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A Theoretical Study On Soft Set Operations With An Application To Candidate Screening

[1]Deepti Tripathi, [2]Rajni Bhargav

[1]Research Scholar, [2]Associate Professor,

[1, 2]Mathematics, MLB Govt Girls' PG Autonomous college, Bhopal, Madhya Pradesh, India,

Abstract: The parametric framework of soft set theory was developed by and it is a representation of uncertainty that does not entail the complicated ties of fuzzy and rough set theories. Therefore, it is suitable for multi-attribute decision-making processes. In this paper, a theoretical exploration of core soft set operations, which include AND, OR, union, intersection, complement, and their algebraic properties, is presented. In this process, the practical usefulness of core soft set operations in candidate screening is illustrated and demonstrated as part of the recruitment process. It makes some formalizations of preliminaries, such as soft subsets and null and absolute sets, and proves some major propositions, such as the laws of De Morgan, idempotency, and relations with extremal sets. It uses these operations, through illustrative examples, in a table form to compare job applicants with criteria such as experience, qualification, and ability, and refined selections are made using conjunctive criteria, gap analysis, and broaden search. We find that soft set algebra is consistent with classical theory and has greater uses in decision sciences. The publication is a bridge between the theoretical rigor and practical HR issues of the real world, and it has set the stage of automated transparent recruitment tools.

Index Terms **-** Soft Set, Complement, NOT Set, Null Soft Set, Absolute Soft Set, AND/OR Operations, Union, Intersection, De Morgan's Laws, Candidate Screening.

1. INTRODUCTION

Conventional set theory is often insufficient for solving uncertainties and ambiguities inherent to real-life problems in a world characterized by complex decision-making structures [4,9,10]. Our perception of human behaviors, traits, and preferences rarely follows a simple pattern; they tend to fall within the scope of various factors [7]. This limitation has led to the development of sophisticated mathematical machines that can simulate imprecise information [2,3,12]. The most powerful of these innovations is the so-called soft set theory, which is conceptualized through [12,14], and which enables flexible and approximation-based representations of subsets of the universe, free of the strict restrictions of fuzzy or rough sets [5,8]. Soft sets can be described as a parameterized set of subsets, which has been found to be highly useful in areas of decision analysis, data mining, and multi-criteria based evaluations, where variables such as skills experience or qualifications must be evaluated concurrently [9,14,15].

This theory has been enhanced through the use of new innovations such as fuzzy soft sets, which combine the level of membership and bridge the gap between classical soft sets and fuzzy logic [3,5,13]. These hybrid structures have been adopted across different industries, including medical assessments to analyze patterns, financial projections, and human resource management [11,12]. One of the most applicable uses is in the recruitment process of candidates where the reviewer has to consider the subjectivity of the evaluation, as well as incomplete information, and assess the suitability of the candidate based on multifactorial criteria, including technical skills, education qualifications, interpersonal skills, and certifications [11,16]. Traditional methods, such as strict scoring systems, do not take into account the interactions of these different parameters, thus producing less than optimal hiring results. Soft-set approaches result in a structured method for integrating, developing, and enhancing these benchmarks, which subsequently facilitates more complex and open-minded selection procedures [1,6,16].

Although the use of soft set applications is on the increase, a thorough theoretical exploration of the basic operations such as AND, OR, union, and intersection, as well as complement and its algebraic properties, is important in strengthening the conceptual standing of these applications. Even though generalized operations under relaxed settings and decision-making methods have been considered previously, very few studies have combined a focused algebraic analysis with a practical domain application [3,6,17]. This paper seeks to fill this gap by conducting a theoretical review of soft set operations, proving the key propositions including De Morgan law and idempotency, and showing that they can be applied to candidate screening based on useful examples.

