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Decoding Abstraction through Visualization: A Narrative Review on Enhancing Mathematical Comprehension Beyond Symbolic Notation

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Abstract: Mathematics is commonly communicated through symbols, formulas, definitions, graphs, and formal procedures. Although symbolic representation is essential for mathematical communication, an exclusive dependence on symbolic forms may create difficulties when learners attempt to understand abstract mathematical relationships. Visual learning provides a complementary pathway through diagrams, graphs, patterns, geometric representations, images, dynamic models, and interactive digital environments. This narrative review examines the role of visual learning in supporting the understanding of abstract mathematical concepts by synthesizing selected literature on visual mathematics, mathematical cognition, visual representations, analytical thinking, geometry learning, and visual learning approaches. The reviewed literature indicates that visualization can support conceptual understanding by making mathematical relationships more accessible and by helping learners move between concrete, pictorial, and symbolic representations. Visual approaches are also associated with spatial reasoning, mathematical communication, problem solving, engagement, and retention. Digital technologies further extend the role of visualization by enabling learners to manipulate mathematical objects, observe changes, explore patterns, and investigate relationships dynamically. However, visual learning should not be regarded as a replacement for symbolic mathematics or as a method applicable only to students classified as visual learners. Rather, visualization should be integrated with symbolic, verbal, and analytical forms of mathematical reasoning. The review identifies an important gap in the existing literature: although the benefits of visual representations have been examined in several mathematical learning contexts, relatively limited attention has been given to how visualization mediates the transition from symbolic mathematical information to abstract conceptual understanding, particularly in advanced mathematics and mathematically dense academic texts. Based on the synthesis, this paper proposes a conceptual pathway in which visual representation supports perception, relationship recognition, mathematical reasoning, symbolic connection, and ultimately abstract mathematical understanding.

Index Terms - Visual learning, mathematical visualization, abstract mathematics, visual representations, mathematical reasoning, conceptual understanding, digital mathematics learning.

I. Introduction

Mathematics occupies a central position in scientific, technological, engineering, and educational development. However, mathematics is also characterized by a high degree of abstraction. Mathematical ideas are frequently expressed through symbols, equations, definitions, formulas, graphs, and formal arguments that require learners to understand relationships that may not be directly observable. For this reason, students may sometimes be able to manipulate mathematical symbols procedurally without developing a corresponding conceptual understanding.

The difficulty becomes particularly apparent when mathematical information is presented predominantly in symbolic form. A formula may provide a precise mathematical relationship, but a learner may still struggle to understand what that relationship represents. Similarly, a theorem or equation can be mathematically correct while remaining conceptually difficult when the reader cannot construct an appropriate mental representation of the underlying idea.

Visual learning offers a complementary approach to this problem. Visual representations can include diagrams, graphs, geometric figures, patterns, spatial arrangements, images, physical models, animations, and interactive digital representations. Such representations can make relationships more observable and can provide learners with another means of organizing mathematical information.

The literature reviewed in the present study suggests that visual mathematics should not be considered merely an introductory technique used before learners progress to more advanced symbolic mathematics. Boaler et al. emphasize the importance of visual thinking in mathematical activity and argue that mathematics has traditionally been presented too heavily through numbers and symbols, while the potential of visual mathematics has often been underestimated.

The role of visualization is also supported by research on geometry learning. Žakelj and Klančar found that students learning geometry with digital resources and interactive visual representations performed better on several post-test measures than students in the control group. They further emphasize that interactive representations can support movement between concrete, pictorial, and symbolic levels of mathematical knowledge.

Recent qualitative research has similarly reported positive associations between visual learning strategies and mathematics comprehension. Sandi and Utama found that students showed greater interest, faster recall, and improved comprehension when mathematical information was presented through images, videos, and visually rich worksheets.

Therefore, the central issue is not simply whether visual materials are useful, but how visual learning contributes to the development of mathematical understanding and the movement from representation to abstraction**.**

This paper presents a narrative review of selected literature to examine this question. The review particularly focuses on the role of visual representation in conceptual understanding, mathematical reasoning, digital mathematics learning, and the comprehension of abstract mathematical ideas.

