EOQ model with constant deterioration rate and selling price & time dependent demand. We considered holding cost as inventory dependent. Shortages are not allowed. Mathematical formulation is provided to find the optimal solution. We show total profit is a convex function with respect to cycle time. Numerical examples and sensitivity analysis are given to validate the proposed model in the paper.

Key words: Selling price, Deterioration rate, EOQ model, holding cost, Time dependent, Inventory

INTRODUCTION

Most of the inventory modelers considered, the demand rate is constant. However, in real market, and in real life the demand rate of commodity is always in a fluctuating state. Many researchers have considered their orientation for time-dependent demand rate. In the classical EOQ model, it was assumed that the supplier is paid for the items as soon as the products/items are received. However in daily life the supplier offers the customer a permissible delay in payments to attract new customer who consider it to be a type of price reduction. The vendor can sell goods to accumulate revenue and earn interest before the fixed period. But if the payment is delayed beyond this period a higher interest will be charged. Deterioration rate affects the inventory cost. In real life situation, for certain types of consumer goods like vegetables, fruits, bread etc, the consumption rate is influenced by the demand and deterioration both. Many researchers have paid their attention for time- dependent demand. Silver and Meal [1] first presented an EOQ model for the case of a varying demand. Donaldon [2] developed an EOQ model for the case of linearly time dependent demand. Silver [3] developed a simple replenishment decision rule for a linear trend in demand. Dave and Patel [4] presented an inventory model for deteriorating items with time proportion demand. Tripathi and Tomar [5] considered optimal order policy for deteriorating items with time dependent demand linked to order quantity. Tripathi and Misra [6] developed a model on credit financing in economic ordering policies of non-deteriorating items with time dependent demand rate. Tripathi and Kumar [7] also developed a modelon credit financing in economic ordering policies of time dependent deteriorating items. Many researchers made valuable contribution in this direction like Mitra [8], Ritchie [9], [10], [11]. Recently Khanra*et al.* [12] developed an EOQ model for deteriorating items having time-dependent demand when delay in payment is permissible. Teng*et al.* [13] developed an EOQ model for non-decreasing demand function of time, which is suitable not only for growth stage but also for the maturity stage of a product life cycle.

The permissible delay in payments produces benefits to the supplier such as it should attract new purchasers who consider it to be a type of price reduction. In this direction, number of research articles appeared which contain the EOQ model under trade credit. Goyal [14] considered the effects of trade credit on the optimal inventory policy. Chand and Ward [15] extended Goyal's [14] model under assumption of the classical economic order model and obtained different results. Aggarwal and Jagg [16] considered the inventory model with an exponential deterioration rate under the condition of permissible delay in payment. Cheng *et al.* [17] extended Goyal's [14] model to the case for deteriorating items. Many related articles can be found in Jamal *et al.* [18], Hwang and Shim [19], Abad and Jaggi [20], Chang [21], Davis and Gaither [22], Mandal and Phanjder [23], Ching and Liao [24], Onyang*et al.* [25], [26] and their references.

Hsieh *et al.* [27] generalized the demand rate for an EOQ model with upstream and downstream trade credits to an increasing function of time, and found that a unique minimum cost per unit of time exists. Tsau*et al.* [28] developed a single item and joint multi-item replenishment models and develop some theorems to solve these problems. Hou and Lin [29] developed an inventory model to determine an optimal ordering policy for deteriorating items with delay in payments permitted to supplier under inflation and time discounting. Jaggi*et al.* [30] incorporates the concept of credit-linked demand and developed a new inventory model under two levels of trade credit policy to reflect the real life situation. Ghiami*et al.* [31] established a two echelon supply chain model for deteriorating EOQ models in which warehouse capacity is limited. Pal and Chandra [32] developed a model with stock dependent demand under shortages and permissible delay in payments. Wee *et al.*[33] considered an economic production quantity model with partial backlogging.

