



A STATISTICAL ANALYSIS OF $(m \times n)$ ASYMMETRICAL FACTORIAL EXPERIMENTS USING m -ple LATTICE DESIGN

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Abstract: The factorial experiments are of two types, symmetrical and asymmetrical. In symmetrical factorial experiments the factors occur at the same number of levels. In asymmetrical factorial experiments all the factors are not at same number of levels. A disadvantage of balanced incomplete block designs is that not every treatment has an appropriate design. These designs typically require a significant number of replications, which may not always be convenient for the studies. As a result, efforts were undertaken to develop patterns with fewer replications, which are referred to as lattice designs. A lattice design is a kind of incomplete block experimental design that is commonly used in agricultural trials to control variability. It may be used to compare a large number of treatments or to arrange them into smaller, homogenous blocks. In this paper, a statistical analysis (2×3) asymmetrical factorial experiments using m -ple lattice design with application.

Index Terms - Asymmetrical Factorial Experiments, m -ple Lattice Design, BIBD, PBIBD.

I. INTRODUCTION

The symmetrical factorial experiments are inflexible, because, all the factors in the symmetrical factorial experiments must have the same number of levels. This may not be possible always in practical situations and further it may even be unrealised to take all the factors under investigation at the same number of levels. The above limitations can be estimated by adopting asymmetrical factorial experiments. The accommodate different factors with different levels. The asymmetrical factorial experiment is more flexible to meet their equipment. Constructions of confounded asymmetrical factorial experiment and the analysis is some bought complicated as a matter of fact, very little things are known about the construction. Later on several confounded asymmetrical factorial experiments. Where obtain individually rather than general method of construction. In the sections that follow, each of the many constructional and analytical approaches for the incomplete block designs has been covered independently. It was discovered that the balanced incomplete block designs were not always appropriate for varietal trials due to the fact that these designs need a high number of replications and that appropriate designs are not accessible for every number of treatments. Yates (1936) developed a series of unfinished block patterns that he named Lattice designs in order to get around these challenges. A different kind of incomplete block designs was later developed by Bose and Nair (1939), who named them partially balanced incomplete block (PBIB) designs. Perfect squares are one kind of lattice pattern that may be used for many other types. These can support any

number of replications, starting with two. For certain pairings of treatments, there is one replication; for other pairs, there are none. These designs are actually a type of confounded symmetrical factorial designs involving two factors as we shall see afterwards. Likewise, there are lattice designs corresponding to confounded symmetrical factorial designs involving three factors. These are called cubic lattice. As these designs follow from factorial designs these are also called quasi factorial designs. For many more treatments, the Partially Balanced Incomplete Block (PBIBD) designs are available with fewer replications. In these designs, the number of replications of treatment pairings varies. These were generally specified for various replications of distinct treatment pairings, where m might be any integer. That is, in these designs some pairs of treatments can be replicated, say, λ_1 times, some others λ_2 times and so on up to m such types of pairs. From considerations of analysis of these designs, these have to possess some more properties as well. These are taken care of by defining some more parameters which we shall discuss subsequently. Two more series of incomplete block designs were defined in order to provide incomplete block designs for all treatment numbers with fewer replications. These are circular designs and reinforced incomplete block designs. The reinforced incomplete block designs, which are actually augmented incomplete block designs, are created by adding a specific number of extra treatments to each block of any typical incomplete block design, say, α so that the block size becomes $(k + \alpha)$ and the number of treatment $(\nu + \alpha)$. By suitably choosing ν and α any number of treatments can be accommodated in such designs. Circular designs are a particular type of *PBIB* designs which are available for any number of treatments. They have no constructional problems. When the block size is 2 or 3, their analysis is also not complicated. For ν treatments and block size n , these are obtained by developing the initial block $1, 2, \dots, n \bmod \nu$. Even before Yates [3] used the *BIB* designs for agricultural experiments, some of them were known in combinatorial mathematics in the form of groups possessing special properties. Bose [5] exploited the group properties for construction of these designs. Since then considerable amount of work has been done in the name of incomplete block designs. But, it seems, much of these activities has very little to do with designs required for actual experiments. They are more curiosity guided than problem oriented. The present activities have penetrated much into combinatorial mathematics and are, therefore, more concerned with group properties rather than problems of precision or availability of designs to suit particular experiments. We have so far tried to view the incomplete block designs from the angle of precision as related to the replication of pairs of treatments. However, the evolution of different incomplete block designs has really been more influenced by analytical concerns. We get two-way data categorised by blocks and treatments from incomplete block designs. If there are b blocks in a design with ν treatments and k as block size, then there are $(b \times \nu)$ cells in the two-way table with frequencies 0 or 1. The frequencies in the k matching cells in the row for the block are unity, while the frequencies in the other cells of the row are zero since k of the ν treatments occur in a block. As a result, the data derived from these designs are not orthogonal. The incidence matrix of the matching design, often indicated by N , is the $(b \times \nu)$ cell frequency table. These patterns are known as binary designs as the cells can have two values: 0 or 1, while acquiring the non-orthogonal data analysis approach. We have seen that the analysis involves the solution of as many reduced normal equations as the number of treatments. In balanced incomplete block designs, these equations take the form $c_1 t_i + c_2 \sum_{m=1}^{\nu} t_m = Q$ ($i = 1, 2, 3, \dots, \nu$). So, taking the restriction $\sum_m t_m = 0$ the solution of these equations becomes very easy. While evolving new designs similar simplicity of the solutions of normal equations has also been kept in view along with the other considerations discussed previously. A further consideration that is kept in view is that all the treatment effects should be estimable as deviates from their means. This in turn implies that the reduced normal equations should be solvable taking one additional restriction among the treatment effects. Usually, this criterion is called connectedness of the design. If a design is not connected, all the $(\nu - 1)$ mutually orthogonal contrasts of the ν treatments effects cannot be estimated. Hence, it is necessary to know if a design is connected before it is recommended. Efforts have, therefore, been made to ascertain if a design is connected by examining its blocks. Bose (1943) in his unpublished class notes gave the following method. If for every pair of treatments a and b , it is possible to get other treatments a_1, a_2, \dots, a_n such that every pair of consecutive treatments in the series a_1, a_2, \dots, a_n, b occurs in some block or other of a design, then the