II. THEORETICAL FOUNDATION: VISUAL LEARNING AND MATHEMATICAL UNDERSTANDING

Visual learning in mathematics should not be reduced to the use of pictures or attractive teaching materials. Mathematical visualization involves the construction, interpretation, manipulation, and use of representations to understand mathematical relationships.

The reviewed literature identifies several forms of mathematical representation. These include graphical representations, diagrams, mental images, physical models, and virtual or digital representations. Žakelj and Klančar distinguish between static representations, such as diagrams and images, and dynamic representations, such as animations and interactive digital environments.

This distinction is important because different mathematical concepts require different forms of visualization. A static diagram may be sufficient for showing the structure of a geometric object, whereas an interactive representation may be more useful when the learner needs to investigate how a mathematical object changes.

A particularly important theoretical idea is the transition between concrete, pictorial, and symbolic forms. The reviewed literature indicates that interactive representations can support movement between these levels and thereby contribute to conceptual understanding.

The relationship can be represented as:

Concrete experience → Visual representation → Symbolic representation → Abstract understanding

However, this should not be interpreted as a rigid one-way progression. Mathematical understanding can move in both directions. A learner may begin with a symbolic equation and create a graph to understand it, or begin with a visual pattern and subsequently derive an algebraic expression.

This reciprocal movement is particularly important in advanced mathematics. Abstract mathematical ideas may generate visual representations, while visual representations may help learners construct or interpret abstract mathematical ideas.

The literature also provides a cognitive basis for the importance of visualization. Boaler et al. discuss research suggesting that visual and spatial processes are involved in mathematical activity and argue that visual thinking should be regarded as an important part of mathematical competence.

This perspective challenges the traditional separation between "visual mathematics" and "abstract mathematics." Instead, visualization can be regarded as one of the mechanisms through which abstract mathematical relationships become conceptually accessible.

An important caution, however, concerns the classification of learners according to fixed learning styles. Boaler et al. question the practice of labeling students according to rigid categories such as visual, auditory, or kinesthetic learners. They argue that visual mathematical thinking has broader relevance and should not be restricted to a particular group of learners.

Therefore, the present review adopts a broader interpretation of visual learning. The focus is not on identifying a particular "type" of learner, but on understanding when and how visual representations support mathematical thinking**.**

III. ROLE OF VISUAL REPRESENTATIONS IN UNDERSTANDING ABSTRACT MATHEMATICS

The central contribution of visual learning is its ability to provide an alternative representation of mathematical information. Abstract concepts that appear difficult in symbolic form may become more accessible when relationships are represented spatially or graphically.

Several studies have investigated different aspects of this relationship. However, the studies differ in their objectives, educational levels, research approaches, and emphasis. A comparative synthesis of the selected literature is therefore important for identifying both the established findings and the areas that remain insufficiently explored.

Table 1: Comparative Literature Review On Visual Learning And Mathematics

Author(s) & Year	Focus of Study	Research Approach	Major Findings	Major Limitation / Gap	Contribution to Present Review
Boaler et al. (2016)	Visual mathematics, mathematical cognition, and the importance of visual thinking	Review/conceptual discussion based on mathematical cognition and educational research	Visual processing is an important component of mathematical activity. Visual approaches can contribute to mathematical understanding and should not	Does not specifically examine a structured framework for using visualization to understand advanced abstract mathematics or	Provides the **cognitive** and theoretical foundation for considering visual mathematics as an integral part of mathematical thinking.

			be treated merely as a preliminary stage before symbolic mathematics.	mathematical research texts.	
Marikyan (2023)	Teaching mathematics through visual strategies and development of analytical thinking	Educational/review-oriented study	Visual strategies can support analytical thinking, spatial reasoning, mathematical communication, engagement, and motivation.	Primarily addresses instructional use of visuals and does not specifically examine advanced mathematics or mathematical research-paper comprehension.	Establishes the relationship between visual strategies and analytical mathematical reasoning.
Žakelj & Klančar (2022)	Role of visual representations in geometry learning	Empirical study involving digital learning resources and interactive representations	Digital and interactive representations supported students' learning and improved performance in several geometry-related tasks. The approach supported movement between concrete, pictorial, and symbolic representations.	Focused mainly on school-level geometry; findings are not directly tested across broader areas of abstract mathematics.	Provides empirical evidence for the effectiveness of purposeful visual and interactive representations.
Sandi & Utama (2025)	Visual learning style and mathematics comprehension	Qualitative study involving interviews and classroom-related evidence	Images, videos, colors, and visually rich learning materials were associated with increased interest, recall, and comprehension among the students studied.	Conducted at elementary level and focuses on visual learning style rather than the broader cognitive mechanisms of visualization in abstract mathematics.	Provides recent evidence connecting visual approaches with mathematics comprehension.

