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Reliability Of Time – Dependent Stress – Strength System For Finite Mixture Model

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ABSTRACT:

In reliability, a life time distribution can be characterized by the reliability function. In this paper considered a situation when stress follow mixture of weibull distribution to find reliability of a system. It has been studied when strength follow weibull distribution and stress follow finite mixture of weibull distribution. The general expression for the reliability of a system is obtained. The reliability is computed numerically for different values of the stress strength parameters.

Keywords: Reliability, Stress – strength system, weibull distribution.

INTRODUCTION

Reliability of a system is the probability that a system will adequately perform its intended purpose for a given period of time under stated environmental conditions [1]. In some cases system failures occur due to certain type of stresses acting on them. Thus system composed of random strengths will have its strength as random variable and the stress applied on it will also be a random variable. A system fails whenever an applied stress exceeds strength of the system.

Reliability engineering deals with the design and construction of systems and products, taking into account the unreliability of its parts and components. It also Includes testing and programs to improve reliability. Good engineering results in a more reliable end product. In the real world , all products and systems are unreliable in the sense that they degrade with age and usage and ultimately fail. The reliability of a product depends on a complex interaction of the laws of physics, engineering design, manufacturing processes, management decisions, random events and usage. Reliability theory is concerned with the study of the various aspects of product reliability. Results of reliability studies are important inputs to decisions regarding design, manufacturing, marketing, strategy and post sale support requirements, all of which impact costs. Significant involvement of management in this activity is therefore also essential. This is a convergent process oriented to solve specific problems or achieve specified goals. A repairable system is a system that , when a failure occurs, can be restored to a satisfactory operating condition in a specified interval of active repair time[4].

In a finite mixture model, the distribution of random quantity of interest is modeled as a mixture of a finite number of component distributions in varying proportions [2]. The flexibility and high degree of accuracy of finite mixture models have been the main reason for their successful applications in a wide range of fields in the biological physical and social sciences. The estimation of reliability based on finite mixture of pareto distributions was studied by Sankaran, P.G. et.al. (2005)[3].

Notations:

Rn reliability of the nth repair system

X stress component

Y strength component

pi mixture of i components

λ stress parameter

μ strength parameter

In reliability theory, the mixture distributions are used for the analysis of the failure times of a sample of items. Mixture of two weibull distributions with probability density function of the form .

$$f(x) = \lambda_1 \alpha x^{\alpha-1} e^{-\lambda_1 x^\alpha} + \lambda_2 \alpha x^{\alpha-1} e^{-\lambda_2 x^\alpha}, x, \lambda_i, \alpha > 0, i = 1,2$$

The above model will be useful to predict how long all manufactured lasers should be life tested to assure that the final product contained no device from the infant mortality population.

In the present paper the statistical analysis of finite mixture of weibull distribution in the context of reliability theory. We derive the reliability, when stress X follows finite mixture of weibull distribution and the strength Y follows weibull distribution.

If X denotes the stress of the component and Y is the strength imposed on it, then the reliability of the component is given by [1],

$$R = P(X < Y)$$
$$= \int_0^\infty f(x) \left(\int_x^\infty g(y)\, dy \right) dx$$

A finite mixture of exponential distribution with k- component can be represented in the form

$$f(x) = p_1 f_1(x) + p_2 f_2(x) + \;\dots\dots.. + p_k f_k(x)$$

Where $p_i > 0, i = 1,2, \dots k: \sum_{i=1}^{k} p_i = 1$

Statistical Model

In the case of Random fixed stress and random independent strength , the reliability of nth repair system is

$$R_n = \int_0^\infty f(x) \left(\int_x^\infty g(y)\, dy \right)^n dx$$

When stress is mixture of one component

$$f(x) = p_1\lambda_1\alpha x^{\alpha-1}e^{-\lambda_1 x^\alpha} \; \& \; g(y) = \mu\alpha y^{\alpha-1}e^{-\mu y^\alpha}$$

$$R_n = \int_0^\infty p_1\lambda_1\alpha x^{\alpha-1}e^{-\lambda_1 x^\alpha}\left(\int_x^\infty \mu\alpha y^{\alpha-1}e^{-\mu y^\alpha}\,dy\right)^n dx$$

