AN EOQ MODEL FOR STOCK DEPENDENT DEMAND RATE WITH PROFIT MAXIMIZATION APPROACH AND FLAT DISCOUNT OFFERS

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Abstract:

This paper is concerned with an inventory model with instantaneous stock replenishment and stock-dependent consumption rate. In this paper we derived total cost in which demand rate is constant and tried to change the demand of producer as the change of demand of consumer. Naturally this could not take care of stock dependent demand rate which is depending on replenishment size i. e. replenishment should continuously come so that shortages do not take place. Gupta and Vrat suggested an EOQ model through cost minimization technique to take care of stock-dependent demand rate. They substituted the expression for variable demand rate in the total cost per unit time derived under the assumption of constant demand, and Mandal and Phaujdar suggested an EOQ model through profit maximization criteria considering the demand rate depending upon the current stock level to yield the optimal solution, which is more realistic. They assumed a linear dependence of the demand rate on the current stock level. In a current paper, profit maximization criteria is used for the producer to earn more profit. In this paper an EOQ model is established in the case where the replenishment of the stock is instantaneous, flat discount offered and shortages are not allowed and the demand rate depends upon the current stock level. Profit maximization criteria are employed to yield the maximum profit per time of the system.

KEYWORDS:

EOQ Model, Flat discount offer, Replenishment rate, Profit Maximization

INTRODUCTION

In earlier we derived the total cost in which demand rate was constant while here we trying to change the demand of producer as the change of demand of consumer. Naturally this could not take care of stock dependent demand rate is depending on replenishment size i. e. replenishment should continuously come so that shortages do not take place. So there is no chance to availability of shortages because here stock will not be keep in the go-down more times.

Gupta and Vrat suggested an EOQ model through cost minimization technique to take care of stock-dependent demand rate. They substituted the expression for variable demand rate in the total cost per unit time derived under the assumption of constant demand. Naturally, this could not take care of stock-dependent demand rate except where the demand rate is dependent on replenishment size.

Mandal and Phaujdar suggested an EOQ model through profit maximization criteria considering the demand rate depending upon the current stock level to yield the optimal solution, which is more realistic. They assumed a linear dependence of the demand rate on the current stock level. Here a profit maximization criteria is used for the producer to earn more profit. So this is the best fitted EOQ model for the producer.

In this paper EOQ model is derived with functional relationship between the demand rate and current stock level.

FORMULATION OF THE MODEL

Here the EOQ model is established in the case where the replenishment of the stock is instantaneous, shortages are not allowed and the demand rate depends upon the current stock level. The total profit per unit time during time T, obtained by Mandal and Phaujdar is if "q" denotes the inventory level at time t, then in this case we have

$$\frac{dq}{dt} = -D(q) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

Where, D is the demand rate at time t. The length T of each cycle is given by

$$T = \int_0^S \frac{dq}{D(q)} = F(S_h) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

Where, S_h is highest stock level. The carrying cost during overall time T is $C_C.G(S)$ where C_C is the unit carrying cost per unit time and

$$G(S) = \int_0^S \frac{q}{D(q)} dq \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

The total profit per unit time during T is thus

$$G(S) = \frac{pS - [S_c - (C_p - d_1).S + C_C.G(S)]}{F(S)} \ldots\ldots\ldots\ldots\ldots\ldots (4)$$

where,

$p = \text{unit selling price of the item}$

$S = \text{highest stock level}$

$S_c = \text{setup cost for each cycle}$

$C_C = \text{carring cost per unit of the item}$

$C_p = \text{unit cost price of the item}$

$d_1 = \text{percentage of flat discount per unit } (d_1 = C_p \times \%)$

$F(S) = \text{Function of highest stock level}$

$G(S) = Function\ of\ highest\ stock\ level$

$Z(S) = Proft\ Function\ per\ unit\ time\ during\ T$

The optimal value of S for maximum total profit per unit time is a solution of $Z'(S) = 0$, provided $Z''(S) < 0$, for that value of S. Thus for optimal value S, expression (4) implies,

$$F(S).\{[p-(C_p-d_1)].S - C_C.G'(S)\}$$
$$= F'(S).\{[p-(C_p-d_1)].S - S_c - C_C.G(S)\}.\dots\dots\dots..(5)$$