design is connected. The criterion of connectedness can be viewed from the following convenient angle also. An unfinished block design is disconnected if all of the blocks can be split into two groups of blocks so that none of the treatments in one group are present in the other. Thus, total confusion in factorial designs is comparable to the idea of connectivity. The fundamental concepts of experimental design, including randomisation, replication, and factorial trials, are discussed in a few reviews by Fisher, R.A. [2]. All contemporary statistical designs are based on his work. Yates, F. provided the techniques for setting up and evaluating factorial experiments [3]. Oscar Kempthorne's discussion of factorial and asymmetrical designs was influenced by the notion of experimental design in detail [6]. Das M. N. and Giri, N.C [14] have provided practical applications of experimental designs. Statistical methods, especially ANOVA, which is widely used for analysing factorial experiments developed by George W. Snedecor and William G. Cochran [8]. Montgomery, D.C [9] has introduced modern approaches to experimental design, including industrial applications and software-based analysis. Klaus Hinkelmann and Oscar Kempthorne [10] explained advanced concepts of experimental design and factorial structures. Banerjee A.K [13] studied the construction of asymmetrical factorial designs with confounding techniques Sreenath P.R. [12] has developed methods for designing asymmetrical factorial experiments using confounded symmetrical designs Adepoju J.A and Ipinyomi R.A [11] has worked on constructing asymmetric fractional factorial designs to reduce experimental cost. Bahr K M et al. [16] applied asymmetrical factorial experiments in agriculture to identify important factors affecting production recent studies by Bharati Y. Taware and Chhaya D. Sonar [15] focus on improved construction methods for asymmetrical designs. This paper, we study a statistical analysis (2×3) asymmetrical factorial experiments using m -ple lattice design with application.

II. PRELIMINARIES

2.1 Asymmetrical Factorial Experiments

An asymmetrical factorial experiment is one in which at least two of the factors have a different number of levels. More complex to design and analyse compared to symmetrical designs, but often more practical in real-world scenarios where factors naturally have different numbers of levels. The notation describes the number of levels for each factor, for example ($2 \times 3 \times 4$).