$$R_n = \int_0^\infty p_1\lambda_1\alpha x^{\alpha-1}e^{-\lambda_1 x^\alpha}\left(e^{-\mu x^\alpha}\right)^n dx$$

$$R_n = \frac{p_1\lambda_1}{(\lambda_1 + n\mu)}$$

When Stress is mixture of one component,

Reliability of 1st repair system is

$$R_1 = \frac{p_1\lambda_1}{(\lambda_1 + \mu)}$$

2nd repair system is

$$R_2 = \frac{p_1\lambda_1}{(\lambda_1 + 2\mu)}$$

3rd repair system is

$$R_3 = \frac{p_1\lambda_1}{(\lambda_1 + 3\mu)}$$

4th repair system is

$$R_4 = \frac{p_1\lambda_1}{(\lambda_1 + 4\mu)}$$

When Stress is mixture of two components and strength is weibull distribution

$$f(x) = p_1\lambda_1\alpha x^{\alpha-1}e^{-\lambda_1 x^\alpha} + p_2\lambda_2\alpha x^{\alpha-1}e^{-\lambda_2 x^\alpha} \; \& \; g(y) = \mu\alpha y^{\alpha-1}e^{-\mu y^\alpha}$$

$$R_n = \int_0^\infty \left(p_1\lambda_1\alpha x^{\alpha-1}e^{-\lambda_1 x^\alpha} + p_2\lambda_2\alpha x^{\alpha-1}e^{-\lambda_2 x^\alpha}\right)\left(\int_x^\infty \mu\alpha y^{\alpha-1}e^{-\mu y^\alpha}\,dy\right)^n dx$$

$$R_n = \int_0^\infty \left(p_1\lambda_1\alpha x^{\alpha-1}e^{-\lambda_1 x^\alpha} + p_2\lambda_2\alpha x^{\alpha-1}e^{-\lambda_2 x^\alpha}\right)\left(e^{-\mu x^\alpha}\right)^n dx$$

$$R_n = \frac{p_1\lambda_1}{(\lambda_1 + n\mu)} + \frac{p_2\lambda_2}{(\lambda_2 + n\mu)}$$

When stress is mixture of two components,

Reliability of 1st repair system is

$$R_1 = \frac{p_1\lambda_1}{(\lambda_1+\mu)} + \frac{p_2\lambda_2}{(\lambda_2+n\mu)}$$

2nd repair system is

$$R_2 = \frac{p_1\lambda_1}{(\lambda_1+2\mu)} + \frac{p_2\lambda_2}{(\lambda_2+2\mu)}$$

3rd repair system is

$$R_3 = \frac{p_1\lambda_1}{(\lambda_1+3\mu)} + \frac{p_2\lambda_2}{(\lambda_2+3\mu)}$$

4th repair system is

2

$$R_4 = \frac{p_1\lambda_1}{(\lambda_1+4\mu)} + \frac{p_2\lambda_2}{(\lambda_2+4\mu)}$$

Stress is mixture of three components and strength is weibull distribution.

$$f(x) = p_1\lambda_1\alpha x^{\alpha-1}e^{-\lambda_1x^\alpha} + p_2\lambda_2\alpha x^{\alpha-1}e^{-\lambda_2x^\alpha} + p_3\lambda_3\alpha x^{\alpha-1}e^{-\lambda_3x^\alpha}\ \&$$

$$g(y) = \mu\alpha y^{\alpha-1}e^{-\mu y^\alpha}$$

$$R_n = \int_0^\infty \left(p_1\lambda_1\alpha x^{\alpha-1}e^{-\lambda_1x^\alpha} + p_2\lambda_2\alpha x^{\alpha-1}e^{-\lambda_2x^\alpha} + p_3\lambda_3\alpha x^{\alpha-1}e^{-\lambda_3x^\alpha}\right)\left(\int_x^\infty \mu\alpha y^{\alpha-1}e^{-\mu y^\alpha}\,dy\right)^n dx$$

$$R_n = \int_0^\infty \left(p_1\lambda_1\alpha x^{\alpha-1}e^{-\lambda_1x^\alpha} + p_2\lambda_2\alpha x^{\alpha-1}e^{-\lambda_2x^\alpha} + p_3\lambda_3\alpha x^{\alpha-1}e^{-\lambda_3x^\alpha}\right)\left(e^{-\mu x^\alpha}\right)^n dx$$

$$R_n = \frac{p_1\lambda_1}{(\lambda_1+n\mu)} + \frac{p_2\lambda_2}{(\lambda_2+n\mu)} + \frac{p_3\lambda_3}{(\lambda_3+n\mu)}$$

