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Analytical Structure Of Sumudu-Fourier Transform In The Distributional Sense

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Abstract: This paper introduces and studies the Sumudu transform with parameter p, the Fourier transform with parameter s, and their combined structure, the Sumudu–Fourier transform (SFT). A generalized formulation of the SFT is developed in the framework of distributions of compact support, enabling its extension to a broad class of generalized functions in the dual space . The analyticity properties of both the classical and distributional SFT are investigated. By applying Cauchy’s integral formula and exploiting appropriate growth conditions on corresponding kernels, we demonstrate that the Sumudu–Fourier transform is analytic in complex domains of the parameters p and s. The analyticity proofs rely on convergence of incremental ratios in the dual space and on uniform bounds for transform kernels. These results establish a rigorous mathematical foundation for parameter-dependent Sumudu–Fourier transforms and extend their applicability to generalized function spaces.

Keywords - Sumudu Transform, Fourier Transform, Sumudu –Fourier Transform, Generalized function, Testing function space.

I. INTRODUCTION

Integral transforms play a central role in solving differential equations, studying dynamical systems, and understanding the structure of signals and operators. Among these transforms, the Fourier transform is classical and extensively developed, while the Sumudu transform, introduced more recently, has gained attention due to its useful operational properties, especially in engineering and applied mathematics. When extended to the framework of generalized functions (distributions), these transforms provide a powerful analytical tool it for handling singularities, impulsive signals, and differential equations with distributional data.

The Sumudu Transform with parameter p of $f(x)$ denoted by $S[f(x)] = F(p)$ and is given by

$$S\{f(x)\} = F(p) = -\frac{1}{p}\int_0^\infty e^{-\frac{x}{p}} f(x)\,dx \quad \text{for parameter } p>0 \tag{1.1}$$

The Fourier transform with parameter s of $f(t)$ is denoted by $F[f(t)] = F(s)$ and is given by

$$F[f(t)] = F(s) = \int_{-\infty}^{\infty} e^{-ist} f(t)\,dt \,, \qquad \text{for parameter } s>0 \tag{1.2}$$

The Sumudu-Fourier Transform is defined as

$$SF\{f(x,t)\} = F(p,s) = \int_0^\infty \int_{-\infty}^{\infty} f(x,t)\, K(x,t)\; dx\, dt \tag{1.3}$$

where, $K(x,t) = \frac{1}{p}\, e^{-i\,(st - i\frac{x}{p})}$

Here we are going to introduce Sumudu- Fourier transform with analytical property in the distributional generalized sense. For the generalization of Sumudu- Fourier Transform and for its analytical structure, various testing function spaces are needed, which we have been defined and discussed in this paper by Gelfand-Shilov technique.

This paper is summarized as follows: The Testing Function Space are defined in section 2, Distributional Sumudu-Fourier Transform is defined in section 3. The main aim of this paper is to prove Analyticity Theorem with parameter ‘p’ fixed and ‘s’ fixed which we have to proved in section 4. Lastly paper concludes in section 5.

The notations and terminology are as per A. H. Zemanian [5], [6].

II. TESTING FUNCTION SPACES

2.1. The Space $SF_{a,b,\alpha}$:-

Let I be the open set in $R_+ \times R_+$ and E_+ denotes the class of infinitely differentiable function defined on I . the space $SF_{a,b,\alpha}$ is given by,

$$SF_{a,b,\alpha} = \left\{ \phi : \phi \in E_+ / \gamma_{a,b,k,q,l}\ \phi(x,t) = \sup_{\substack{0<x<\infty \\ 0<t<\infty}} \left| K_{a,b}(x) t^k D_x^l\ D_t^q\ \phi(x,t) \right| \leq C_{lq} A^k k^{k\alpha} \right\}$$

where the constants A and C_{lq} depend on the testing function ϕ. Also, where

$K_{a,b}(x) = \begin{cases} e^{ax}, & 0 \leq x < \infty \\ e^{bx}, & -\infty < x < 0 \end{cases}$ is the kernel of sumudu transform and t^k is the kernel for Fourier transform for testing function space.