2.2 Lattice Design

Let there be k^2 treatment numbered by $1, 2, 3, \dots, k^2$. Let these treatment number be arranged in the form of a $k \times k$ square. The contents of each of the k rows of this square are taken to form a block. Thus, k blocks are obtained from k rows. Similarly, taking the contents of the columns to form blocks, another k blocks are obtained. A Latin square of order k (arrangement with latin letters) is now taken and is superimposed on the $k \times k$ square written with the treatment numbers. The treatment numbers which fall on a symbol of the latin square are taken to form a block. Thus, from k symbols of the latin square k blocks are obtained. Next, another latin square orthogonal to the previous one is taken and is superimposed on the square. From this latin square also, another set of k blocks can be obtained likewise. Similarly, by using $(m-2)$ mutually orthogonal latin squares of order k , $(m-2)k$ blocks are obtained where $m \leq k+1$. These blocks along with the $2k$ blocks formed by the rows and columns of the arrangement of the treatment numbers, give a lattice design with k^2 treatment, mk blocks each of size k and m replications. This is called m -ple lattice design. A square lattice design with two replications is called a simple lattice and one with three replications, a triple lattice.

When k is a prime or a power of a prime by using all the $(k-1)$ mutually orthogonal latin squares for obtaining the blocks as indicated above, a lattice design in $(k+1)$ replications is obtained. This is called a balanced lattice. Balanced lattices are also balanced incomplete block designs belonging to the series $b = s + s^2$, $v = s^2$, $k = s+1$, $k = s$, $\lambda = 1$. By taking any two sets of blocks of the design is a simple lattice design for 16 treatments is obtained.

If the k^2 treatments are coded by the combinations of the k^2 factorial, that is, the combinations of two factors each at k levels, then a confounded design in blocks of size k obtained by confounding m main effects and interactions in m different replications, gives an m -ple lattice design. These designs

are, therefore, also called quasifactorial designs. Extending this analogy to factorials with three factors each at k levels, (k^3 factorials) two types of designs corresponding to block sizes k and k^2 can be obtained by adopting suitable confounding. These designs are called cubic lattices.

III. A STATISTICAL ANALYSIS OF $(m \times n)$ ASYMMETRICAL FACTORIAL

EXPERIMENTS USING m -ple

LATTICE DESIGN

Consider the asymmetrical factorial experiments for the first factor F_1 and F_2 ($m = 2$) each with two levels (1 and 2) and the second factor F_1, F_2 and F_3 ($m = 3$) each with three levels (0, 1 and 2), then these constitute (2×3) asymmetrical factorial experiment are discussed below.

3.1 A Statistical Analysis of (2×3) Asymmetrical Factorial Experiments

The levels of factor $F_1 = f_{11}, f_{12}$ and the levels of factors $F_2 = f_{21}, f_{22}, f_{23}$. Then, there are $(2 \times 3) = 6$ treatment combinations in this asymmetrical factorial experiment they are given by

$$\begin{array}{ccc} f_{11}f_{21} & f_{11}f_{22} & f_{11}f_{23} \\ f_{12}f_{21} & f_{12}f_{22} & f_{12}f_{23} \end{array}$$

Or
11, 12, 13, 21, 22, 23

Since, the number of treatment combinations is not large. The data can be analysed an appropriate for Lattice design. This is achieved by partitioning (the treatment). The treatment sum of square into components which gives the sum of square due to main effect F_1 and F_2 as well as the sum of squares due to interaction $(F_1 \times F_2)$ of the 2 factor F_1 and F_2 . Suppose there are r replicates in this design. Then, the total degrees of freedom partitions as follows.

Table 3.1: Partitions of the Total Degrees of Freedom in (2×3) Asymmetrical Factorial Experiment

Source of Variation (SV)	Degrees of Freedom (df)
Blocks	$(r - 1)$
Treatments	$6 - 1 = 5$
Main Effect F_1	$2 - 1 = 1$
Main Effect F_2	$(3 - 1) = 2$
Interaction Effect $(F_1 \times F_2)$	$(1 \times 2) = 2$
Error	$5(r - 1)$
Total	$(6r - 1)$

The sum of squares due to blocks, treatments, errors can be obtained as in the analysis of Lattice design. The sum of squares due to main effect F_1 and main effect F_2 and the interactions $(F_1 \times F_2)$ can be obtain by forming the following consider, the total with 6 treatment total.

