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Multi-Star Decomposition of Complete Bipartite Graphs

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Abstract: We give some necessary and sufficient condition for decomposing some special type of complete bipartite graph $K_{m,n}$ into multi-star subgraphs. In particular the condition for $\alpha K_{t,\alpha}$ with $t = 1,2,3,\ldots,$ admits multi-star decomposition exist. The graph of the form $K_{1,\alpha}$ has multi-star decomposition if and only if $\alpha = n(2n+1)$ for any natural number n. A Complete bipartite graph K_{t,α_t} admits Multi-star Decomposition $(S_1, S_2, S_3, \ldots, S_n)$ if and only if $n = Kt^2$ or $2n = Kt^2 - 1, t, K(\neq 0) \in N$.

Keywords: Complete bipartite graph, Arithmetic Decomposition, Multi decomposition, Multi-Star Decomposition.

1.Introduction

Let $G = (V, E)$ be a simple connected graph. If $G_1, G_2, \ldots, G_n$are connected edge-disjoint subgraphs of G with $E(G) = E(G_1) \cup E(G_2) \cup \ldots \cup E(G_n)$, then $(G_1, G_2, \ldots, G_n)$ is a Decomposition of G. Different types of decompositions of graphs are available in literature such as path decomposition, cycle decomposition, triangle decomposition and few papers are available in diamond decomposition .Akiyama and Kano [2] introduced the concept of path factors in graphs and their applications. The notions of even star decomposition and odd star decomposition were discussed by Merly [4] in Even Star Decomposition of Complete Bipartite Graphs and by Purwanto [7] in Odd Star Decomposition of Complete Bipartite Graphs, respectively. In this paper, we explore the multi-star decomposition of certain complete bipartite graphs.

1.1 Star

A complete bipartite graph of the form $K_{1,n}$ is called a star, and is denoted by S_n.

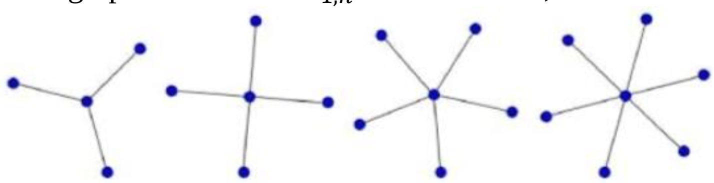
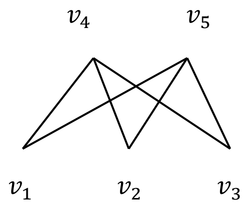


Fig 1: A Star $K_{1,3}, K_{1,4}, K_{1,5}, K_{1,6}$

1.2 Complete bipartite graph

A bipartite graph G which contains every edge joining V_1 and V_2 then G is a complete bipartite graph. If V_1 and V_2 have *m* and *n* vertices, we write $G = K_{(m,n)}$. Clearly $K_{m,n}$ has *mn* edges.



$Fig\ 2: Complete\ Bipartite\ Graph\ K_{2,3}$

1.3 Decomposition of a graph

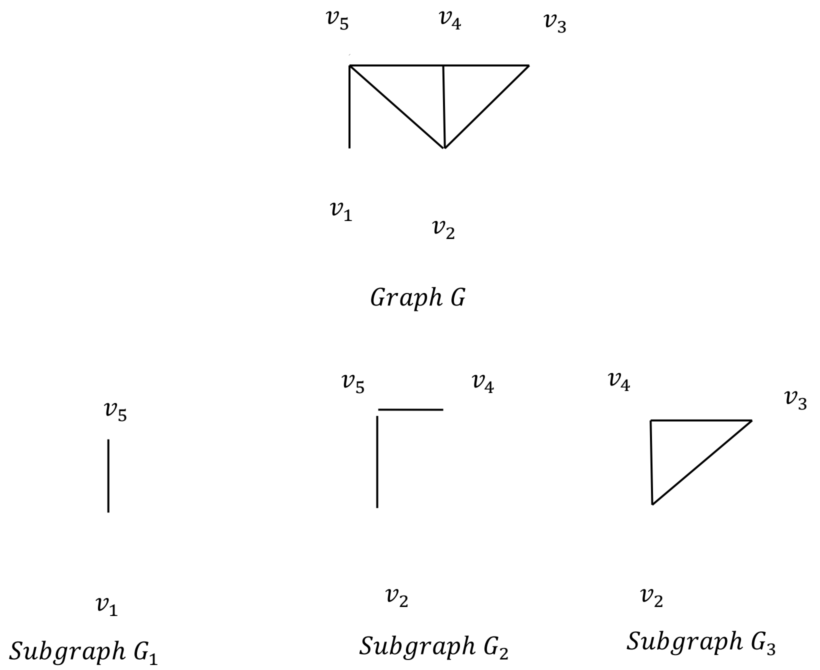
Let *G* = *(V, E)* be a simple connected graph . If $G_1, G_2, \dots, G_n$are connected edge-disjoint subgraphs of *G* with *E(G) = E*$(G_1) \cup E(G_2) \cup \dots \cup E(G_n)$, then $(G_1, G_2, \dots, G_n)$ is a Decomposition of G. For the Decomposition referred to Sujatha. M [6] A Study on Decomposition of Graph.