The main objectives of the study are threefold: (1) to formalize and extend the definition of the soft sets and operations to ensure that they align with the traditional set theory; (2) to develop significant algebraic properties to support reliable calculations; and (3) to apply these operations to a candidate-screening process, which shows how this can be used to intersect the skill sets, unite the qualifications, and analyze any gaps. By exploring this research topic, the study contributes to the mathematical foundation of soft set theory and aligns theoretical concepts with practical situations that can be used by HR professionals to optimize their recruitment procedures.

The structure of future sections will be presented in this paper. In Section 2, the dotted line represents fundamental concepts, the definition of fuzzy soft sets, subsets, their complements, null sets, and absolute sets. Section 3 examines the operations on soft sets, such as AND, OR, union, and intersection, and determines the properties of these operations and their proofs. Section 4 is the concluding section and provides insights and possible directions for further research, as well as a reference list.

2. PRELIMINARIES

Definition 2.1 (Soft Set)[10]**:** A soft set is the fundamental building block of multiset theory. It provides a method for modeling real-world objects using multiple parameters simultaneously. Unlike conventional sets, where an element either belongs or does not belong, a soft set allows us to describe objects through various attributes or properties, making it ideal for complex decision-making scenarios.
Let U be a universal set and E be the set of parameters. Let $A \subseteq E$ and $P(U)$ be the power set of U. A pair (F, A) is called a soft set over U. where F is a mapping given by:

$$F : A \to P(U) \qquad [1]$$

In essence, a soft set is a parameterized family of subsets of the universe U.
Example 2.1. This example demonstrates how soft sets can be applied for candidate screening. Each parameter represents a desirable attribute for job candidates, and the mapping identifies the candidates that possess each attribute. Tabular representation provides a clear, computer-friendly format for storing and processing this information.
Let be a set of job candidates and $E = \{q_1, q_2, q_3.q_4, q_5\}$ be parameters representing highly experienced; holds a PhD; knows Python; leadership quality; certified in project management, respectively. Now Defining (F, E) as:

$F(q_1) = \{c_2, c_4\}$ (Highly Experienced Candidate)

$F(q_2) = \{c_1, c_3\}$ (Candidate with PhD)

$F(q_3) = \{c_3, c_4, c_5\}$ (Candidate Who Known Python)

$F(q_4) = \{c_1, c_3, c_5\}$ (Candidate with Leadership Quality)

$F(q_5) = \{c_1\}$ (Candidate Certified in Project Management)

This soft set can be represented in tabular form, as shown below:

Table 1: Representation of (F,E) soft set for candidate screening

Candidate	*Highly Experienced*	*Holds PhD*	*Knows Python*	*Leadership*	*PM Certified*
c_1	0	1	0	1	1
c_2	1	0	0	0	0
c_3	0	1	1	1	0
c_4	1	0	1	0	0
c_5	0	0	1	1	0
c_6	0	0	0	0	0

Definition 2.2 (Soft Subset & Equality)[9]**:** It extends the classical set theory concepts of subset and equality to soft sets. A soft subset requires both parameter containment and identical approximations for common parameters. This is crucial for comparing different soft-set models of the same real-world system**.**

For two sets (F,A) and (G,B) over U, (F,A) is a soft subset of (G,B), which is denotes as $(F,A)\subseteq(G,B)$, if:

1. $A\subset B$, and
2. $\forall\varepsilon\in A, F(\varepsilon)=G(\varepsilon)$.

They are soft equal if each is a soft subset of the other.

Definition 2.3 (NOT Set): The NOT set operation allows us to work with the negations of parameters. This is particularly useful when it wants to consider what attributes are absent rather than present, enabling more comprehensive analysis.

Let $E=\{q_1,q_2,.....q_n\}$ be a parameters set. The NOT set of E is defined as $|E=\{\neg q_1,\neg q_2,.....,\neg q_n\}$, where $\neg q_i$ denote "not q_i"

Definition 2.4 (Complement): The complement of a soft set represents all elements that do not satisfy the given parameters. It is computed using the NOT set and provides a way to identify candidates lacking certain attributes, which can be as important as identifying those who have them.