Table 3.2: Table of Treatment Totals

		Level of F_2			
		f_{21}	f_{22}	f_{23}	Total
Level of F_1	f_{11}	T_{11}	T_{12}	T_{13}	F_{11}
	f_{12}	T_{21}	T_{22}	T_{23}	F_{12}
	Total	F_{21}	F_{22}	F_{23}	G

Sum of Squares (SS) due to main effect $F_1 = \frac{F_{11}^2 + F_{12}^2}{3r} - cf = Q_{F_1}$, Sum of squares due to main effect $F_2 = \frac{F_{21}^2 + F_{22}^2 + F_{23}^2}{2r} - cf = Q_{F_2}$, Sum of squares due to total = $\sum_{i,j} y_{ij}^2 - cf = Q$, Where $cf = \frac{G^2}{N}$; $G = F_{11} + F_{12} = F_{21} + F_{22} + F_{23}$ is the grand total , $N = 6r$ is the total number of observations. The sum of squares due to interactions $(F_1 \times F_2) = (SS \text{ due to cells} - SS \text{ due to ME } F_1 - SS \text{ due to ME } F_2)$.Where,
 Sum of Squares due to cells = $\frac{\sum_j T_{ij}^2}{r} - cf$, Sum of Squares due to replicate cells = $\frac{\sum_{j=1}^r y_{.j}^2}{6} - cf = Q_R$. Then the error sum of squares can be obtained by $Q_E = Q - Q_R - Q_{F_1} - Q_{F_2} - Q_{F_1 \times F_2}$. The ANOVA table relates to each of these values, and an inference is made.

Table 3.3: ANOVA for 2X3 Asymmetrical Factorial Experiments

SV	Df	SS	MSS	F- Ratio
Blocks	$(r-1)$	Q_R	$M_R = \frac{Q_R}{(r-1)}$	$F_R = \frac{M_R}{M_E} \sim F_{(r-1),5(r-1)}$
Treatment	$6-1=5$	-	-	-
Main effect F_1	$2-1=1$	$Q_{F_{11}}$	$M_{F_{11}} = Q_{F_{11}}$	$F_{F_{11}} = \frac{M_{F_{11}}}{M_E} \sim F_{1,5(r-1)}$
Main effect F_2	$3-1=2$	$Q_{F_{12}}$	$M_{F_{21}} = \frac{Q_{F_{21}}}{2}$	$F_{F_{21}} = \frac{M_{F_{21}}}{M_E} \sim F_{2,5(r-1)}$
Interaction $(F_1 \times F_2)$	$1 \times 2 = 2$	$Q_{F_{11} \times F_{12}}$	$M_{F_{11} \times F_{12}} = \frac{Q_{F_{11} \times F_{12}}}{2}$	$F_{F_{11} \times F_{12}} = \frac{M_{F_{11} \times F_{12}}}{M_E} \sim F_{2,5(r-1)}$
Error	$5(r-1)$	Q_E	$M_E = \frac{Q_E}{5(r-1)}$	-
Total	$(6r-1)$	Q	-	-

Inference: If $F_o > F_e$ then the null hypothesis of in significance of any factorial effects is rejected otherwise, it is accepted.

3.2 A Statistical Analysis of (3X4) Asymmetrical factorial Experiments

The levels of factor $F_1 = f_{11}, f_{12}, f_{13}$ and the levels of factors $F_2 = f_{11}, f_{12}, f_{13}, f_{14}$. Then, there are $(3X4) = 12$ treatment combinations in this asymmetrical factorial experiment they are given by

$$f_{11}f_{21} \quad f_{11}f_{22} \quad f_{11}f_{23} \quad f_{11}f_{24}$$

$$\begin{matrix}
 f_{12} f_{21} & f_{12} f_{22} & f_{12} f_{23} & f_{12} f_{24} \\
 f_{13} f_{21} & f_{13} f_{22} & f_{13} f_{23} & f_{13} f_{24} \\
 \text{Or} \\
 11, 12, 13, 14; & 21, 22, 23, 24; & 31, 32, 33, 34
 \end{matrix}$$

Since, the number of treatment combinations is not large. Hence, the data can be analysed as appropriate for Lattice design. This is achieved by partitioning the (treatment) sum of square into components which gives the sum of square due to main effects F_1 and F_2 as well as the sum of squares due to interactions ($F_1 \times F_2$) of the 2 factor F_1 and F_2 . Suppose there are ‘r’ replicates in the design. Then, the total degrees of freedom can be partition as follows.