2.2. The space $SF_{a,b}^{\beta}$:-

The space $SF_{a,b}^{\beta}$ is given by

$$SF_{a,b}^{\beta} = \left\{ \phi : \phi \in E_+ / \sigma_{a,b,k,q,l}\ \phi(x,t) = \sup_{\substack{0<x<\infty \\ 0<t<\infty}} \left| K_{a,b}(x) t^k D_x^l\ D_t^q\ \phi(x,t) \right| \leq C_{kq} B^l l^{l\beta} \right\}$$

Where the constants B and C_{kq} depend on the testing function ϕ.

2.3. The space $SF_{a,b,\alpha}^{\beta}$:-

The space $SF_{a,b,\alpha}^{\beta}$ is given by

$$SF_{a,b,\alpha}^{\beta} = \left\{ \phi : \phi \in E_+ / \rho_{a,b,k,q,l}\ \phi(x,t) = \sup_{\substack{0<x<\infty \\ 0<t<\infty}} \left| K_{a,b}(x) t^k D_x^l\ D_t^q\ \phi(x,t) \right| \leq C A^k k^{k\alpha} B^l l^{l\beta} \right\}$$

III. DISTRIBUTIONAL GENERALIZED SUMUDU-FOURIER TRANSFORM (SFT)

For $f(x,t) \in SF_{a,b,\alpha}^{*\beta}$, where $SF_{a,b,\alpha}^{*\beta}$ is the dual space of $SF_{a,b,\alpha}^{\beta}$. It contains all distributions of compact support. The Distributional Sumudu–Fourier transform is a function of $f(x,t)$ and is defined as

$$SF\{f(x,t)\} = F(p,s) = \langle f(x,t), \tfrac{1}{p}\, e^{-i(st - i\frac{x}{p})} \rangle \qquad (3.1)$$

where for each fixed $x(0 < x < \infty)$, $t(-\infty < t < \infty)$, $p > 0$ and $s > 0$ The right-hand side of (3.1) has a sense as an application of $f(x,t) \in SF_{a,b,\alpha}^{*\beta}$ to $\frac{1}{p}\, e^{-i(st - i\frac{x}{p})} \in SF_{a,b,\alpha}^{\beta}$.

IV. ANALYTICITY THEOREM

Let $f(x,t) \in SF_{a,b,\alpha}^{*\beta}$ and its Sumudu-Fourier $F(p,s)$ is defined by $SF\{f(x,t)\} = F(p,s) = \langle f(x,t), \frac{1}{p}\, e^{-i(st - i\frac{x}{p})} \rangle$

Then $F(p,s)$ is analytic for some fixed $p > 0, s > 0$ on Ω_f ,

where $\Omega_f = \{(p,s) / \sigma_1 < s < \sigma_2\}$, $\Omega_f = \{(p,s) / \sigma_1 < p < \sigma_2\}$ and

i) $D_s F(p,s) = \langle f(x,t), \frac{1}{p}\, e^{-(\frac{x}{p}+ist)}\, (-it)\rangle$

ii) $D_p F(p,s) = \left\langle f(x,t), \frac{\partial}{\partial p}\frac{1}{p} e^{\frac{-x}{p}}\, e^{-ist}\right\rangle$

Proof: i) let s be an arbitrary but fixed point in Ω_f . Choose the real positive numbers a, b, and r such that $\sigma_1 < a < s-r < s+r < b < \sigma_2$. Also let Δs be a complex increment such that $0<|\Delta s|<r$.

For $\Delta s \neq 0$, we write

$$\frac{F(p,s+\Delta s)-F(p,s)}{\Delta s} - < f(x,t),\ \frac{\partial}{\partial s}\frac{1}{p} e^{-(\frac{x}{p}+ist)} >$$

$$=< f(x,t),\ \frac{1}{\Delta s}\frac{1}{p} e^{-(\frac{x}{p}+i(s+\Delta s)t)} > - < f(x,t),\ \frac{1}{\Delta s}\frac{1}{p} e^{-(\frac{x}{p}+ist)} > - < f(x,t),\ \frac{\partial}{\partial s}\frac{1}{p} e^{-(\frac{x}{p}+ist)} >$$

$$= < f(x,t),\ \frac{1}{\Delta s}\frac{1}{p} e^{-\frac{x}{p}}[e^{-i(s+\Delta s)t} - e^{-ist}]> - < f(x,t),\ \frac{\partial}{\partial s}\frac{1}{p} e^{-(\frac{x}{p}+ist)} >$$

$$= < f(x,t),\ \frac{1}{\Delta s}\frac{1}{p} e^{-\frac{x}{p}}[e^{-i(s+\Delta s)t} - e^{-ist}] - \frac{\partial}{\partial s}\frac{1}{p} e^{-(\frac{x}{p}+ist)} >$$

$$= < f(x,t),\ \psi_{\Delta s}(x,t) >$$

where $\psi_{\Delta s}(x,t) = \frac{1}{\Delta s}\frac{1}{p} e^{-\frac{x}{p}}[e^{-i(s+\Delta s)t} - e^{-ist}] - \frac{\partial}{\partial s}\frac{1}{p} e^{-(\frac{x}{p}+ist)}$