Where prime denotes derivative with respect to S. Equation (5) is in general a non-linear equation which can be solved numerically by Newton Raphson method (second derivation), if the explicit from D(q) is known. The optimal cycle length is given by F(S*), where S* is the optimal value of S.

$$\boldsymbol{Case: D(q) = \alpha + \frac{\beta}{q}}$$

By putting the value of D(q) in to equation (2), we get;

$$F(S) = \int_0^S \frac{1}{D(q)} dq = \int_0^S \frac{1}{\alpha + \frac{\beta}{q}} dq$$

$$= \int_0^S \frac{1}{\frac{\alpha.q+\beta}{q}} dq$$

$$= \int_0^S \frac{q}{\alpha.q+\beta} dq \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots.(6)$$

$$F(S) = \int_0^S \frac{S}{\alpha.S+\beta} dq \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots.(7)$$

Also from equation (3), we get;

$$G(S) = \int_0^S \frac{q}{D(q)} dq = \int_0^S \frac{q.q}{\alpha.q+\beta} dq$$

$$G(S) = \int_0^S \frac{q^2}{\alpha.q+\beta}dq$$

$$G'(S) = \left\{\frac{S^2}{2.\alpha} - \frac{\beta}{\alpha^2}.S + \frac{\beta}{\alpha^3}log\left[1+\frac{\alpha.S}{\beta}\right]\right\} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (8)$$

So,

$$G'(S) = \frac{S^2}{\alpha.S+\beta} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (9)$$

Now, putting the values of F(S), F'(S), G(S) and G'(S) from equation (6), (7), (8) and (9) respectively in to equation (5) and by simplifying it, we get;

$$\Rightarrow \left\{S_C + \left(\frac{\beta}{\gamma}\left[\frac{C_C}{\alpha} - \frac{\{p-[C_p-d_1]\}}{S}\right] \times \left[\frac{1}{\alpha}(\alpha.S+\beta).log\left(1+\frac{\alpha}{\beta}.S\right) - S\right]\right) - \frac{C_C.S^2}{2.\alpha}\right\} = 0 \ldots\ldots\ldots\ldots\ldots\ldots (10)$$

If we take $\beta = 0$, then equation (10) reduces to,

$$\Rightarrow \left\{S_C + \frac{(0)}{\gamma}\left[\frac{C_C}{\alpha} - \frac{\{p-[C_p-d_1]\}}{S}\right] \times \left[\frac{1}{\alpha}(\alpha.S+(0)).log\left(1+\frac{\alpha}{\beta}.S\right) - S\right] - \frac{C_C.S^2}{2.\alpha}\right\} = 0$$

$$\Rightarrow \left\{S_C - \frac{C_C.S^2}{2.\alpha}\right\} = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (11)$$

$$\Rightarrow S_C = \frac{C_C.S^2}{2.\alpha},$$

$$\Rightarrow \frac{2.\alpha.S_C}{C_C} = S^2$$

This gives,

$$\Rightarrow S^* = \left(\frac{2.\alpha.S_C}{C_C}\right)^{1/2} \quad OR \quad \Rightarrow S^* = \sqrt{\frac{2.\alpha.S_C}{C_C}}$$

This is the classical EOQ formula with uniform demand rate but new thing is that in classical EOQ model, they didn't consider time, while the time for all constant values $(\alpha \; and \; \beta)$ taken in to the consideration in this formula that is differentiate classical formula and this formula.

Equation (10) is a transcendental equation which can be solved by Newton Raphson (using derivative) method. The solution of equation (10) which satisfies Z''(S) < 0 gives the optimal value of S.

CONCLUSION:

Here EOQ model is derived in which production of items in the system in instantaneous, flat discount offered and shortages do not occur i. e. for production of items, all component materials should come timely. So there is no hindrance in producing the final items required by customer.

Profit maximization criteria are employed to yield the maximum profit per time of the system. A functional form of demand rate is taken in order to formulate the model and all constants will be converted in to variables because it's changes in long period.

In some cases when conditions on demand rate are used, the model reduces to corresponding already established inventory model for that demand rate. The model presented here can be further extended for finite rate of replenishment and / or allowing very limited shortages.

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