Table 3.4: Degrees of Freedom

Source of Variation	Degree of Freedom
Blocks	(r-1)
Treatments	12-1=11
Main effect F_1	3-1=2
Main effect F_2	4-1=3
Interaction effect $F_1 \times F_2$	2x3=6
Error	11(r-1)
Total	12r-1

The sum of squares due to total ,blocks, treatments and error can be obtain as in the analysis Lattice design .The Sum of square due to main effect F_1 and main effect F_2 as well as their interactions ($F_1 \times F_2$) can be obtained by forming the following consider, total with 12 treatments total.

Table 3.5: Treatment Totals

		Level of F_2				
		f_{21}	f_{22}	f_{23}	f_{24}	Total
Level of F_1	f_{11}	T_{11}	T_{12}	T_{13}	T_{14}	F_{11}
	f_{12}	T_{21}	T_{22}	T_{23}	T_{24}	F_{12}
	f_{13}	T_{31}	T_{32}	T_{33}	T_{34}	F_{13}
	Total	F_{21}	F_{22}	F_{23}	F_{24}	G

Sum of squares due to total = $\sum_{i,j} y_{ij}^2 - CF = Q = \sum_{i=1}^{12} \sum_{j=1}^r y_{ij}^2 - CF$, Where $CF = \frac{G^2}{N}$

$G = F_{11} + F_{12} + F_{13} = F_{21} + F_{22} + F_{23} + F_{24}$, $N = 12r$ is the total number of observations,

Sum of Squares to replicate cells = $\frac{\sum_{j=1}^r y_{.j}^2}{12} - C.F = Q_R$, sum of squares due to main effect $F_1 =$

$\frac{F_{11}^2 + F_{12}^2 + F_{13}^2}{4r} - C.F = Q_{F_1}$ sum of squares due to main effect $F_2 = \frac{F_{21}^2 + F_{22}^2 + F_{23}^2 + F_{24}^2}{2r} - C.F = Q_{F_2}$.

The sum of squares due to interactions ($F_1 \times F_2$) = $Q_{F_1 \times F_2} = [SS \text{ due to cells} - SS \text{ due to main}$

effect F_1 - S.S due to main effect F_2], where Sum of Squares due to cells $= \frac{\sum_j T_{ij}^2}{r} - C.F$
 $= \sum_{i=1}^3 \sum_{j=1}^4 \frac{T_{ij}^2}{r} - CF = Q_c$. Then, the error sum of squares can be obtained by subtractions, ESS
 $= Q - Q_R - Q_{F_1} - Q_{F_2} - Q_{F_1 \times F_2}$ All these values are referring in the ANOVA table and the inference is drawn.

Table 6: ANOVA for (3X4) Asymmetrical Factorial Experiment

Source of variation	Degrees of freedom	Sum of squares	Mean sum of squares	F- Ratio
Blocks	$(r-1)$	Q_R	$M_R = \frac{Q_R}{(r-1)}$	$F_R = \frac{M_R}{M_E} \sim F_{(r-1), 11(r-1)}$
Treatment	$12-1=11$	-	-	
Main effect F_1	$3-1=2$	$Q_{F_{11}}$	$M_{F_{11}} = \frac{Q_{F_{11}}}{2}$	$F_{F_{11}} = \frac{M_{F_{11}}}{M_E} \sim F_{2, 11(r-1)}$
Main effect F_2	$4-1=3$	$Q_{F_{12}}$	$M_{F_{21}} = \frac{Q_{F_{21}}}{3}$	$F_{F_{21}} = \frac{M_{F_{21}}}{M_E} \sim F_{3, 11(r-1)}$
Interaction ($F_1 \times F_2$)	$2 \times 3 = 6$	$Q_{F_{11} \times F_{12}}$	$M_{F_{11} \times F_{12}} = \frac{Q_{F_{11} \times F_{12}}}{6}$	$F_{F_{11} \times F_{21}} = \frac{M_{F_{11} \times F_{21}}}{M_E} \sim F_{6, 11(r-1)}$
Error	$11(r-1)$	Q_E	$M_E = \frac{Q_E}{11(r-1)}$	
Total	$12r-1$	Q	-	-

IV. APPLICATION

The data was collected for the factor F_1 is gender as f_{11} -Male and f_{12} - Female. The factor F_2 as stage at 3 levels are f_{21} -stage 1 and 2, f_{22} - stage 3, f_{23} - stage 4. The response of Bilirubin $f_{11}f_{21}$ - 49.15, 26.25, 58.3. $f_{11}f_{22}$ - 15.38333, 11.1, 10.07561. $f_{12}f_{21}$ -41.86667, 23.825, 62.33333 $f_{12}f_{22}$ - 10.42444. 7.545283, 10.77317 $f_{11}f_{23}$ -58.6875, 64.52, 69.45, $f_{12}f_{23}$ -10.70444, 10.68649, 9.721739 are given. Test there is any significance difference between the two factors of asymmetrical factorial experiments.