To prove $\psi_{\Delta s}(x,t) \in SF^{\beta}_{a,b,\alpha}$, we shall show that as $|\Delta s| \to 0$, $\psi_{\Delta s}(x,t)$ converges in $SF^{\beta}_{a,b,\alpha}$ to zero. To proceed, Let C denotes the circle with center at s and radius r_1 where $0<r<r_1<\min(s-a,\ b-s)$ .We may interchange differentiation on s with differentiation on t and by using Cauchy's integral formula, we get

$$(-D_t)^q \psi_{\Delta s}(x,t) = \frac{1}{\Delta s}\frac{1}{p} e^{-\frac{x}{p}}[\{-i(s+\Delta s)\}^q e^{-i(s+\Delta s)t} - (-is)^q e^{-ist}] - \frac{\partial}{\partial s}\frac{(-is)^q}{p} e^{-(\frac{x}{p}+ist)}$$

Now applying Cauchy's integral formula, we get

$$=(-i)^q \frac{e^{\frac{-x}{p}}}{p\Delta s}\left[\frac{1}{2\pi i}\int_c \frac{\xi^q e^{-i\xi t}}{(\xi-s-\Delta s)}d\xi - \frac{1}{2\pi i}\int_c \frac{\xi^q e^{-i\xi t}}{(\xi-s)}d\xi\right] - \frac{e^{\frac{-x}{p}}}{p}\frac{1}{2\pi i}(-i)^q \int_c \frac{\xi^q e^{-i\xi t}}{(\xi-s)^2}d\xi$$

$$=(-i)^q \frac{1}{2\pi i}\frac{e^{\frac{-x}{p}}}{p\Delta s}\left[\int_c \frac{1}{(\xi-s-\Delta s)} - \frac{1}{(\xi-s)}\right]\xi^q e^{-i\xi t} - \frac{e^{\frac{-x}{p}}}{p}\frac{1}{2\pi i}(-i)^q \int_c \frac{\xi^q e^{-i\xi t}}{(\xi-s)^2}d\xi$$

$$=(-i)^q \frac{1}{2\pi i}\frac{e^{\frac{-x}{p}}}{p}\left[\int_c \frac{1}{(\xi-s-\Delta s)(\xi-s)}\right]\xi^q e^{-i\xi t} - \frac{e^{\frac{-x}{p}}}{p}\frac{1}{2\pi i}(-i)^q \int_c \frac{\xi^q e^{-i\xi t}}{(\xi-s)^2}d\xi$$

$$=(-i)^q \frac{1}{2\pi i}\frac{e^{\frac{-x}{p}}}{p}\left[\int_c \frac{1}{(\xi-s-\Delta s)(\xi-s)} - \frac{1}{(\xi-s)^2}\right]\xi^q e^{-i\xi t} d\xi$$

$$=(-i)^q \frac{1}{2\pi i}\frac{e^{\frac{-x}{p}}}{p}\left[\int_c \frac{\Delta s}{(\xi-s-\Delta s)(\xi-s)^2}\right]\xi^q e^{-i\xi t} d\xi$$

Now $D_x^{\,l}(-D_t)^q \psi_{\Delta s}(x,t) = =(-1)^l (-i)^q \frac{\Delta s}{2\pi i}\frac{e^{\frac{-x}{p}}}{p^{l+1}}\left[\int_c \frac{\xi^q e^{-i\xi t}}{(\xi-s-\Delta s)(\xi-s)^2}\right]d\xi$

Now for all $\xi \in C$ and $0 < x < \infty$, $\sup\limits_{I_1} \left| e^{ax} t^k (-1)^l (-\mathrm{i})^q \frac{e^{\frac{-x}{p}}}{p^{l+1}} \right| \leq K,$

where K is constant independent of ξ and t

Moreover $|\xi - s - \Delta s| > r_1 - r > 0$ and $|\xi - s| = r_1$, $C_1 = \max\{ \left|\xi^q e^{-i\xi t}\right|, \xi \in C\}$