Present Review	Visual learning as a bridge between mathematical representation and abstract understanding	Narrative synthesis of selected studies	Integrates cognitive, pedagogical, representational, and technological perspectives to explain how visualization may support conceptual understanding and mathematical abstraction.	Further empirical research is required to test the proposed framework, especially in advanced mathematics and mathematical-text comprehension.	Provides an integrated conceptual perspective linking visual representation, reasoning, symbolic connection, and abstract mathematical understanding.

i. Critical Synthesis of the Reviewed Literature

The comparative analysis demonstrates that the selected studies generally support the value of visualization in mathematics, but they approach the subject from substantially different perspectives.

Boaler et al. provide the broadest theoretical foundation. Their work challenges the traditional tendency to treat visual mathematics as something appropriate only at early stages of mathematical learning. Instead, visual processing is presented as an important component of mathematical activity itself. This perspective is particularly important because it challenges the assumption that abstraction and visualization are opposing forms of mathematical thinking.

Marikyan focuses more directly on instructional strategies and analytical thinking. The study emphasizes the contribution of visual approaches to spatial reasoning, communication, engagement, and analytical processes. This adds a pedagogical dimension to the cognitive argument presented by Boaler et al.

Žakelj and Klančar provide an empirical perspective through geometry learning. Their findings indicate that digital and interactive representations can contribute to improved performance and can help learners move between different forms of representation. This is particularly relevant to the present review because it provides evidence that visualization can operate as a mechanism for connecting concrete, pictorial, and symbolic mathematics.

Sandi and Utama provide a recent qualitative perspective concerning visual learning and mathematics comprehension. Their findings indicate positive student responses to images, videos, and visually rich materials. However, their study is conducted at the elementary level, which limits direct generalization to advanced mathematical learning.

Taken together, the studies reveal an important pattern. The usefulness of visualization is relatively well established, but the mechanisms through which visualization supports the development of abstract mathematical understanding are less systematically integrated across the literature.

The studies also differ in their educational contexts. Some focus on mathematical cognition, others on classroom instruction, geometry, digital learning, or learner experience. This fragmentation creates an opportunity for a review that connects these perspectives.

ii. Visualization as a Bridge to Abstraction

Mathematical abstraction requires learners to identify general relationships beyond individual examples. Visual representations can support this process by allowing learners to observe patterns and relationships before expressing them formally.

For example, a growing geometric pattern can reveal a relationship between position and quantity. A learner can first observe the pattern, identify how it grows, and subsequently express the relationship using an algebraic formula.

The literature on visual mathematics provides examples in which students investigate patterns visually and then develop algebraic generalizations. Such activities demonstrate that visual exploration can contribute to the development of formal mathematical relationships.

This suggests that visual representation can function as a bridge between observation and abstraction.

The process may be described as:

Observation → Pattern recognition → Relationship identification → Generalization → Symbolic expression

The importance of this process is particularly relevant when learners encounter abstract concepts for the first time.

iii. Visualization and Conceptual Understanding

Conceptual understanding requires more than remembering mathematical procedures. A learner should understand what a mathematical expression means, how different quantities are related, and why a particular procedure is appropriate.

Visual representations can support such understanding because they externalize relationships.

For example, a graph can show the behavior of a function. A geometric diagram can demonstrate relationships among sides and angles. A number line can provide a spatial representation of numerical relationships.

The qualitative study by Sandi and Utama reports that visual materials such as images, videos, and colorful worksheets were associated with improved comprehension and recall in the context examined.

These findings support the broader argument that visual representation can make mathematical information more meaningful by connecting symbolic information with perceptual relationships.

IV. Visual Learning, Mathematical Reasoning, and Digital Technologies

The reviewed literature indicates that visual learning is associated not only with comprehension but also with mathematical reasoning and problem solving.

i. Visual Learning and Analytical Thinking

Mathematical reasoning requires learners to identify relationships, compare alternatives, recognize patterns, make assumptions, and evaluate conclusions.