ASSUMPTIONS AND NOTATIONS

While developing mathematical model we made certain assumptions and notations as per the requirements of our mathematical model.

- Lead time is taken as zero.
- Shortages are not allowed.
- hUnit holding cost per unit time.
- D (p,t) = a. P(t).t Demandrate.
- P(t) = p. $e^{\gamma t}$ is the selling price per unit at time t. p is the selling price of the item at time t = 0.
- θ is deterioration rate.
- $I_d(t)$ inventory without decay.
- C_2 Constant ordering cost per unit/unit time.
- C_3 Unit purchase cost per unit time.

DEVLOPMENT OF MODEL AND OPTIMAL SOLUTION

In this paper we developed a model as its inventory level is I(0) at time (t = 0). Now the differential equation at time [0, T] is given by:

$$\frac{dI(t)}{dt} + \theta . I(t) = -D\{p,t\} \quad 0 < t < T \qquad (1)$$

We use initial condition as I (t) = I (0) and I(0) = Q_T at t = 0. (2)

By the help of condition given above, the solution of equation is:

$$I(t) = e^{-\theta t}\left[-a\,p\,\frac{e^{(\theta+r)t}}{(\theta+r)}\left(t-\frac{1}{(\theta+r)}\right)+\left(\frac{1}{(\theta+r)^2}-\frac{I(0)}{a\,p}\right)\right] \qquad (3)$$

From eq. (3) we have;

$$I(0) = -a\,p\left[e^{\theta t}\,I(t)+a\,p\,\frac{e^{(\theta+r)t}}{(\theta+r)}\left(t-\frac{1}{(\theta+r)}\right)-\left(\frac{1}{(\theta+r)^2}\right)\right] \qquad (4)$$

Now inventory without decay $I_d(t)$ at time ‘t’ is given by:

$$\frac{dI_d(t)}{dt} = -D\{p,t\} \qquad (5)$$

Using the initial condition at t = 0 as I_d (t) = I (0).
By the help of above condition, the solution of equation is:

$$I_d(t) = -a\,p\left[\frac{t\,e^{rt}}{r}-\frac{e^{rt}}{r^2}\right]+\frac{a\,p}{r^2}+I(0) \qquad (6)$$

Now we find total stock loss S(t) due to decay in [0,T] is given by:

$S(t) = I_d(t) - I(t)$ (7)

$$= a\,p\left[\frac{1}{r^2}-\frac{t\,e^{rt}}{r}+\frac{e^{rt}}{r^2}\right]+I(t)\left(a\,p\,e^{\theta t}+1\right)-a\,p\left[a\,p\left(\frac{e^{(\theta+r)t}}{(\theta+r)}\right)\left(t-\frac{1}{(\theta+r)}\right)-\frac{1}{(\theta+r)^2}\right] \tag{8}$$

We use boundary condition I(T) = 0, for t = T.

$$S(T)=a\,p\left[\frac{1}{r^2}-\frac{T\,e^{rT}}{r}+\frac{e^{rT}}{r^2}\right]-a\,p\left[a\,p\left(\frac{e^{(\theta+r)T}}{(\theta+r)}\right)\left(T-\frac{1}{(\theta+r)}\right)-\frac{1}{(\theta+r)^2}\right] \tag{9}$$

The order quantity is:

$$Q_T = S(T)+\int_0^T D\{p,t\}\,dt \tag{10}$$

$$=a\,p\left[\frac{1}{r^2}-\frac{T\,e^{rT}}{r}+\frac{e^{rT}}{r^2}\right]-a\,p\left[a\,p\left(\frac{e^{(\theta+r)T}}{(\theta+r)}\right)\left(T-\frac{1}{(\theta+r)}\right)-\frac{1}{(\theta+r)^2}\right]$$

(11)
$$+a\,p\left\{\frac{T\,e^{rT}}{r}-\frac{e^{rT}}{r^2}+\frac{1}{r^2}\right\}$$

