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Ancient Indian Rules For Finding The Segment And Altitude Of A Triangle

S. K. Dahal

Department of Mathematics, K.M.C. College, Bagbzar, Kathmandu, Nepal

Abstract. We find the notion of the triangle in Indian literature for the first time, perhaps, in the Vedas. The terms used are *trirasri* in the *Rigveda* and *tribhuja* in the *Atharvaveda*. *Sulba-sutras* were also familiar with important types of triangle - mostly the right-angled triangle. We have discussed here the rules of *Bhaskaracharya* and *Brahmagupta* for finding the segment and altitude of the triangles and critically reviewed the rules in terms of modern mathematics.

Key words: *Trirasri*, *Tribhuja*, *Sulba-sutra*, Segment, Altitude.

1. Introduction

In this paper we discuss the rules of *Bhaskaracharya* and *Brahmagupta* for finding the segment and altitude of a triangle - especially scalene triangle - and review them in the light of modern mathematics [2].

2. Ancient Indian Rules

Bhaskaracharya has written in his commentary, "In a triangle the difference of the squares of the two sides (or the product of their sum and difference) is equal to the product of the sum and difference of their segments of the base. Divide it by the base (or the sum of the segments). Add and subtract this quotient to and from the base and make them half. This will give the value of the two segments. From the segments of the base the altitude of a scalene triangle can be found out."

Let ABC be a scalene triangle in which BC = a & AC = b are two sides and BD = c_1 & AD = c_2 their segments respectively of the base AB (vide Fig. 1).

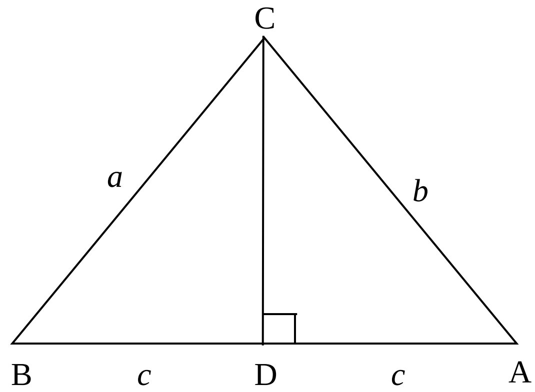

Fig. 1

Then by the *Bhaskaracharya* rule

$$\begin{aligned} a^2 - b^2 &= (a+b)(a-b) \\ &= c_1^2 - c_2^2 \\ &= (c_1 + c_2)(c_1 - c_2) \end{aligned}$$

$$\because c_1 + c_2 = c$$

$$\therefore c_1 - c_2 = \frac{a^2 - b^2}{c}$$

Hence,

$$c_1 = \frac{1}{2}\left(c + \frac{a^2 - b^2}{c}\right)$$

and $$c_2 = \frac{1}{2}\left(c - \frac{a^2 - b^2}{c}\right)$$

Now,
$$h=\sqrt{a^2-c_1^2}$$
$$=\sqrt{b^2-c_2^2}$$

By means of these formulae *Bhaskaracharya* has found the segments of the base, altitude and area of a scalene triangle [1].

Brahmagupta has also given the same set of formulae. He has written, "The difference of the squares of the two sides of a triangle being divided by its base, when the quotient is added to and subtracted from the base, the results, divided by two, give the segments of the base. The square root of the square of a side as diminished by the square of the corresponding segment is the altitude."

Let AD be the altitude of the triangle ABC and BD & CD the segments of its base BC (vide Fig. 2). Then according to *Brahmagupta*,

$$\text{BD or CD}=\frac{1}{2}\left(a\pm\frac{b^2-c^2}{a}\right)$$

Further,

$$\text{AD}=\sqrt{(\text{AB}^2-\text{BD}^2)}$$
$$=\sqrt{(\text{AC}^2-\text{CD}^2)}$$

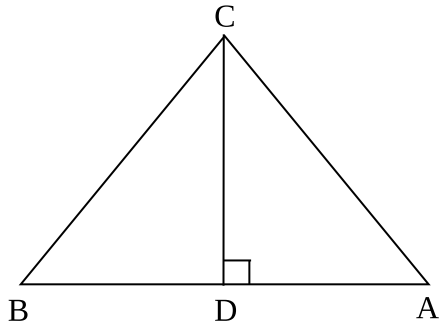

Fig. 2

These results are obtained from a consideration of the right-angled triangles ABD and ACD in the *Yuktibhasa* [2]. From the above formulae for altitude we have

BD^2 = c^2 – AD^2 and (1)

CD^2 = b^2 – AD^2. (2)

Subtracting (2) from (1) we get

BD2 – CD^2 = c^2 – b^2

or, (BD + CD)(BD – CD) = c2 – b2

or, a(BD – CD) = c^2 – b^2 [∵ BD + CD = BC = a]

$$\text{or, BD}-\text{CD}=\frac{c^2-b^2}{a} \qquad (3)$$

$$\text{or, BD}+\text{BD}-\text{BD}-\text{CD}=\frac{c^2-b^2}{a}$$

$$2\text{BD}-a=\frac{c^2-b^2}{a}$$

or,

$$\therefore \text{BD}=\frac{a+\dfrac{c^2-b^2}{a}}{2}$$

Now, from (3) we can write

$$\text{BD}+\text{CD}-\text{CD}-\text{CD}=\frac{c^2-b^2}{a}$$

$$\text{or, } a-2\text{CD}=\frac{c^2-b^2}{a}$$

$$\text{or, } 2\text{CD}=a-\frac{c^2-b^2}{a}$$

$$\text{or, CD}=\frac{a-c^2+b^2}{2a}$$

or, $2a\mathrm{CD} = a^2 - c^2 + b^2$

or, $2\mathrm{BC} \cdot \mathrm{CD} = \mathrm{BC}^2 - \mathrm{AB}^2 + \mathrm{AC}^2$

or, $\mathrm{AB}^2 - \mathrm{AC}^2 = \mathrm{BC}^2 - 2\mathrm{BC} \cdot \mathrm{CD}$

$= \mathrm{BC}^2 - 2\mathrm{BC} \cdot \mathrm{CD} + \mathrm{CD}^2 - \mathrm{CD}^2$

$= (\mathrm{BC} - \mathrm{CD})^2 - \mathrm{CD}^2$

$= \mathrm{BD}^2 - \mathrm{CD}^2$

or, $\mathrm{AB}^2 - \mathrm{BD}^2 = \mathrm{AC}^2 - \mathrm{CD}^2$

or, $\mathrm{AD}^2 = \mathrm{AC}^2 - \mathrm{CD}^2$ [by (1)]

$\therefore \mathrm{AD} = \sqrt{[(\text{Side})^2 - (\text{Its segment})^2]}$

3. Conclusion

Considering a triangle, among the altitude, side and corresponding segment the above is the relation which coincides with the modern mathematics. So, we can boldly conclude that the ancient Indian mathematicians were quite rich in geometry. At present use of this method various field of mathematics & physics.

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[2] Sen, S.N. (1971). History of Science in India, S. Chand, New Delhi.