2. Multi-Star Decomposition of a Graph

2.1 Arithmetic Decomposition:

A decomposition $(G_1, G_2, \dots, G_n)$ of a graph *G* is an Arithmetic Decomposition (AD) if the sequence of number of edges in the decomposed subgraphs in increasing order is an arithmetic sequence.

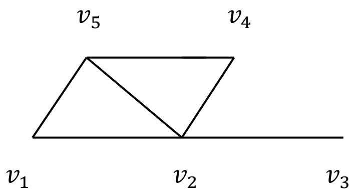
G is a simple connected graph and $(G_1, G_2, \dots, G_n)$ is a Decomposition of G.$|E(G_1)| = 1$, $|E(G_2)| = 2$, $|E(G_3)| = 3$. Also, they are in arithmetic progression with first term 1 and common difference 1. So the decomposition thus (G_1, G_2, G_3) is an arithmetic decomposition.



$Fig\ 3 : Arithmetic\ Decomposition$

2.2 Multi Decomposition (MD)

An arithmetic decomposition is said to be Multi Decomposition if the first term and common difference of the sequence of number of edges in the subgraphs are 1 and 2. Since the number of edges of each subgraph of *G*is Odd and Even, we denote the multi decomposition as (G_1, G_2, G_n). In below example G_1,$G_2$and G_3 are arithmetic decomposition of *G* with $a = 1$ and 2, $d = 1$ and 2. So (G_1, G_2) are Odd and (G_1, G_3) are Even. This is called a Multi decomposition of *G*.



Graph G

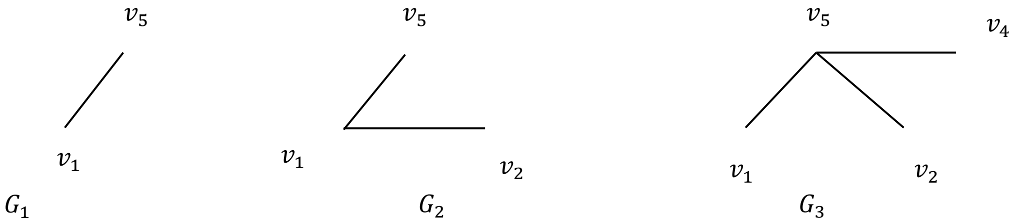


Fig 4 : *Multi Decomposition*

Theorem 2.1

Any graph G admits Multi Decomposition $(G_1, G_2, G_3, \dots, G_n)$ where $G_i = (V_i, E_i)$ and $|E(G_i)| = i$, *i* =*1,2, 3, 4. . . ., n)* if and only if $q = n(2n+1)$ where n is any natural number.

2.3 Star Decomposition

A decomposition $(G_1, G_2, \dots, G_n)$ of $G$is said to be star decomposition if each $(G_1, G_2, \dots, G_n)$ are some stars.