Visual representations can support these processes by providing an external structure for reasoning.

Marikyan's work emphasizes the potential relationship between visual strategies and analytical thinking, including spatial reasoning and mathematical communication.

When students construct a diagram, they must decide which information is relevant and how different mathematical elements are connected. This process itself can encourage analytical thinking.

Visualization can therefore serve two functions:

1. **Representational function** — showing mathematical information.
2. **Reasoning function** — supporting the process of thinking about that information.

This distinction is important because the educational value of visualization does not arise merely from making information attractive. Its value lies in how the representation changes the learner's interaction with the mathematical problem.

ii. Visual Learning and Problem Solving

Problem solving often requires learners to construct a mental representation of a problem before selecting a solution strategy.

Visual representations can help organize the information contained in a problem.

For example, a geometric problem may become clearer when the relevant relationships are drawn. A word problem may be represented through a diagram. A function problem may be investigated through a graph.

The reviewed geometry research provides empirical support for this role. Students using digital resources and interactive representations demonstrated better performance in several geometry-related tasks, including problems involving area and perimeter.

Visualization can therefore support problem solving by helping learners transform a problem from a purely verbal or symbolic form into a representation that exposes its mathematical structure.

iii. Digital Visualization

Digital technology has substantially expanded the possibilities of mathematical visualization.

Digital environments can provide:

- dynamic graphs;
- interactive geometry;
- simulations;
- animations;
- virtual manipulatives;
- mathematical models;
- parameter manipulation; and
- interactive representations.

According to Žakelj and Klančar, digital technologies can support modelling, simulation, experimentation, mathematical investigation, and problem solving.

The major advantage of digital visualization is its dynamic nature.

A static figure provides one representation. An interactive representation can allow the learner to change the figure and immediately observe the consequences.

For example, a learner studying a function can change a parameter and observe how the graph responds. Similarly, a learner studying geometry can manipulate a vertex and investigate whether particular properties remain unchanged.

This creates an environment in which mathematical relationships can be explored rather than simply described.

iv. Visual Learning and Engagement

Visual representations may also influence learner engagement.

The reviewed literature reports positive associations between visual approaches and student interest, motivation, and participation. Sandi and Utama found that students responded positively to images, videos, and visually rich worksheets, with reported benefits in interest, recall, and comprehension.

However, engagement should not be treated as equivalent to understanding.

An attractive animation may capture attention without producing mathematical learning. Therefore, visual materials must be connected to a clearly defined mathematical objective.

The most effective visual representation is not necessarily the most colorful or technologically advanced. It is the representation that makes the intended mathematical relationship clearer.

v. Visual and Symbolic Representations Must Work Together

A major conclusion emerging from the reviewed studies is that visualization should complement rather than replace symbolic mathematics.

Students ultimately need to understand mathematical notation, definitions, formulas, logical arguments, and proofs.

The role of visualization is to support the construction and interpretation of meaning.

The relationship can therefore be expressed as:

Visual representation ↔ *Symbolic representation*

rather than:

Visual representation → *Replacement of symbolic mathematics*

This distinction is particularly important for advanced mathematics, where formal symbolic reasoning is unavoidable.

V. Research Gap and Proposed Conceptual Framework

Although the literature establishes the importance of visualization in mathematics learning, several gaps remain.

First, existing studies examine visual learning from different perspectives, including mathematical cognition, analytical thinking, geometry education, digital learning, and visual learning approaches. However, these perspectives remain relatively fragmented.

Second, much of the empirical research focuses on school-level mathematics. The reviewed studies provide valuable evidence concerning elementary mathematics and geometry, but there is less evidence concerning advanced abstract mathematics.

Third, existing research focuses primarily on classroom learning. Relatively little attention has been given to how visual representations might support the comprehension of mathematically dense academic texts and research papers containing definitions, equations, theorems, propositions, and proofs.

Fourth, although existing studies demonstrate the importance of moving between representations, there remains a need for a clearer conceptual explanation of the process through which visualization mediates the transition from mathematical information to abstract understanding.