The holding cost is

$$H(t)=\int_0^T h\ I(t)\ dt$$

$$=\gamma.h\left[\left(t_1-\frac{\delta t_1^{\,2}}{2}-\frac{\alpha\,t_1^{\,\beta+1}}{(\beta+1)}+\frac{\alpha.\delta.t_1^{\,\beta+2}}{(\beta+2)}+\frac{\delta^2\,t_1^{\,3}}{6}-\frac{\alpha.\delta^2.t_1^{\,\beta+3}}{2(\beta+3)}\right).\left(t_1+\frac{\delta t_1^{\,2}}{2}+\frac{\delta^2\,t_1^{\,3}}{6}\right.\right.$$

$$\left.+\frac{\alpha\,t_1^{\,\beta+1}}{(\beta+1)}+\frac{\alpha.\delta.t_1^{\,\beta+2}}{(\beta+2)}+\frac{\alpha.\delta^2.t_1^{\,\beta+3}}{2(\beta+3)}\right)-\left\{\frac{t_1^{\,2}}{2}+\frac{\delta.t_1^{\,3}}{6}+\frac{\delta^2.t_1^{\,4}}{24}+\frac{\alpha.t_1^{\,(\beta+2)}}{(\beta+1).(\beta+2)}+\right.$$

$$\frac{\alpha.\delta.t_1^{\,(\beta+3)}}{(\beta+2).(\beta+3)}+\frac{\alpha.\delta^2.t_1^{\,(\beta+4)}}{2.(\beta+2).(\beta+4)}-\frac{\delta.t_1^{\,3}}{3}-\frac{\delta^2.t_1^{\,4}}{8}-\frac{\delta^3.t_1^{\,5}}{30}-\frac{\alpha.\delta.t_1^{\,(\beta+3)}}{(\beta+1).(\beta+3)}$$

$$-\frac{\alpha.\delta^2.t_1^{\,(\beta+4)}}{(\beta+2).(\beta+4)}-\frac{\alpha.\delta^3.t_1^{\,(\beta+5)}}{2.(\beta+3).(\beta+5)}-\frac{\alpha.\,t_1^{\,(\beta+2)}}{(\beta+2)}-\frac{\alpha.\delta.t_1^{\,(\beta+3)}}{2.(\beta+3)}-\frac{\alpha.\delta^2.t_1^{\,(\beta+4)}}{6.(\beta+4)}-$$

$$\frac{\alpha^2.t_1^{\,(2\beta+2)}}{(\beta+1).(2\beta+2)}-\frac{\alpha^2.\delta.t_1^{\,(2\beta+3)}}{(\beta+2).(2\beta+3)}-\frac{\alpha^2.\delta^2.t_1^{\,(2\beta+4)}}{2.(\beta+3).(2\beta+4)}+\frac{\alpha.\delta.t_1^{\,(\beta+3)}}{(\beta+3)}+$$

$$\frac{\alpha.\delta^2.t_1^{\,(\beta+4)}}{2.(\beta+4)}+\frac{\alpha.\delta^3.t_1^{\,(\beta+5)}}{6.(\beta+5)}+\frac{\alpha^2.\delta.t_1^{\,(2\beta+3)}}{(\beta+1).(2\beta+3)}+\frac{\alpha^2.\delta^2.t_1^{\,(2\beta+4)}}{(\beta+2).(2\beta+4)}+\frac{\alpha^2.\delta^3.t_1^{\,(2\beta+5)}}{2.(\beta+3).(2\beta+5)}$$

$$+\frac{\delta^2.t_1^{\,4}}{8}+\frac{\delta^3.t_1^{\,5}}{20}+\frac{\delta^4.t_1^{\,6}}{72}+\frac{\alpha.\delta^2.t_1^{\,(\beta+4)}}{2.(\beta+1).(\beta+4)}+\frac{\alpha.\delta^3.t_1^{\,(\beta+5)}}{2.(\beta+2).(2\beta+5)}+\frac{\alpha.\delta^4.t_1^{\,(\beta+6)}}{4.(\beta+3).(\beta+6)}$$