The complement of (F,A) is $(F,A)^c=(F^c,|A)$, where $F^c:|A\to P(U)$ is given by $F^c(\alpha)=U-F(\neg\alpha)$ for all $\alpha\in|A$.

Example 2.2. This shows how the complementary operation works in practice. The complement identifies all candidates who do not possess a specific attribute, which could be useful for identifying skill gaps or candidates who might require additional training.

Using Example 2.1, $(F,E)^c$ includes:

$F^c(\text{not highly experienced})=U-F(\text{highly experienced})=\{c_1,c_3,c_5,c_6\}$

Definition 2.5 (Null Soft Set): A null soft set represents a situation where none of the parameters apply to any elements in the universe. This can represent an empty search result or a set of requirements that no candidate satisfies.

A soft set (F,A) over U is a null soft set which is denoted by Φ, if $\forall\varepsilon\in A, F(\varepsilon)=\varnothing$.

Definition 2.6 (Absolute Soft Set): An absolute soft set represents the opposite extreme, where all parameters apply to every element. This might represent a universal truth or baseline condition in the system being modeled.

A soft Set (F,A) over U is a absolute soft set, which is denoted by $\tilde{A}$, if $\forall\varepsilon\in A, F(\varepsilon)=U$. It holds that $\tilde{A}^c=\Phi$, and $\Phi^c=\tilde{A}$

3. OPERATIONS ON SOFT SETS

Definition 3.1 (AND Operation)[10]**:** The AND operation combines two soft sets by requiring that elements satisfy both parameters simultaneously. This is equivalent to a logical AND operation, and is useful for finding elements that meet multiple criteria simultaneously.

If (F,A) and (G,B) are two soft sets then $(F,A)\wedge(G,B)=(H,A\times B)$,where $H(\alpha,\beta)=F(\alpha)\cap G(\beta)$ for all $(\alpha,\beta)\in A\times B.$

Definition 3.2 (OR Operation): The OR operation combines soft sets by including elements that satisfy at least one of the parameters. This broadens the search criteria and is useful when flexibility in the requirements is acceptable.

If (F,A) and (G,B) are two soft set, then $(F,A)\vee(G,B)=(O,A\times B)$, where $O(\alpha,\beta)=F(\alpha)\cup G(\beta)$ for all $(\alpha,\beta)\in A\times B.$

Example 3.1: This example demonstrates how "AND" and "OR" operations work in a hiring context. AND finds candidates who meet both technical and soft skill requirements simultaneously, whereas OR finds candidates who have at least one of the desired attributes, potentially casting a wider net.

Let $U=\{c_1,c_2,....,c_{10}\}$ be a set of candidates. Let (F,A) describe "Technical Skill" with parameters A={Expert Level. Intermediate, Beginner}, and (G,B) describe "Soft Skill" with B ={Excellent Communicator, Team Player, Creative}.

- $F(\text{Expert Level})=\{c_2,c_4,c_7,c_8\}$
- $G(\text{Excellent Comminicator})=\{c_2,c_3,c_7\}$

The, for the AND operation $(H,A\times B)$:

$$H(\text{Expert Level, Excellent Communicator})=F(\text{Expert Level})\cap G(\text{Excellent Communicator})$$
$$=\{c_2,c_7\}(\text{Candidate who are both Expert and Excellent Communicators})$$

For OR Operation $(O,A\times B)$:

$$O(\text{Expert Level. Excellent Communicator})=F(\text{Expert Level})\cup G(\text{Excellent Communicator})$$
$$=\{c_2,c_3,c_4,c_7,c_8\}(\text{Candidate who are either Expert Level OR anExcellent Communicator})$$

Proposition 3.1 (De Morgan's Laws): De Morgan's Laws establish important relationships between AND, OR, and complement operations in soft set theory, similar to their counterparts in classical set theory and Boolean algebra. These laws allow for the simplification of complex soft set expressions.