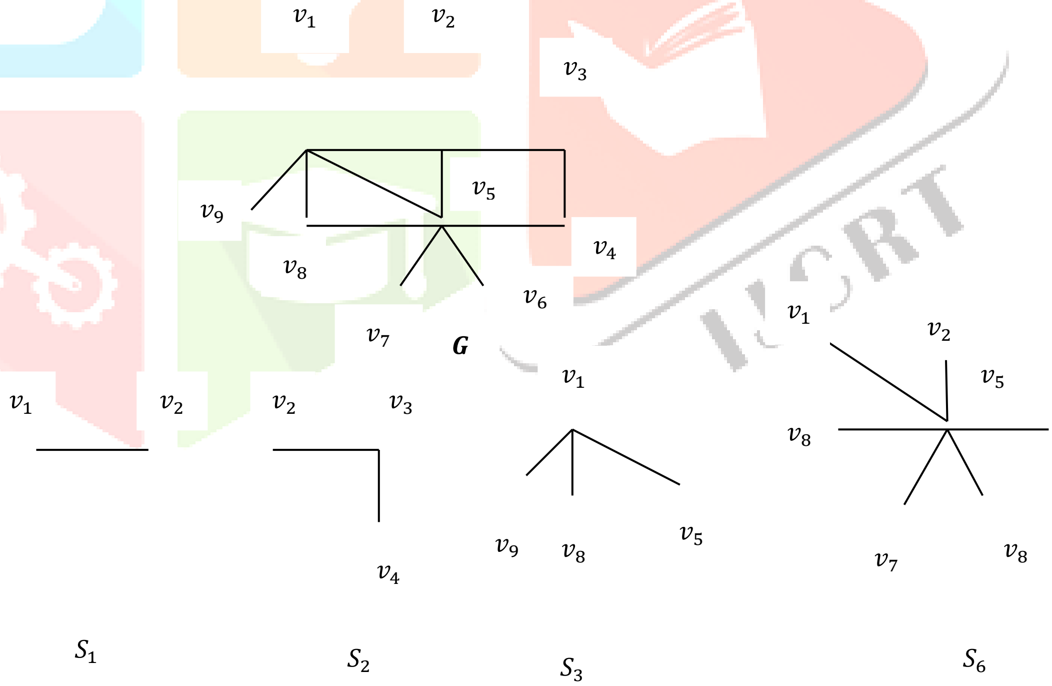
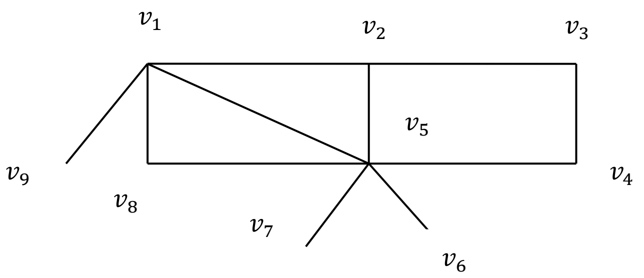


Fig 5 : *Star Decomposition* (S_1, S_2, S_3, S_6) *of G*

2.4 Multi-Star Decomposition

A Multi-star decomposition is a connected graph with one central vertex where every other vertex is connected to the central vertex by one or more edges (multiple edges are allowed)



G

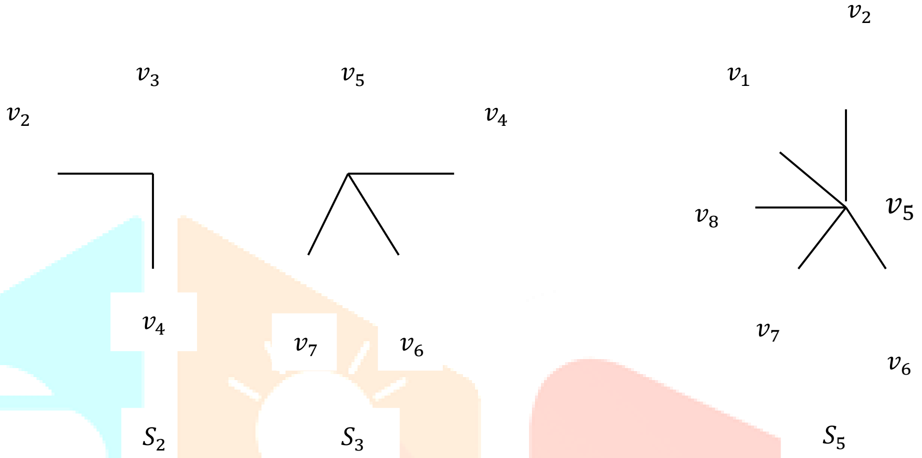


Fig 6 : Multi − Star Decomposition (S_2, S_3, S_5) *of G*

3. Multi-Star Decomposition of Complete Bipartite Graph

Number of edges in $K_{1,1}$ is 1. This number 1 cannot be expressed as $n(2n+1)$ for any natural number n. So it is not a Multi Decomposition, and hence is not a Multi-Star Decomposition. In the similar manner $K_{1,1}, K_{1,2}, K_{1,4}, K_{1,5}$ etc has no Multi Star Decomposition.