$$-\frac{\alpha.\delta^2.t_1^{\,(\beta+4)}}{2.(\beta+4)}-\frac{\alpha.\delta^3.t_1^{\,(\beta+5)}}{4.(\beta+5)}-\frac{\alpha.\delta^4.t_1^{\,(\beta+6)}}{12.(\beta+3)}-\frac{\alpha^2.\delta^2.t_1^{\,(2\beta+4)}}{2.(\beta+1).(2\beta+4)}-\frac{\alpha^2.\delta^3.t_1^{\,(2\beta+5)}}{2.(\beta+2).(2\beta+5)}$$

$$\left.\left.-\frac{\alpha^2.\delta^4.t_1^{\,(2\beta+6)}}{4.(\beta+3).(2\beta+6)}\right\}\right]$$

(12)

The maximum backorder unit:-

$$Q_2 = - I_2(T) = A(T - t_1) \tag{13}$$

Hence the order size during [0, T]

$$M(t) = Q_1 + Q_2$$

$$= \gamma . \left[\left\{ t_1 + \frac{\delta t_1^{\ 2}}{2} + \frac{\delta^2 t_1^{\ 3}}{6} + \frac{\alpha t_1^{\ \beta+1}}{(\beta+1)} + \frac{\alpha.\delta.t_1^{\ \beta+2}}{(\beta+2)} + \frac{\alpha.\delta^2.t_1^{\ \beta+3}}{2(\beta+3)} \right\} \right] + A(T - t_1) \tag{14}$$

The total shortage cost during interval [t_1 , T] is :-

$$SC = -s \int_{t_1}^{T} I_2(t)\, dt \tag{15}$$

$$= \frac{A.s}{2}[T - t_1]^2 \tag{16}$$

Now the purchase cost is as follows:-

$$PC = C_3 .M(t) \tag{17}$$

$$= C_3 \left[\gamma . \left\{ t_1 + \frac{\delta t_1^{\ 2}}{2} + \frac{\delta^2 t_1^{\ 3}}{6} + \frac{\alpha t_1^{\ \beta+1}}{(\beta+1)} + \frac{\alpha.\delta.t_1^{\ \beta+2}}{(\beta+2)} + \frac{\alpha.\delta^2.t_1^{\ \beta+3}}{2(\beta+3)} \right\} + A(T - t_1) \right] \tag{18}$$

Ordering cost is taken as constant

$OC = C_4$

The total average cost is as follows:-

$$Z = \frac{1}{T}(OC + HC + SC + PC) \tag{19}$$

$$= \frac{1}{T}\Bigg[C_4 + \gamma.h \Bigg[\left(t_1 - \frac{\delta t_1^{\ 2}}{2} - \frac{\alpha t_1^{\ \beta+1}}{(\beta+1)} + \frac{\alpha.\delta.t_1^{\ \beta+2}}{(\beta+2)} + \frac{\delta^2 t_1^{\ 3}}{6} - \frac{\alpha.\delta^2.t_1^{\ \beta+3}}{2(\beta+3)} \right) . \left(t_1 + \frac{\delta t_1^{\ 2}}{2} + \frac{\delta^2 t_1^{\ 3}}{6} \right.$$

$$\left. + \frac{\alpha t_1^{\ \beta+1}}{(\beta+1)} + \frac{\alpha.\delta.t_1^{\ \beta+2}}{(\beta+2)} + \frac{\alpha.\delta^2.t_1^{\ \beta+3}}{2(\beta+3)} \right) - \left\{ \frac{t_1^{\ 2}}{2} + \frac{\delta.t_1^{\ 3}}{6} + \frac{\delta^2.t_1^{\ 4}}{24} + \frac{\alpha.t_1^{\ (\beta+2)}}{(\beta+1).(\beta+2)} + \frac{\alpha.\delta.t_1^{\ (\beta+3)}}{(\beta+2).(\beta+3)} \right.$$