Solution:

Arrange the given data in Blocks and find its total.

Blocks I	Blocks II	Blocks III
49.15	26.25	58.30
15.38	11.10	10.07
41.86	23.82	62.33
10.42	07.50	10.77
58.68	64.52	69.45
10.70	10.68	09.72

Total of Block I =186.19, Total of Block II=143.87 and Total of Block III=220.65

Null Hypothesis H_0 : There is no significance difference between the factor A and factor B also the three blocks of asymmetrical experiments.

		Stage (Level of F_2)			
		f_{21}	f_{22}	f_{23}	Total
Gender (Level of F_1)	f_{11}	133.7	128.03	192.66	454.4
	f_{12}	36.55	28.74	31.11	96.4
	Total	170.25	156.76	223.77	550.8

$$\text{Correction Factor (CF)} = \frac{(550.8)^2}{18} = 16854.48$$

$$\text{Raw Sum of Squares} = (49.15)^2 + (26.25)^2 + (58.3)^2 + \dots + (9.72)^2 = 26202.4646$$

$$\text{SS Due to Main Effect } F_1 = \frac{(454.39)^2}{9} + \frac{(96.41)^2}{9} - 16854.48 = 7119.43$$

$$\text{SS Due to Main Effect } F_2 = \frac{(170.45)^2}{6} + \frac{(156.77)^2}{6} + \frac{(223.77)^2}{6} - 16854.48 = 429.36$$

$$\text{Total Sum of Squares} = \text{RSS} - \text{CF} \\ = 20202.4646 - 16854.48, = 9347.98$$

$$\text{Sum of Squares Due to Replicate Cells } Q_R = \frac{(186.19)^2}{6} + \frac{(143.87)^2}{6} + \frac{(220.65)^2}{6} - 16854.48 \\ = 474.1125$$

Sum of Squares Due to Interaction

$$F_1XF_2 = \frac{(133.7)^2}{3} + \frac{(128.25)^2}{3} + \frac{(192.65)^2}{3} + \frac{(36.55)^2}{3} + \frac{(28.74)^2}{3} + \frac{(31.11)^2}{3} - 16854.48 - 429.36 - 7119.43 \\ = 470.4024$$

$$\text{Sum of Squares Due To Error} = \text{TSS} - \text{SSQ} - \text{SS } F_1 - \text{SS } F_2 - \text{SS } (F_1XF_2) \\ = 9347.98 - 474.1125 - 7119.43 - 429.36 - 470.4024 \\ = 854.6751$$

Table 4.1: ANOVA for (2X3) Asymmetrical Factorial Experiments

SV	df	SS	MSS	F- Ratio
Blocks	2	474.1125	237.05	2.77
Treatment	6-1 = 5	-	-	-
Main effect F_1	2-1 = 1	7119.43	7119.43	83.303
Main effect F_2	3-1=2	429.36	214.68	2.512
Interaction ($F_1 \times F_2$)	1x2 = 2	470.4024	235.2	2.75
Error	5(3-1) = 10	854.67	85.464	-
Total	(6r-1) = 17	9347.98	-	-

Table Value $F_{0.05}$ for 2 and 10 $df = 4.10$ and $F_{0.05}$ for 1 and 10 $df = 4.96$.

5. CONCLUSION

In this paper, study a statistical analysis (2×3) asymmetrical factorial experiments using m -ple lattice design with application. To test whether there is any significance difference between the two factors (F_1 and F_2) of asymmetrical factorial experiments. From the above ANOVA table, it is seen that the three blocks (Block I, Block II and Block III) are not significant, indicating variation among replications. Interaction effect between ($F_1 \times F_2$) and Main effect F_2 is also not significant. On the other hand, we found Main effect F_1 is found to be significant, it has a strong effect on the response. Hence, it is concluded that only Main effect F_1 significantly influences the response variable, while all other effects are not significant. Hence, it is concluded that only Main effect F_1 significantly influences the response variable, while all other effects are not significant.

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