Number of edges in $K_{1,3}$ is 3. This can be expressed as $1(2(1)+1)$. So it is MD, and hence $K_{1,3}$ have an decomposition $G_3 = S_3$. In the similar manner $K_{1,7}$ and $K_{1,12}$ etc has MSD.

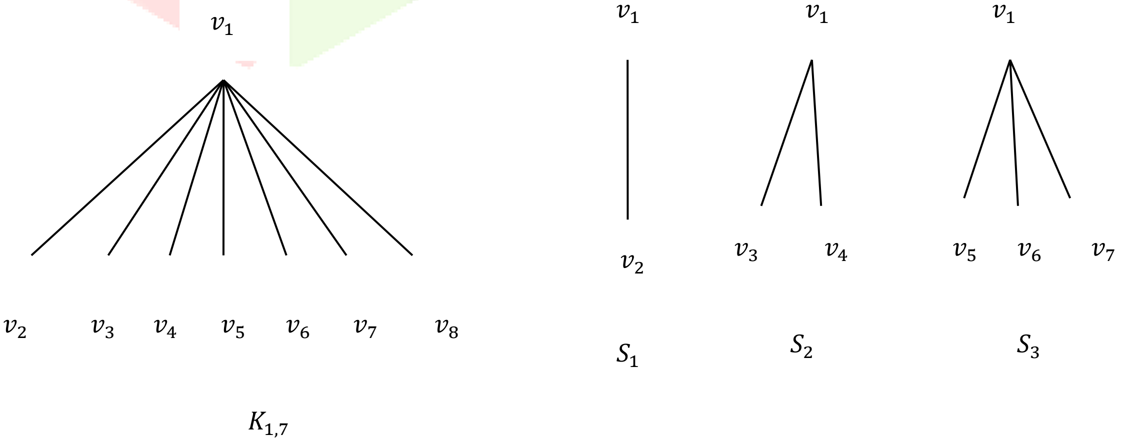


Fig 7 : Multi Star Decomposition of Complete Bipartite Graph

Theorem 3.1

The graph $K_{1,\alpha}$ has a Multi-Star Decomposition $(S_1, S_2, \ldots, S_n)$ if and only if $\alpha = n(2n+1)$, $n \in N$.

Proof

Assume that $K_{1,\alpha}$ has Multi-Star Decomposition $(S_1, S_2, \dots, S_n)$. Then

$$|E(K_{1,\alpha})| = \sum_{i=1}^{n} |E(S_i)|$$

$$\Rightarrow \alpha = \sum_{i=1}^{n} i$$
$$\Rightarrow \alpha = n(2n+1)$$

Conversely assume that $\alpha = n(2n+1)$. We have to prove that $K_{1,\alpha} = K_{1,n(2n+1)}$has a MSD $(S_1, S_2, \dots, S_n)$. Let us prove this by mathematical induction. For $n = 1, \alpha = 1 * (2+1) = 3$ clearly $K_{1,3}$ has MSD S_3. Assume that the result is true for $n = m$. That is $K_{1,m(2m-1)}$ has Multi-Star Decomposition $(S_1, S_2, \dots, S_m)$. We have to show that it is true for $n = m+1$. That is $K_{1,m+1(2(m+1)+1)}$ has MSD $(S_1, S_2, \dots, S_{m+1})$. We have $(m+1)(2m+3) = 2m(m+1) + 3(m+1)$.
Also, $K_{1,(m+1)(2m+3)} = K_{1(m+1)(2m+3)} = K_{1(m+1)(2m+3)} = K_{1,(m+1)(2m+3)}$
$K_{1,(m+1)(2m+3)}$ can be decomposed into $K_{1,2m(m+1)}, K_{1,3(m+1)}$.
By hypothesis $K_{1,2m(m+1)}$ has MSD $(S_1, S_2, \dots, S_m)$.
$(K_{1,2m(m+1)}, K_{1,3(m+1)})$ can be decomposition into $(S_1, S_2, \dots, S_m, K_{1,3(m+1)})$.
$(K_{1,2m(m+1)}, K_{1,3(m+1)})$ has the MSD $(S_1, S_2, \dots, S_m, K_{3(m+1)})$.
Hence the proof.