$$+ \frac{\alpha.\delta^2.t_1^{\ (\beta+4)}}{2.(\beta+2).(\beta+4)} - \frac{\delta.t_1^{\ 3}}{3} - \frac{\delta^2.t_1^{\ 4}}{8} - \frac{\delta^3.t_1^{\ 5}}{30} - \frac{\alpha.\delta.t_1^{\ (\beta+3)}}{(\beta+1).(\beta+3)} - \frac{\alpha.\delta^2.t_1^{\ (\beta+4)}}{(\beta+2).(\beta+4)} - \frac{\alpha.\delta^3.t_1^{\ (\beta+5)}}{2.(\beta+3).(\beta+5)}$$

$$- \frac{\alpha.t_1^{\ (\beta+2)}}{(\beta+2)} - \frac{\alpha.\delta.t_1^{\ (\beta+3)}}{2.(\beta+3)} - \frac{\alpha.\delta^2.t_1^{\ (\beta+4)}}{6.(\beta+4)} - \frac{\alpha^2.t_1^{\ (2\beta+2)}}{(\beta+1).(2\beta+2)} - \frac{\alpha^2.\delta.t_1^{\ (2\beta+3)}}{(\beta+2).(2\beta+3)} - \frac{\alpha^2.\delta^2.t_1^{\ (2\beta+4)}}{2.(\beta+3).(2\beta+4)}$$

$$+ \frac{\alpha.\delta.t_1^{\ (\beta+3)}}{(\beta+3)} + \frac{\alpha.\delta^2.t_1^{\ (\beta+4)}}{2.(\beta+4)} + \frac{\alpha.\delta^3.t_1^{\ (\beta+5)}}{6.(\beta+5)} + \frac{\alpha^2.\delta.t_1^{\ (2\beta+3)}}{(\beta+1).(2\beta+3)} + \frac{\alpha^2.\delta^2.t_1^{\ (2\beta+4)}}{(\beta+2).(2\beta+4)} + \frac{\alpha^2.\delta^3.t_1^{\ (2\beta+5)}}{2.(\beta+3).(2\beta+5)}$$

$$+ \frac{\delta^2.t_1^{\ 4}}{8} + \frac{\delta^3.t_1^{\ 5}}{20} + \frac{\delta^4.t_1^{\ 6}}{72} + \frac{\alpha.\delta^2.t_1^{\ (\beta+4)}}{2.(\beta+1).(\beta+4)} + \frac{\alpha.\delta^3.t_1^{\ (\beta+5)}}{2.(\beta+2).(2\beta+5)} + \frac{\alpha.\delta^4.t_1^{\ (\beta+6)}}{4.(\beta+3).(\beta+6)} - \frac{\alpha.\delta^2.t_1^{\ (\beta+4)}}{2.(\beta+4)}$$

$$\left. - \frac{\alpha.\delta^3.t_1^{\ (\beta+5)}}{4.(\beta+5)} - \frac{\alpha.\delta^4.t_1^{\ (\beta+6)}}{12.(\beta+3)} - \frac{\alpha^2.\delta^2.t_1^{\ (2\beta+4)}}{2.(\beta+1).(2\beta+4)} - \frac{\alpha^2.\delta^3.t_1^{\ (2\beta+5)}}{2.(\beta+2).(2\beta+5)} - \frac{\alpha^2.\delta^4.t_1^{\ (2\beta+6)}}{4.(\beta+3).(2\beta+6)} \right\} \Bigg]$$

$$+ \frac{A.s}{2}[T - t_1]^2 + C_3 \left[\gamma . \left\{ t_1 + \frac{\delta t_1^{\ 2}}{2} + \frac{\delta^2 t_1^{\ 3}}{6} + \frac{\alpha t_1^{\ \beta+1}}{(\beta+1)} + \frac{\alpha.\delta.t_1^{\ \beta+2}}{(\beta+2)} + \frac{\alpha.\delta^2.t_1^{\ \beta+3}}{2(\beta+3)} \right\} + A(T - t_1) \right] \Bigg]$$