The literature therefore supports the identification of the following central research gap:

Existing studies establish the usefulness of visual representations in mathematics learning, but there is insufficient synthesis of how visualization mediates the transition from symbolic mathematical information to abstract conceptual understanding, particularly in advanced mathematical learning and the comprehension of mathematically dense academic texts.

i. Proposed Conceptual Framework

3. Based on the synthesis of the selected literature, the present review proposes the following pathway:

Abstract Mathematical Information

↓

Visual Representation

↓

Perception and Organization of Information

↓

Pattern and Relationship Recognition

↓

Mathematical Reasoning

↓

Connection with Symbolic Representation

↓

Conceptual Understanding

↓

Abstract Mathematical Reasoning

The framework suggests that visualization may function as a mediating mechanism between mathematical information and conceptual understanding.

The first stage involves transforming mathematical information into a suitable visual representation. The learner then interprets the representation and identifies relevant patterns or relationships. These relationships become the basis for mathematical reasoning. The learner subsequently connects the visual representation with symbolic mathematical notation and develops a more integrated conceptual understanding.

This framework is consistent with the literature emphasizing movement between concrete, pictorial, and symbolic representations.

The framework also extends the existing discussion by proposing that this process may be relevant not only to classroom mathematics but also to the interpretation of mathematically dense academic material.

ii. Significance of the Proposed Framework

The proposed framework has three major implications.

First, it positions visualization as part of mathematical reasoning rather than merely a teaching aid.

Second, it emphasizes the importance of connecting representations rather than presenting isolated visual materials.

Third, it provides a conceptual basis for future research investigating whether visual representations can improve the comprehension of advanced mathematical concepts and mathematical research papers.

Future empirical research could test this framework by comparing conventional symbolic presentations with presentations that integrate carefully designed visual representations.

Potential outcomes for investigation could include:

- Conceptual understanding;
- Mathematical reasoning;
- Problem-solving ability;
- Retention;
- Interpretation of mathematical notation;
- Understanding of proofs; and
- Ability to explain mathematical relationships.

Such research would help determine whether the conceptual relationship proposed in this review can be demonstrated experimentally.

VI. Conclusion and Future Directions

The present narrative review examined the role of visual learning in understanding abstract mathematical concepts through a synthesis of selected literature on visual mathematics, mathematical cognition, visual representations, analytical thinking, geometry learning, and visual learning approaches.

The reviewed literature consistently indicates that visualization can contribute to mathematical understanding by making relationships more accessible and by providing learners with representations that complement symbolic mathematical forms. Visual representations can support pattern recognition, spatial reasoning, conceptual understanding, mathematical communication, problem solving, and engagement.

The evidence from geometry learning demonstrates that digital and interactive representations can support students' movement between concrete, pictorial, and symbolic forms and can contribute to improved performance in mathematical tasks. Similarly, the reviewed qualitative research indicates that visual materials can support interest, recall, and comprehension in mathematics learning.

At the theoretical level, visual mathematics should not be regarded as an inferior or preliminary form of mathematics. Visual and symbolic forms can operate together in the development of mathematical understanding. Boaler et al. particularly emphasize the importance of visual mathematical thinking and challenge the traditional tendency to treat visual mathematics as merely a precursor to more important abstract mathematics.

Nevertheless, visualization is not a substitute for symbolic reasoning, formal mathematical language, or proof. Its value depends on the quality and purpose of the representation and on the connections established between visual, symbolic, verbal, and concrete forms.

An important gap remains in understanding how visualization can support advanced mathematical learning and the comprehension of mathematically dense academic texts. Existing research has largely examined classroom mathematics, particularly geometry and elementary learning, while comparatively less attention has been given to the use of visual representations for interpreting abstract mathematical literature.

The present review therefore proposes that visual learning can be conceptualized as a mediating bridge between mathematical representation and abstraction. Through visual representation, learners may organize mathematical information, identify patterns and relationships, develop reasoning, connect visual structures with symbolic expressions, and ultimately construct abstract mathematical understanding.

Future research should empirically investigate this proposed relationship, particularly among undergraduate, postgraduate, and research-level mathematics learners. Studies could examine whether structured diagrams, concept maps, graphical representations, proof-flow diagrams, dynamic mathematical models, and other visualization techniques improve the comprehension of advanced mathematical texts.

The broader implication is that mathematics education and mathematical communication need not choose between visual and symbolic approaches. Instead, the integration of both may provide a more comprehensive pathway toward mathematical understanding.

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