Theorem 3.2

A complete bipartite graph $K_{1,\alpha}$ admits multi-star decomposition $(S_1, S_2, \dots, S_{t^2k})$ if and only if $\alpha = (t^2k)(2(t^2k+1)$, where $n = t^2k$, $(k \neq 0) \in N$.

Proof

Assume that$K_{1,\alpha}$ admits multi-star decomposition $(S_1, S_2, \dots, S_{t^2k})$
Then

$$|E(K_{1,\alpha})| = \sum_{i=1}^{n} |E(S_i)|$$
$$1\alpha = \sum_{i=1}^{n} i$$
$$1\alpha = n(2n+1)$$
$$\alpha = (t^2k)(2(t^2k+1).$$

Conversely, Assume that $\alpha = (t^2k)(2(t^2k+1)$. Let $\{u_1, u_2, v_1, v_2, \dots, v_\alpha\}$ be the vertex set $K_{1,\alpha}$. For every α, $K_{1,\alpha}$ can be decomposed into $K_{1,\alpha}^{u_1}$ with vertex set $\{u_1, v_1, v_2, \dots, v_\alpha\}$ and $K_{1,\alpha}^{u_2}$ with vertex set $\{u_2, v_1, v_2, \dots, v_\alpha\}$. Since number of edges in $K_{1,\alpha}$ can be expressed in the form $n(2n+1)$, it has an multi-star decomposition $(G_1, G_2, \dots, G_n)$. This multi decomposition can be partitioned into two sets $U = \{G_1, G_2, \dots, G_{k-1}, G_{n-(k-1)}, \dots, G_n\}$ and $V = \{G_k, G_{k+1}, \dots, G_{k+n}\}$. Using these two sets U and V, we can decompose $K_{1,\alpha}^{u_1}$ and $K_{1,\alpha}^{u_2}$, whose union gives the decomposition of $K_{1,\alpha}$. $K_{1,\alpha}^{u_1}$ can be decomposed as $\{S_1, S_2, \dots, S_{k-1}, S_{n-(k-1)}, \dots, S_n\}$ and $K_{1,\alpha}^{u_2}$ can be decomposed as $\{S_k, S_{k+1}, \dots, S_{k+n}\}$, whose union gives the MSD of $K_{1,\alpha}$. Hence the Proof.

Theorem 3.3

A complete bipartite graph $K_{t,\alpha}$ admits multi-star decomposition $(S_1, S_2, \dots, S_{t^2k})$ if and only if $\alpha = (t^2k)(2(t^2k+1)$, where $n = t^2k$, $(k \neq 0) \in N$.

Proof

Assume that$K_{t,\alpha}$ admits multi-star decomposition $(S_1, S_2, \dots, S_{t^2k})$

Then

$$|E(K_{t,\alpha})| = \sum_{i=1}^{n} |E(S_i)|$$
$$t\alpha = \sum_{i=1}^{n} i$$
$$t\alpha = n(2n+1)$$
$$\alpha = (t^2k)(2(t^2k+1), (t=1)$$

Conversely, Assume that $\alpha = (t^2k)(2(t^2k+1)$. Let $\{u_1, u_2, v_1, v_2, \dots, v_\alpha\}$ be the vertex set $K_{1,\alpha}$. For every α, $K_{1,\alpha}$ can be decomposed into $K_{1,\alpha}^{u_1}$ with vertex set $\{u_1, v_1, v_2, \dots, v_\alpha\}$ and $K_{1,\alpha}^{u_2}$ with vertex set $\{u_2, v_1, v_2, \dots, v_\alpha\}$. Since number of edges in $K_{1,\alpha}$ can be expressed in the form $n(2n+1)$, it has an multi-star decomposition $(G_1, G_2, \dots, G_n)$. This multi decomposition can be partitioned into two sets $U = \{G_1, G_2, \dots, G_{k-1}, G_{n-(k-1)}, \dots, G_n\}$ and $V = \{G_{k+1}, G_{k+2}, \dots, G_{n-k}\}$. Using these two sets U and V, we can decompose $K_{1,\alpha}^{u_1}$ and $K_{1,\alpha}^{u_2}$, whose union gives the decomposition of $K_{1,\alpha}$. $K_{1,\alpha}^{u_1}$ can be decomposed as $\{S_1, S_2, \dots, S_{k-1}, S_{n-(k-1)}, \dots, S_n\}$ and $K_{1,\alpha}^{u_2}$ can be decomposed as $\{S_{k+1}, S_{k+2}, \dots, S_{n-k}\}$, whose union gives the MSD of $K_{t,\alpha}$. Hence the Proof.