(20)

Now to minimize the total cost Z per unit time, we obtain the optimum value of t_1 and T with

help of following equations $\frac{\partial Z}{\partial t_1} = 0 \; and \; \frac{\partial Z}{\partial T} = 0.$

Now $\frac{\partial Z}{\partial t_1} = 0$

$$-\frac{\alpha.\delta^2.(\beta+4).t_1^{(\beta+3)}}{2.(\beta+3)} - \frac{\alpha.\delta^3.(\beta+5).t_1^{(\beta+4)}}{4.(\beta+3)} - \frac{\alpha.\delta^4.(\beta+6).t_1^{(\beta+5)}}{12.(\beta+3)} - \frac{\alpha^2.\delta^2.(2\beta+4).t_1^{(2\beta+3)}}{2.(\beta+3).(\beta+1)} - \frac{\alpha^2.\delta^3.(2\beta+5).t_1^{(2\beta+4)}}{2.(\beta+3)(\beta+2)} - \frac{\alpha^2.\delta^4.(2\beta+6).t_1^{(2\beta+5)}}{4.(\beta+3)^2}\Bigg\} - \Bigg\{t_1 + \frac{\delta.t_1^2}{2} + \frac{\delta^2.t_1^3}{6} + \frac{\alpha.t_1^{(\beta+1)}}{(\beta+1)} + \frac{\alpha.\delta.t_1^{(\beta+2)}}{(\beta+2)}$$

$$+\frac{\alpha.\delta^2.t_1^{(\beta+3)}}{2.(\beta+2)} - \delta.t_1 - \frac{\delta^2.t_1^3}{2} - \frac{\delta^3.t_1^4}{6} - \frac{\alpha.\delta.t_1^{(\beta+2)}}{(\beta+1)} - \frac{\alpha.\delta^2.t_1^{(\beta+3)}}{(\beta+2)} - \frac{\alpha.\delta^3.t_1^{(\beta+4)}}{2.(\beta+3)} - \alpha.t_1^{(\beta+1)} -$$

$$= \frac{1}{T}\Bigg[\gamma.h\Bigg[\Bigg\{2t_1 + \frac{3.\delta.t_1^2}{2} + \frac{2.\delta^2.t_1^3}{3} + \frac{\alpha.(\beta+2).t_1^{(\beta+1)}}{(\beta+1)} + \frac{\alpha.\delta.(\beta+3).t_1^{(\beta+2)}}{(\beta+2)} + \frac{\alpha.\delta^2.(\beta+4).t_1^{(\beta+3)}}{2.(\beta+3)}$$

$$-\frac{3.\delta.t_1^2}{2} - \delta^2.t_1^3 - \frac{5.\delta^3.t_1^4}{12} - \frac{\alpha.\delta.(\beta+3).t_1^{(\beta+2)}}{2.(\beta+1)} - \frac{\alpha.\delta^2.(\beta+4).t_1^{(\beta+3)}}{2.(\beta+2)} - \frac{\alpha.\delta^3.(\beta+5).t_1^{(\beta+4)}}{4.(\beta+3)}$$

$$-\frac{\alpha.(\beta+2).t_1^{(\beta+1)}}{(\beta+1)} - \frac{\alpha.\delta.(\beta+3).t_1^{(\beta+2)}}{2.(\beta+1)} - \frac{\alpha.\delta^2.(\beta+4).t_1^{(\beta+3)}}{6.(\beta+1)} - \frac{\alpha^2.(2\beta+2).t_1^{(2\beta+1)}}{(\beta+1)^2} -$$