Consequently,

$$\sup_I \left| e^{ax} t^k D_x{}^l D_t{}^q \psi_{\Delta s}(x,t) \right| = \sup_I \left| e^{ax} t^k \frac{(-1)^l (i)^q}{p^{l+1}} e^{-\frac{x}{p}} \frac{\Delta s}{2\pi i} \int_c \frac{\xi^q e^{-i\xi t}}{(\xi - s - \Delta s)(\xi - s)^2} d\xi \right|$$

$$\leq \left| \frac{\Delta s}{2\pi i} \right| \int_C \frac{K C_1}{(r_1 - r)(r_1)^2} d\xi$$

$$\leq \frac{|\Delta s|}{2\pi} \frac{C_2}{(r_1 - r)(r_1)^2} 2\pi r_1, \quad \text{where } C_2 = K C_1$$

$$\leq \frac{|\Delta s| C_2}{(r_1 - r)(r_1)}$$

The right-hand side is independent of t and converges to zero as $|\Delta s| \to 0$. This shows that $\psi_{\Delta s}(x,t)$ converges to zero as $|\Delta s| \to 0$.
Which ends the proof.

ii) Let p be an arbitrary but fixed point in Ω_f. choose the real positive number a_2, b_2 *and* h such that $\sigma_1 < a_2 < \mathrm{Re}\, p - h < \mathrm{Re}\ p + h < b_2 < \sigma_2$.
Also let Δp be a complex increment such that $0 < |\Delta p| < h$.
For $\Delta p \neq 0$
We write

$$\frac{F(p + \Delta p, s) - F(p, s)}{\Delta p} - \left\langle f(x,t), \frac{\partial}{\partial p} \frac{1}{p} e^{\frac{-x}{p}} e^{-ist} \right\rangle$$

$$= \frac{1}{\Delta p} \left\langle f(x,t), \frac{1}{p + \Delta p} e^{-(\frac{x}{p+\Delta p} + ist)} \right\rangle - \frac{1}{\Delta p} \left\langle f(x,t), \frac{1}{p} e^{-(\frac{x}{p} + ist)} \right\rangle - \left\langle f(x,t), \frac{\partial}{\partial p} \frac{1}{p} e^{-(\frac{x}{p} + ist)} \right\rangle$$

$$= \left\langle f(x,t), \frac{1}{\Delta p} e^{-ist} \left[\frac{e^{-\frac{x}{p+\Delta p}}}{p + \Delta p} - \frac{1}{p} e^{-\frac{x}{p}} \right] \right\rangle - \left\langle f(x,t), \frac{\partial}{\partial p} \frac{1}{p} e^{-(\frac{x}{p} + ist)} \right\rangle$$

$$= \langle f(x,t), \frac{1}{\Delta p} e^{-ist} \left[\frac{e^{-\frac{x}{p+\Delta p}}}{p + \Delta p} - \frac{1}{p} e^{-\frac{x}{p}} \right] - \frac{\partial}{\partial p} \frac{1}{p} e^{-(\frac{x}{p} + ist)} \rangle$$

$$= < f(x,t), \psi_{\Delta p}(x,t) >$$

where, $\psi_{\Delta p}(x,t) = \frac{1}{\Delta p} e^{-ist} \left[\frac{e^{-\frac{x}{p+\Delta p}}}{p + \Delta p} - \frac{1}{p} e^{-\frac{x}{p}} \right] - \frac{\partial}{\partial p} \frac{1}{p} e^{-(\frac{x}{p} + ist)}$

To prove $\psi_{\Delta p}(x,t) \in SF^{\beta}_{a,b,\alpha}$, we show that $|\Delta p| \to 0$, $\psi_{\Delta p}(x,t)$ converges in $SF^{\beta}_{a,b,\alpha}$ to zero.
To proceed, let C denotes the circle with centre at p and radius r_1,

Where $0 < r < r_1 < \min(p-a, b-p)$.