Theorem 3.4

A Complete bipartite graph K_{t,α_t} admits multi-star decomposition $(S_1, S_2, \dots, S_n)$ if and only if $n = Kt^2$ or $2n = Kt^2 - 1, t, K(\neq 0) \in N$.

Proof

Assume that K_{t,α_t} admits Multi-star decomposition $(S_1, S_2, \dots, S_n)$.

$$t_{\alpha_t} = \sum_{i=1}^{n} |E(S_i)|$$
$$t_{\alpha_t} = \sum_{i=1}^{n} i$$
$$t_{\alpha_t} = n(2n+1)$$
$$\alpha_t = \frac{n(2n+1)}{t}, \text{ where } \alpha_t \in N$$
$$\frac{n(2n+1)}{t} = t^3K, K \in N$$
$$n(2n+1) = Kt^2.$$
$$n = Kt^2 \text{ or } 2n+1 = Kt^2$$
$$n = Kt^2 \text{ or } 2n = Kt^2 - 1.$$

Conversely assume that $n = Kt^2$ or $2n = Kt^2 - 1$.

Case 1 :$n = Kt^2$

When $= Kt^2$, $\alpha_t = \frac{Kt^2(2Kt^2)+1)}{t} = Kt(2(Kt^2)+1)$, we have to prove K_{t,α_t} admits multi-star decomposition. Applying induction on $'t'$ is the result is true when $t = 1$. Suppose the result is true when $t = m$. That is K_{m,α_m} admits MSD $(S_1, S_2, \dots, S_{kt^{m+1}})$. we have to show that that the result is true for $t = m+1$, that is to Prove $K_{m+1,\alpha_{m+1}}$admits MSD $(S_1, S_2, \dots, S_{kt^{m+1}}, S_{kt^{m+1}}+1, \dots, S_{kt^{m+2}})$.

We have, $|E(K_{m+1,\alpha_{m+1}})| = m+1_{\alpha_{m+1}} = K(m+1)(2K(m+1)+1)$ ______________(1)

Also, $\left|E\left(K_{m,\alpha_m}\right)\right| = m_{\alpha_m} = Km(2Km+1)$ ___________________________(2)

Therefore, $|E(K_{1^{m+1},\alpha_{m+1}})| - \left|E\left(K_{1^m,\alpha_m}\right)\right| = 3K^2(m+2) + K(m+1)$

Now $|E(S_{K1^{m+2}+2})| + \cdots + |E(S_{K1^{m+3}})| = 3K^2(m+2) + K(m+1)$ _____________(3)

Therefore, From (2) and (3) $|E(K_{m+1,\alpha_{m+1}})| = \left|E\left(K_{m,\alpha_m}\right)\right| + \left|E\left(S_{K(m+2)}\right)\right| + \cdots + \left|E\left(S_{K(m+3)}\right)\right|$

Therefore result is true for $t = m+1$

Hence, K_{t,α_t} admits multi-star decomposition $(S_1, S_2, \ldots, S_n)$ where $n = Kt^2$ or $2n = Kt^2 - 1, t, K(\neq 0) \in N$.

Case 2 :$2n = Kt^2 - 1$.

The Proof is similar to case 1. Hence the Proof.

4.Conclusion

In this study, we introduce a concept known as multi-star decomposition of graphs and determined this parameter for fews, especially for some complete bipartite graph $K_{1,\alpha}$, has multi star decomposition if and only if $\alpha = n(2n+1) \in N$.A Complete bipartite graph K_{t,α_t} admits Multi star Decomposition $(S_1, S_2, S_3, \ldots, S_n)$ if and only if$n = Kt^2$ or $2n = Kt^2 - 1, t, K(\neq 0) \in N$.

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