$$\frac{\alpha^2.\delta.(2\beta+3).t_1^{(2\beta+2)}}{(\beta+1)(\beta+2)} - \frac{\alpha^2.\delta^2.(2\beta+4).t_1^{(2\beta+3)}}{2.(\beta+1)(\beta+3)} + \frac{\alpha.\delta.(\beta+3).t_1^{(\beta+2)}}{(\beta+2)} + \frac{\alpha.\delta^2.(\beta+4).t_1^{(\beta+3)}}{2.(\beta+2)}$$

$$+\frac{\alpha.\delta^3.(\beta+5).t_1^{(\beta+4)}}{6.(\beta+2)} + \frac{\alpha^2.\delta.(2\beta+3).t_1^{(2\beta+2)}}{(\beta+1).(\beta+2)} + \frac{\alpha^2.\delta^2.(2\beta+4).t_1^{(2\beta+3)}}{(\beta+2)^2} + \frac{\alpha^2.\delta^3.(2\beta+5).t_1^{(2\beta+4)}}{2.(\beta+2).(\beta+3)}$$

$$+\frac{3.\delta^2.t_1^3}{3} - \frac{3.\delta^3.t_1^4}{12} - \frac{\delta^4.t_1^5}{6} - \frac{\alpha.\delta^2.(\beta+4).t_1^{(\beta+3)}}{6.(\beta+1)} - \frac{\alpha.\delta^3.(\beta+5).t_1^{(\beta+4)}}{6.(\beta+2)} + \frac{\alpha.\delta^4.(\beta+6).t_1^{(\beta+5)}}{12.(\beta+3)}$$

$$\frac{\alpha.\delta.t_1^{(\beta+2)}}{2} + \frac{\alpha.\delta^2.t_1^{(\beta+3)}}{6} - \frac{\alpha^2.t_1^{(2\beta+1)}}{(\beta+1)} - \frac{\alpha^2.\delta.t_1^{(2\beta+2)}}{(\beta+2)} + \frac{\alpha^2.\delta^2.t_1^{(2\beta+3)}}{2.(\beta+3)} + \alpha.\delta.t_1^{(\beta+2)} - \frac{\alpha.\delta^2.t_1^{(\beta+3)}}{2}$$

$$+\frac{\alpha.\delta^3.t_1^{(\beta+4)}}{6} + \frac{\alpha^2.\delta.t_1^{(2\beta+2)}}{(\beta+1)} + \frac{\alpha^2.\delta^2.t_1^{(2\beta+3)}}{(\beta+2)} + \frac{\alpha^2.\delta^3.t_1^{(2\beta+4)}}{2.(\beta+3)} + \frac{\delta^2.t_1^3}{2} + \frac{\delta^3.t_1^4}{4} + \frac{\delta^4.t_1^5}{12} +$$

$$\frac{\alpha.\delta^2.t_1^{(\beta+3)}}{2.(\beta+1)} + \frac{\alpha.\delta^3.t_1^{(\beta+4)}}{2.(\beta+2)} + \frac{\alpha.\delta^4.t_1^{(\beta+5)}}{4.(\beta+3)} - \frac{\alpha.\delta^2.t_1^{(\beta+3)}}{2} - \frac{\alpha.\delta^3.t_1^{(\beta+4)}}{4} - \frac{\alpha.\delta^4.t_1^{(\beta+5)}}{12} -$$

$$\frac{\alpha^2.\delta^2.t_1^{(2\beta+3)}}{2.(\beta+1)} - \frac{\alpha^2.\delta^3.t_1^{(2\beta+4)}}{2.(\beta+2)} - \frac{\alpha^2.\delta^4.t_1^{(2\beta+5)}}{4.(\beta+3)}\Bigg\}\Bigg] - A.s(T-t_1) + C_3\{\gamma.(1+\delta.t_1+\alpha.t_1^{\beta} +$$