We may interchange differentiation on p with differentiation on x and by using Cauchy's integral formula, we get

$$(-D_x)^l \psi_{\Delta p}(x,t) = \frac{1}{\Delta p} e^{-ist} \left[\frac{e^{-\frac{x}{p+\Delta p}}}{(p+\Delta p)^{l+1}} - \frac{1}{p^{l+1}} e^{-\frac{x}{p}} \right] - \frac{\partial}{\partial p} \frac{1}{p^{l+1}} e^{-(\frac{x}{p}+ist)}$$

$$= \frac{1}{\Delta p} e^{-ist} \left[\frac{1}{2\pi i} \int_c \frac{(\frac{1}{\xi})^{l+1} e^{-\frac{x}{\xi}}}{(\xi - p - \Delta p)} d\xi - \frac{1}{2\pi i} \int_c \frac{(\frac{1}{\xi})^{l+1} e^{-\frac{x}{\xi}}}{(\xi - p)} d\xi \right] - e^{-ist} \left[\frac{1}{2\pi i} \int_c \frac{(\frac{1}{\xi})^{l+1} e^{-\frac{x}{\xi}}}{(\xi - p)^2} d\xi] \right]$$

$$= \frac{1}{\Delta p} e^{-ist} \frac{1}{2\pi i} \left[\int_c \frac{1}{(\xi - p - \Delta p)} d\xi - \frac{1}{2\pi i} \int_c \frac{1}{(\xi - p)} \right] (\frac{1}{\xi})^{l+1} e^{-\frac{x}{\xi}} d\xi - e^{-ist} \left[\frac{1}{2\pi i} \int_c \frac{(\frac{1}{\xi})^{l+1} e^{-\frac{x}{\xi}}}{(\xi - p)^2} d\xi \right]$$

$$= e^{-ist} \frac{1}{2\pi i} \left[\int_c \frac{1}{(\xi - p - \Delta p)(\xi - p)} d\xi \right] (\frac{1}{\xi})^{l+1} e^{-\frac{x}{\xi}} d\xi - e^{-ist} \left[\frac{1}{2\pi i} \int_c \frac{(\frac{1}{\xi})^{l+1} e^{-\frac{x}{\xi}}}{(\xi - p)^2} d\xi \right]$$

$$= e^{-ist} \frac{1}{2\pi i} \left[\int_c \frac{1}{(\xi - p - \Delta p)(\xi - p)} - \frac{1}{(\xi - p)^2} \right] (\frac{1}{\xi})^{l+1} e^{-\frac{x}{\xi}} d\xi$$

$$= e^{-ist} \frac{1}{2\pi i} \left[\int_c \frac{(\xi - p) - (\xi - p - \Delta p)}{(\xi - p - \Delta p)(\xi - p)^2} \right] (\frac{1}{\xi})^{l+1} e^{-\frac{x}{\xi}} d\xi$$

$$= e^{-ist} \frac{1}{2\pi i} \left[\int_c \frac{\Delta p}{(\xi - p - \Delta p)(\xi - p)^2} \right] (\frac{1}{\xi})^{l+1} e^{-\frac{x}{\xi}} d\xi$$

$$D^q{}_t (-D_x)^l \psi_{\Delta p}(x,t) = (-is)^q \frac{\Delta p}{2\pi i} e^{-ist} \left[\int_c \frac{1}{(\xi - p - \Delta p)(\xi - p)^2} \right] (\frac{1}{\xi})^{l+1} e^{-\frac{x}{\xi}} d\xi$$

Now for all $\xi \in C$ $and - \infty < t < \infty$, $sup\left|e^{ax} t^k (-\mathrm{is})^q \, e^{-ist}\right| \le K$

where K is constant independent of ξ and x . Moreover $|\xi - p - \Delta p| > r_1 - r > 0$ and $|\xi - p| = r_1$, $C_1 = \max\{ \left|\frac{1}{\xi}\right|^{l+1} e^{-\frac{x}{\xi}}, \xi \in C \}$, Consequently

$$\sup_l \left|e^{ax} t^k D_t^q D_x^l \psi_{\Delta p}(x,t)\right|$$

$$= sup \left| e^{ax} t^k (-\mathrm{is})^q (e^{-ist}) \frac{\Delta p}{2\pi i} \int_c \frac{1}{(\xi - p - \Delta p)(\xi - p)^2} \left(\frac{1}{\xi}\right)^{l+1} e^{-\frac{x}{\xi}} d\xi \right|$$