When stress is mixture of three components,

Reliability of 1st repair system is

$$R_1 = \frac{p_1\lambda_1}{(\lambda_1+\mu)} + \frac{p_2\lambda_2}{(\lambda_2+\mu)} + \frac{p_3\lambda_3}{(\lambda_3+\mu)}$$

2nd repair system is

$$R_2 = \frac{p_1\lambda_1}{(\lambda_1+2\mu)} + \frac{p_2\lambda_2}{(\lambda_2+2\mu)} + \frac{p_3\lambda_3}{(\lambda_3+2\mu)}$$

3rd repair system is

$$R_3 = \frac{p_1\lambda_1}{(\lambda_1+3\mu)} + \frac{p_2\lambda_2}{(\lambda_2+3\mu)} + \frac{p_3\lambda_3}{(\lambda_3+3\mu)}$$

4^{th} repair system is

$$R_4 = \frac{p_1\lambda_1}{(\lambda_1 + 4\mu)} + \frac{p_2\lambda_2}{(\lambda_2 + 4\mu)} + \frac{p_3\lambda_3}{(\lambda_3 + 4\mu)}$$

Similarly Stress is mixture of k components then the reliability of nth repair system is

$$R_n = \sum_{i=1}^{k} \frac{p_i\lambda_i}{(\lambda_i + n\mu)}$$

RELIABILITY COMPUTATIONS

Table1

p1	μ	R1	R2	R3	R4
0.2	0.1	0.166667	0.142857	0.125	0.111111
0.4	0.2	0.285714	0.222222	0.181818	0.153846
0.6	0.3	0.375	0.272727	0.214286	0.176471
0.8	0.4	0.444444	0.307692	0.235294	0.190476
1	0.5	0.5	0.333333	0.25	0.2
1.2	0.6	0.545455	0.352941	0.26087	0.206897
1.4	0.7	0.583333	0.368421	0.269231	0.212121
1.6	0.8	0.615385	0.380952	0.275862	0.216216
1.8	0.9	0.642857	0.391304	0.28125	0.219512
2	1	0.666667	0.4	0.285714	0.222222

Figure 1

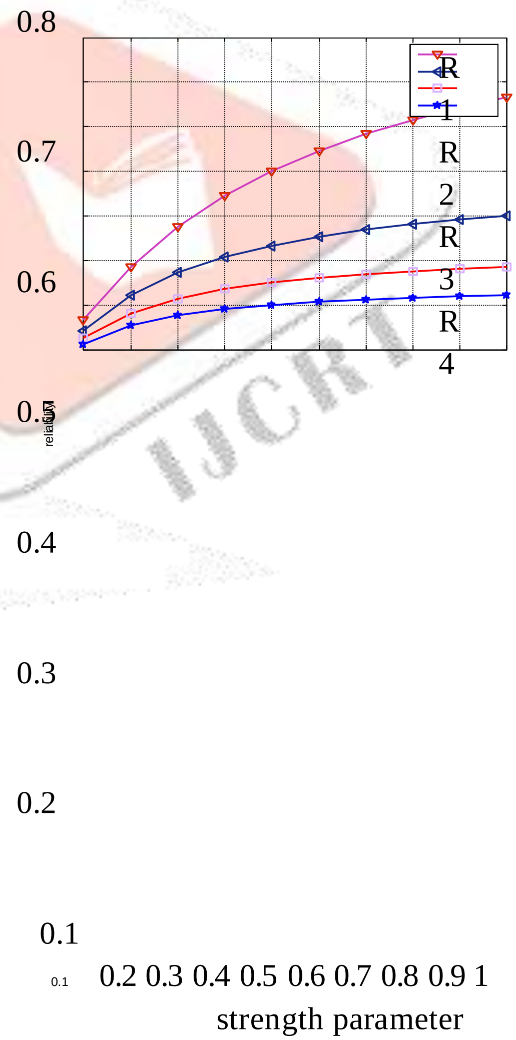


Table 1 & figure 1 shows , the stress is mixture of one component then to find the 1^{st} repair system, 2^{nd} repair system, 3^{rd} repair system & 4^{th} repair system & take λ1=0.5 (stress parameter fixed)