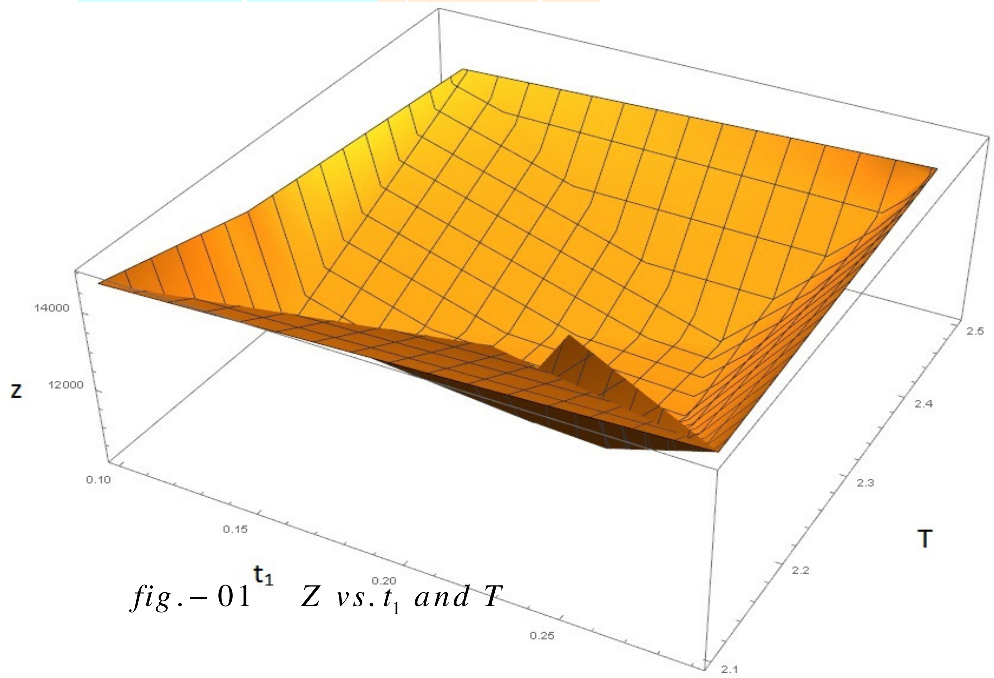
$$\alpha.\delta.t_1^{(\beta+1)} + \frac{\delta^2.t_1^2}{2} + \frac{\alpha.\delta^2.t_1^{(\beta+2)}}{2}) - A\}\Bigg]$$

(21)

As $\dfrac{\partial Z}{\partial T} = 0.$

$$= \frac{1}{T}\left[A.s.(T - t_1) + C_3.A\right] - \frac{1}{T^2}(OC + HC + SC + PC). \tag{22}$$

It is difficult to find second order derivative for sufficient condition of optimality. We can show it by fig.-1.



$fig.-01$ $Z\ vs.\ t_1\ and\ T$

At m. That is, second derivative of $\dfrac{d^2Z}{dt_1^{\ 2}}$ *and* $\dfrac{d^2Z}{dT^2}$ are positive.

NUMERICAL EXAMPLES

The model which is proposed by us is illustrated below by given following example:-

For the analysis of graphical and numerical solution of the developed model, value of parameters is taken as proper units, When α = 0.7, β = 4, γ = 550, δ = 0.7, h = 10, s = 50,

A = 300, C_4 = 150, C_3 = 10. After putting these value we get optimal solution as $t_1 = t_1^* = 1.14597$ and $T = T^* = 3.35888$, Q = Q* = 1136.99 and Z = Z* = 18000.1.

Conclusion

In real life most of the future planning's are uncertain. Uncertainty plays an important role in business. In this paper we developed an inventory model for Weibull distribution deterioration and inventory dependent and inventory dependent demand. Holding cost is considered as exponential function of time. Mathematical model have been presented to find optimal inventory systems. The first and second order approximations have been used for exponential terms. The following inferences can be made for the sensitivity analysis.

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