1. $((F,A)\vee(G,B))^c=(F,A)^c\wedge(G,B)^c$

Proof:

Let $(F,A)\vee(G,B)=(O,A\times B)$ where $O(\alpha,\beta)=F(\alpha)\cup G(\beta)$ for all $(\alpha,\beta)\in A\times B$.

Then, $((F,A)\vee(G,B))^c=(O^c,|(A\times B))$

Where,
$$\begin{aligned}O^c(\alpha,\beta)&=U-O(\alpha,\beta)\\&=U-(F(\alpha)\cup G(\beta))\\&=(U-F(\alpha))\cap(U-G(\beta))\\&=F^c(\alpha)\cap G^c(\beta)\end{aligned}$$

Now, $(F,A)^c\wedge(G,B)^c=(F^c,|A)\wedge(G^c,|B)=(J,|A\times|B)$

Where, $J(\alpha,\beta)=F^c(\alpha)\cap G^c(\beta)$

Since, $|A\times|B=|(A\times B)$ and $O^c(\alpha,\beta)=J(\alpha,\beta)$ for all $(\alpha,\beta)\in|(A\times B)$ the two soft sets are equal.

2. $((F,A)\wedge(G,B))^c=(F,A)^c\vee(G,B)^c$

Proof:

Let $(F,A)\wedge(G,B)=(H,A\times B)$, where $H(\alpha,\beta)=F(\alpha)\cap G(\beta)$ for all $(\alpha,\beta)\in A\times B$.

Then, $\left((F,A)\wedge(G,B)\right)^c=\left(H^c,\rceil(A\times B)\right)$

Where,
$$H^c(\alpha,\beta)=U-H(\alpha,\beta)$$
$$=U-\left(F(\alpha)\cap G(\beta)\right)$$
$$=\left(U-F(\alpha)\right)\cap\left(U-G(\beta)\right)$$
$$=F^c(\alpha)\cap G^c(\beta)$$

Now, $(F,A)^c\vee(G,B)^c=\left(F^c,\rceil A\right)\vee\left(G^c,\rceil B\right)=\left(K,\rceil A\times\rceil B\right)$

Where, $K(\alpha,\beta)=F^c(\alpha)\cup G^c(\beta)$

Since, $\rceil A\times\rceil B=\rceil(A\times B)$ and $H^c(\alpha,\beta)=K(\alpha,\beta)$ for all $(\alpha,\beta)\in\rceil(A\times B)$ the two soft sets are equal.

Definition 3.3 (Union): The union operation combines two soft sets while preserving their original parameter sets. This is different from the OR operation, as it maintains the separate identity of parameters from each soft set rather than creating parameter pairs.

The union of two sets (F,A) and (G,B) over U is (H,C) where $C=A\cup B$ and $\forall e\in C$:

$$H(e)=\begin{cases}F(e), & \text{if } e\in A-B\\ G(e), & \text{if } e\in B-A\\ F(e)\cup G(e), & \text{if } e\in A\cap B\end{cases}$$

Denoted as $(F,A)\dot{\cup}(G,B)=(H,C)$.

Definition 3.4 (Intersection): The intersection operation finds common ground between two soft sets, considering only the parameters they share. This is useful for identifying elements that satisfy the common criteria of both soft sets.

The intersection of (F,A) and (G,B) over U is (H,C), where $C=A\cap B$, and $\forall e\in C$, $H(e)=F(e)$ (Which equals $G(e)$), Denotes as $(F,A)\dot{\cap}(G,B)=(H,C)$.

Proposition 3.2 (Corrected Properties): These properties establish the algebraic structure of soft set operations. The correction to property (3) is particularly important, as it ensures mathematical consistency when combining any soft set with a null soft set.