Table2

p1	p2	μ	R1	R2	R3	R4
0.1	0.9	0.1	0.803333	0.671429	0.576786	0.505556
0.2	0.8	0.2	0.67619	0.511111	0.410909	0.34359
0.3	0.7	0.3	0.5875	0.416364	0.322527	0.263235
0.4	0.6	0.4	0.522222	0.353846	0.267647	0.215238
0.5	0.5	0.5	0.472222	0.309524	0.230263	0.183333
0.6	0.4	0.6	0.432727	0.276471	0.203162	0.160591
0.7	0.3	0.7	0.400758	0.250877	0.182615	0.143561
0.8	0.2	0.8	0.374359	0.230476	0.166502	0.13033
0.9	0.1	0.9	0.352198	0.213834	0.153528	0.119756
1	0	1	0.333333	0.2	0.142857	0.111111

Figure 2

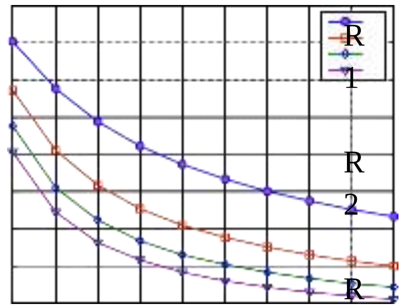


Table 2 & figure 2 shows, the stress is mixture of two components then to find the 1st repair system, 2nd repair system, 3rd repair system & 4th repair system and take λ1=0.5 , λ2=0.4(stress parameters fixed)

Table3

p1	p2	p3	μ	R1	R2	R3	R4
0.01	0.09	0.9	0.1	0.755333	0.607143	0.507679	0.43627
0.02	0.18	0.8	0.2	0.614286	0.443968	0.347758	0.285874
0.03	0.27	0.7	0.3	0.523036	0.35497	0.268791	0.216324
0.04	0.36	0.6	0.4	0.459365	0.299021	0.221765	0.176261
0.05	0.45	0.5	0.5	0.4125	0.260623	0.19057	0.150217
0.06	0.54	0.4	0.6	0.376606	0.232647	0.168368	0.131932
0.07	0.63	0.3	0.7	0.348258	0.211362	0.151762	0.118388
0.08	0.72	0.2	0.8	0.325315	0.194627	0.138872	0.107954
0.09	0.81	0.1	0.9	0.306374	0.181124	0.128579	0.099668
0.1	0.9	0	1	0.290476	0.17	0.120168	0.092929

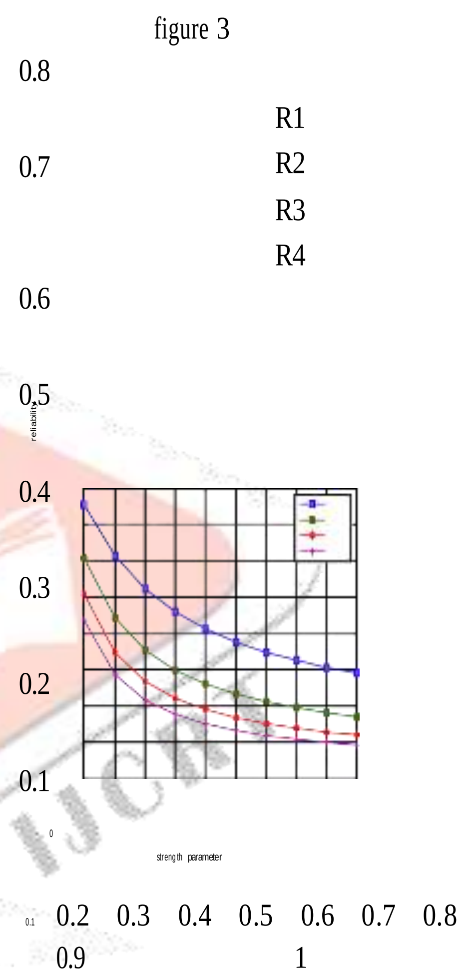
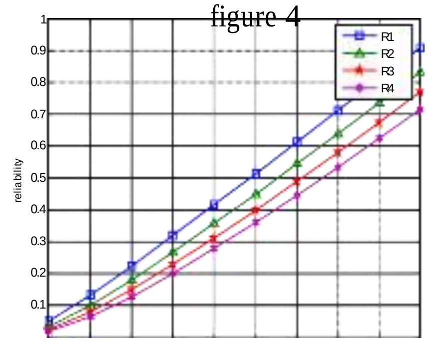