$$\le \left|\frac{\Delta p}{2\pi i}\right| \int_C \frac{K C_1}{(r_1 - r)(r_1)^2} \, d\xi$$

$$\leq \frac{|\Delta p|}{2\pi}\int_{c}\frac{C_2}{(r_1-r)(r_1)^2}\,d\xi \quad , where\ C_2 = KC_1$$

$$\leq \frac{|\Delta p|}{2\pi(r_1-r)(r_1)^2}\,2\pi r_1 C_2$$

$$\leq \frac{|\Delta p|}{(r_1-r)(r_1)}\,C_2$$

The right-hand side converges to zero as $|\Delta p| \to 0$. This shows that $\psi_{\Delta p}(x,t)$ converges to zero as $|\Delta p| \to 0$. Which ends the proof.

V. CONCLUSIONS

In this work, we examined the analytical structure of the parameterized Sumudu transform, the Fourier transform, and their combined Sumudu–Fourier transform. The introduction of a generalized SFT within the distributional setting significantly broadens the scope of the transform, allowing it to operate on distributions of compact support. We demonstrated that both the classical and distributional SFTs possess strong analyticity properties with respect to the transform parameters p and s. By constructing appropriate complex neighborhoods and applying Cauchy's integral formula, we proved that the incremental differences converge to zero in the dual space, establishing holomorphicity in specified domains.

These results highlight the structural robustness of the SFT and confirm its suitability for the analysis of generalized differential models, integral equations, and operator theory. The analyticity theorems also create pathways for future developments, including inversion formulas, operational calculus, and applications to partial differential equations within classical and distributional frameworks.

REFERENCES

[1] Watugala, G.K., 1993, "Sumudu Transform: a new integral transform to solve differential equations and control engineering problems", International Journal of Math. Edu. Sci. Tech. 24, pp. 35-43.

[2] Watugala, G. K., 2002, "Sumudu transform for functions of two variables", Math Engineering Industries, 8, pp. 293-302, 2002.

[3] Weerakoon S., "Complex inversion formula for Sumudu transform", International Journal of Mathematical Education in Science and Technology, Vol. 29(4), 618-621.

[4] Kilicman, A., and Gadain, H. E., 2009, "An Application of Double Laplace Transform and Double Sumudu Transform", Lobachevskii Journal of Mathematics, Vol. 30, No. 3, pp. 214– 223.

[5] Zemanian, A.H., 1965, "Distribution theory and transform analysis", McGraw Hill, New York.

[6] Zemanian, A.H., 1968, "Generalized integral transform", Inter science publisher, New York.

[7] Lokenath Debnath and Dambaru Bhatta, 2007, "Integral Transforms and their Applications", Chapman and Hall/CRC Taylor and Francis Group Boca Raton London, New York.

[8] Sharma, V. D., and Rangari, A. N., 2014, "Generalized Fourier-Laplace Transform and its Analytical Structure", International Journal of Applied Mathematics and Mechanics, Vol. 3, No. 1, pp. 41-46.

[9] Sharma, V. D., and Rangari, A. N., 2011, "Analyticity of Distributional Fourier-Laplace Transform", International Journal of Mathematical Sciences and Engineering Applications, Vol. 5. No. V, pp. 57-62.

[10] Sharma, V. D., and Deshmukh, P. B, 2015, "S-Type spaces for two-dimensional fractional Fourier-Mellin Transform", International Journal of Advances in Science Engineering and Technology, Special Issue 1, pp. 74-76.

[11] Sharma, V. D., and Deshmukh, P. B., 2016, “Analyticity of Two-Dimensional Fractional Fourier-Mellin Transform”, International Journal of Engineering Sciences & Research Technology, Vol. 5, Issue 3, pp. 507-511.

[12] Sharma, V. D., and Dolas, P. D., 2017, “Analytical Structure of Distributional Two-Dimensional Fourier-Mellin Transform”, International Journal in Physical and Applied sciences, Vol. 4, Issue 06, pp. 10-14.

[13] Sontakke, P. K., and Gudadhe, A. S., 2008, “Analyticity and Operation Transform on Generalized fractional Hartley Transform”, Int. Journal of Math. Analysis, Vol. 2, no. 20, 977-986.

[14] Sharma, V.D., and Rangari, A.N., 2013, “Generalized Fourier-Laplace Transform and Topological Properties”, International J. of Modern Mathematical Sciences, 6(2):77-83.