The following properties and their proof hold for soft sets (F,A), (G,B), and (H,C) over U:

1. $(F,A)\dot{\cup}(F,A)=(F,A)$ (Idempotency of Union)

Proof:

Let $(F,A)\dot{\cup}(F,A)=(H,C)$ where $C=A\cup A=A$. For any $e\in C$;

Since, $e\in A\cap A$, $H(e)=F(e)\cup F(e)=F(e)$

Thus, $(H,C)=(F,A)$

2. $(F,A)\dot{\cap}(F,A)=(F,A)$ (Idempotency of Intersection)

Proof:

Let $(F,A)\dot{\cap}(F,A)=(H,C)$ where $C=A\cap A=A$. For any $e\in C$, ;

Since, $H(e)=F(e)\cap F(e)=F(e)$

Thus, $(H,C)=(F,A)$

3. $(F,A)\dot{\cup}\Phi=(F,A)$ (Union with Null Set)

Proof:

Let $(F,A)\dot{\cup}\Phi=(H,C)$ where $C=A\cup A_\Phi$ and A_Φ is the parameters set of Φ. For any $e\in C$:

- If $e\in A-A_\Phi$, Then $H(e)=F(e)$
- If $e\in A_\Phi-A$, Then $H(e)=\Phi(e)=\varnothing$
- If $e\in A\cap A_\Phi$, Then $H(e)=F(e)\cup\Phi(e)=F(e)\cup\varnothing=F(e)$

Since Φ has no parameters that affect non-null values, $(H,C)=(F,A)$

4. $(F,A)\dot{\cap}\Phi=\Phi$ (Intersection with Null Set)

Proof:

Let $(F,A)\dot{\cap}\Phi=(H,C)$ where $C=A\cap A_{\Phi}$. For any $e\in C$, $H(e)=F(e)$ or $\Phi(e)$. Since $\Phi(e)=\varnothing$ for all *e*, and the intersection requires identical approximations, $H(e)=\varnothing$. Thus $(H,C)=\Phi$.

5. $(F,A)\dot{\cup}\tilde{A}=\tilde{A}$ (Union with Absolute Set)

Proof:

Let $(F,A)\dot{\cup}\tilde{A}=(H,C)$ where $C=A\cup A$. For any $e\in C$;

Since, $e\in A\cap A$, $H(e)=F(e)\cup A(e)=F(e)\cup U=U$

Thus, $(H,C)=\tilde{A}$

6. $(F,A)\dot{\cap}\tilde{A}=(F,A)$ (Intersection with Absolute Set)

Proof:

Let $(F,A)\dot{\cap}\tilde{A}=(H,C)$ where $C=A\cap A=A$. For any $e\in C$, $H(e)=F(e)$ or $\tilde{A}(e)$.

Since the intersection requires identical approximations and $\tilde{A}(e)=U$, we must have $F(e)=U$, but this may not be true. However, by definition of interaction, for $e\in A\cap A$, it take $H(e)=F(e)$ (as both should be same set)

Thus, $(H,C)=(F,A)$.

4 CONCLUSION

The current paper has made a conceptual systematic theoretical discussion on soft set operations, which strengthens their premises in uncertainty modeling and decision analysis. With the formalization of definitions of soft sets, subsets, complements, null and absolute sets, and rigorous proof of algebraic properties, such as the laws of De Morgan and the idempotent property, we have obtained a consistent theory, which makes soft set theory consistent with classical set axioms. The conjunctive AND and OR disjunctive assessments are supported by the AND and OR operations, whereas the union and intersection operations maintain the parametric structures to draw fine chances of comparison, as shown in the candidate screening application. The exemplary cases highlight the effectiveness of soft sets in real-life situations: in recruitment, they allow identifying the subset of applicants that fits a variety of specifications (AND), searching more extensive databases (OR), and identifying areas of skill gaps (complements), which are graphically displayed in table formats that can be converted into computationally executable. These not only reduce the biases of standard screening, but also enable scalable and data-driven HR functions, but could also be scaled to group decision-making or hybrid fuzzy models. Future studies could incorporate machine learning to facilitate the dynamic adaptation of parameters or hypersoft sets to hyper-complex attributes. Finally, the research continues to develop soft set theory as an instrument of an imprecise environment, which is open to innovations within decision support systems.

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