Table 3 & figure shows the stress is mixture of three components then to find the 1st repair system, 2nd repair system, 3rd repair system & 4th repair system and take λ1=0.5 , λ2=0.4 , λ3=0.3. (stress parameters fixed)

Table 4

p1	λ1	R1	R2	R3	R4
0.1	0.1	0.05	0.033333	0.025	0.02
0.2	0.2	0.133333	0.1	0.08	0.066667
0.3	0.3	0.225	0.18	0.15	0.128571
0.4	0.4	0.32	0.266667	0.228571	0.2
0.5	0.5	0.416667	0.357143	0.3125	0.277778
0.6	0.6	0.514286	0.45	0.4	0.36
0.7	0.7	0.6125	0.544444	0.49	0.445455
0.8	0.8	0.711111	0.64	0.581818	0.533333
0.9	0.9	0.81	0.736364	0.675	0.623077
1	1	0.909091	0.833333	0.769231	0.714286



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stress parameters

Table 4 & figure 4 shows the stress is mixture of two components then to find the 1st repair system, 2nd repair system, 3rd repair system & 4th repair system and take μ=0.1 (strength parameter fixed)

Table 5

p1	p2	p3	λ1	λ2	λ3	R1	R2	R3	R4
0.01	0.09	0.9	0.1	1	0.01	0.168636	0.12119	0.100763	0.088237
0.02	0.18	0.8	0.2	2	0.02	0.318095	0.246364	0.214522	0.194762
0.03	0.27	0.7	0.3	3	0.03	0.445329	0.362429	0.324091	0.29993
0.04	0.36	0.6	0.4	4	0.04	0.554648	0.469524	0.428329	0.401818
0.05	0.45	0.5	0.5	5	0.05	0.64951	0.568407	0.527207	0.5
0.06	0.54	0.4	0.6	6	0.06	0.732576	0.659888	0.620952	0.594424
0.07	0.63	0.3	0.7	7	0.07	0.805906	0.744722	0.709866	0.685172
0.08	0.72	0.2	0.8	8	0.08	0.871111	0.823582	0.794263	0.772381
0.09	0.81	0.1	0.9	9	0.09	0.929467	0.897062	0.874448	0.856207
0.1	0.9	0	1	10	0.1	0.981998	0.965686	0.950709	0.936813

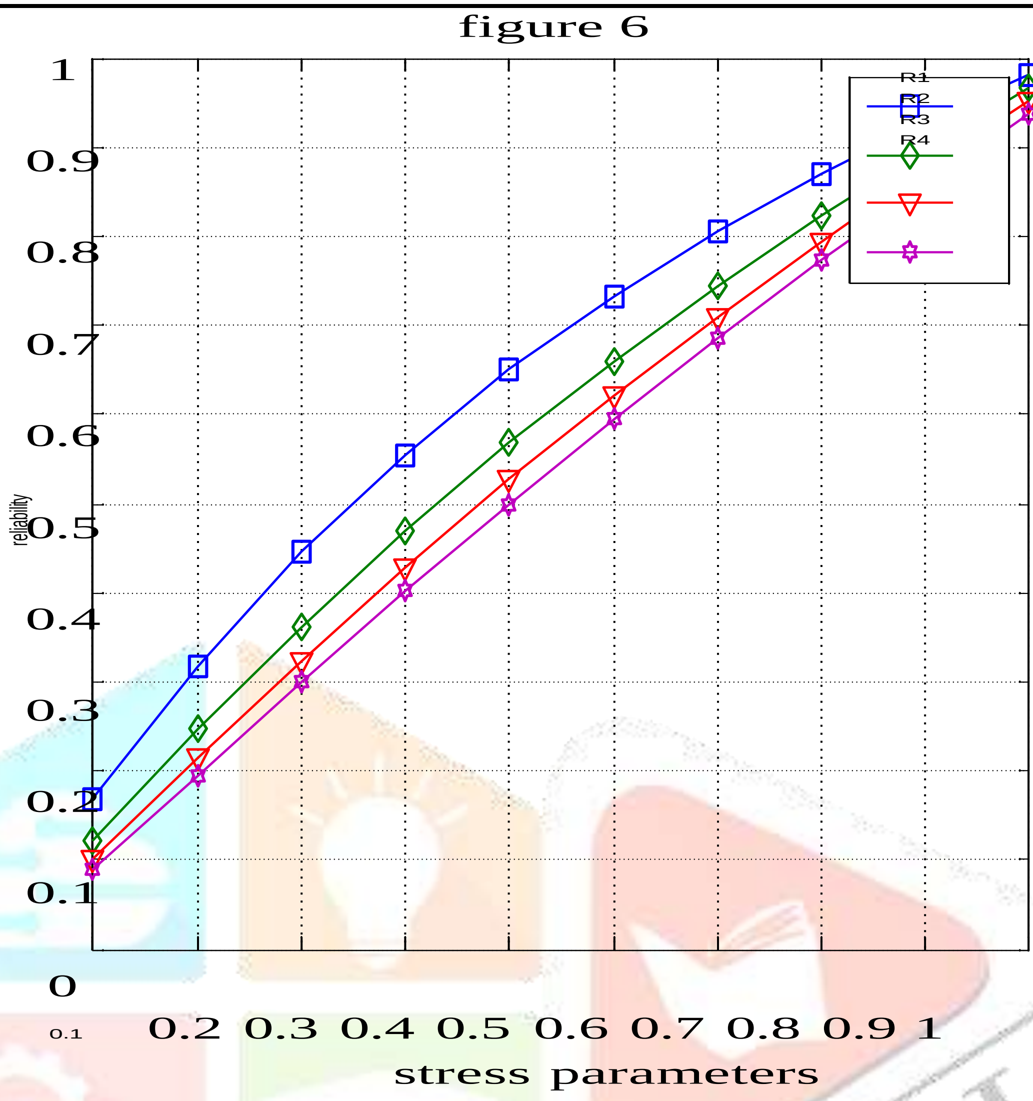

Table 5 & figure 5 shows the stress is mixture of three components then to find the 1st repair system, 2nd repair system, 3rd repair system & s 4th repair system and take μ=0.1(strength parameter fixed)

Table 6

p1	p2	p3	p4	λ1	λ2	λ3	λ4	R1	R2	R3	R4
0.01	0.07	0.02	0.9	0.1	1	0.01	0.1	0.520455	0.362619	0.281991	0.232488
0.02	0.14	0.04	0.8	0.2	2	0.02	0.2	0.686667	0.540909	0.452239	0.391905
0.03	0.21	0.06	0.7	0.3	3	0.03	0.3	0.764572	0.642701	0.561364	0.502337
0.04	0.28	0.08	0.6	0.4	4	0.04	0.4	0.808028	0.706667	0.635591	0.581818
0.05	0.35	0.1	0.5	0.5	5	0.05	0.5	0.834804	0.749396	0.688224	0.640741
0.06	0.42	0.12	0.4	0.6	6	0.06	0.6	0.8524	0.779144	0.726667	0.685402
0.07	0.49	0.14	0.3	0.7	7	0.07	0.7	0.864496	0.800463	0.75535	0.719819
0.08	0.56	0.16	0.2	0.8	8	0.08	0.8	0.873086	0.816056	0.77708	0.746667
0.09	0.63	0.18	0.1	0.9	9	0.09	0.9	0.87934	0.827621	0.793716	0.767791
0.1	0.7	0.2	0	1	10	0.1	1	0.883978	0.836275	0.806535	0.784505

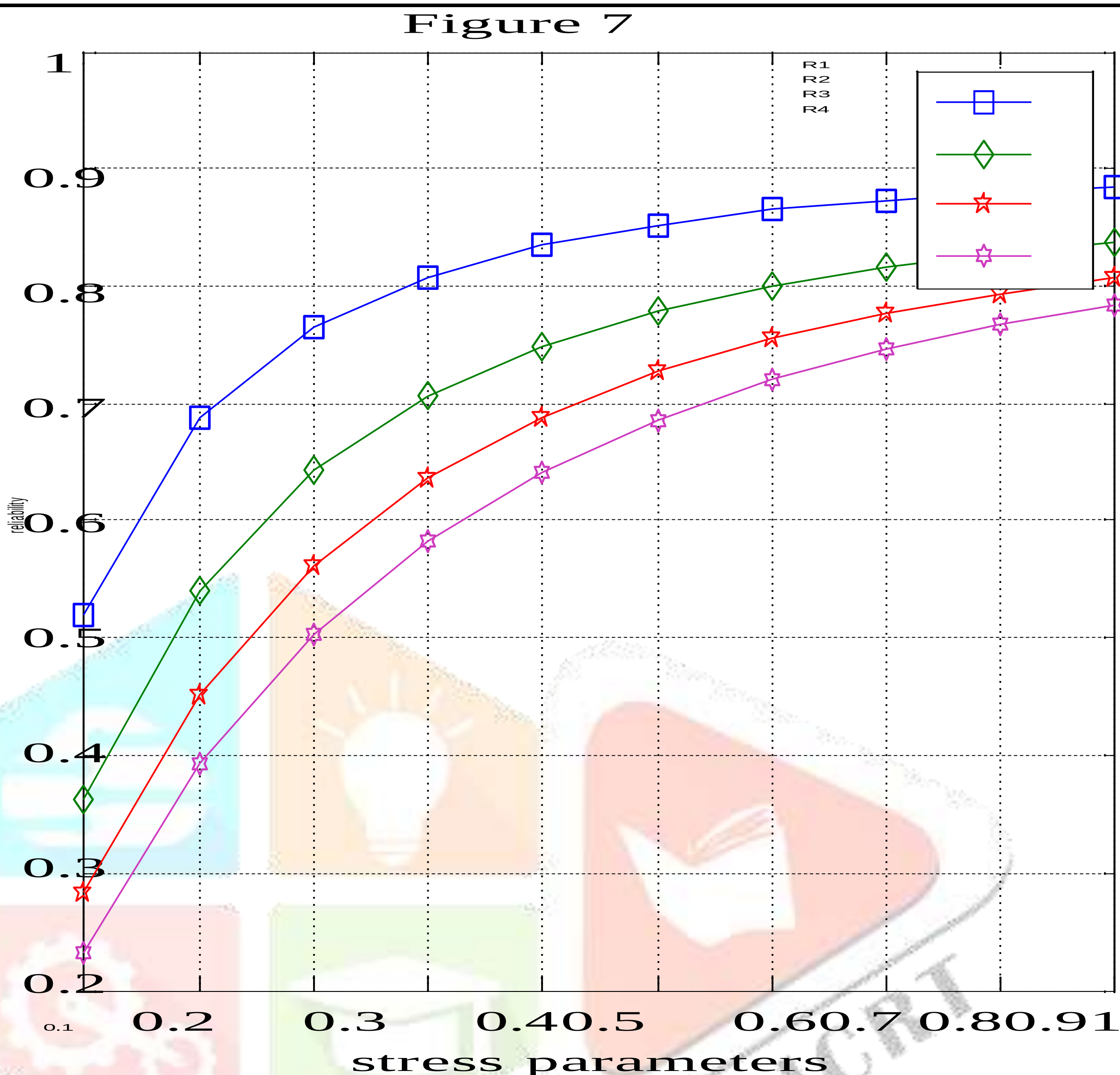

Table 6 & figure 6 shows the stress is mixture of four components then to find the 1st repair system, 2nd repair system, 3rd repair system & s 4th repair system and take μ=0.1 (strength parameter fixed)

CONCLUSION

In this paper, Reliability can be studied when strength follows weibull distribution and stress follows finite mixture of weibull distribution for repairable system. From tables and figures, we can observe, we cannot use mixture components then reliability increase when strength parameter increase. We can use mixture of stress components reliability increases when stress parameter increases and reliability decreases when strength parameter increases. We can find the reliability for different cycles/repairable systems

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