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Mathematical Study of Rheology of Tomato Ketchup Through a Circular Pipe by Power-Law Fluid Model

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Abstract: The nature of product's flow characteristics is significant in transportation system design of food processing operations and equipment. The study aims to investigate mathematically the rheology of tomato ketchup through a circular pipe using power-law fluid model. The rheological flow parameters of tomato ketchup exhibiting pseudoplastic nature of flow under certain prescribed condition are considered. The equations guiding fluid motion are obtained in cylindrical coordinate system using appropriate flow assumptions. Using relevant boundary conditions, the flow dominant equations are solved analytically to get expressions related to flow behaviour computation. MATLAB's "bvp4c" solver is utilised to do the mathematical computations. The impact of rheological flow feature factors is analyzed from graphs and tables. The findings of this investigation will be beneficial for transportation system design and evaluation of fluid food products through a circular pipe.

Index Terms - Tomato Ketchup, Rheology, Pseudoplastic, Power-Law fluid, Flow index.

I. INTRODUCTION

The mathematical study of food rheology has applications in food preparation, quality assurance, and sensory evaluation. The design of the transportation system for fluid food products is heavily influenced by the nature and description of the product's flow characteristics. For the design of products, the assessment of manufacturing procedures, and the formulation of packaging and storage plans, it is pivotal to examine the flow characteristics based on rheological flow parameters of fluid fruit and vegetable products. Most of the food scientists have studied rheology of liquid food products experimentally. Thus, the situation demands to study the flow behavior of fruit juices mathematically considering geometry as per requirement of food processing industries.

One of the most popular vegetable products is the tomato, which is mostly sold in processed forms including pastes, puree and ketchup. The spicy product known as tomato ketchup is made mostly from extracted tomatoes, which can be either hot or cold. Among all tomato products, tomato ketchups were perhaps the first to gain popular favour and continue to hold a sizable portion of the market. Since tomato ketchups are non-Newtonian, time-independent fluids, their rheological characteristics are crucial for handling, storing, processing, and transportation in food industry.

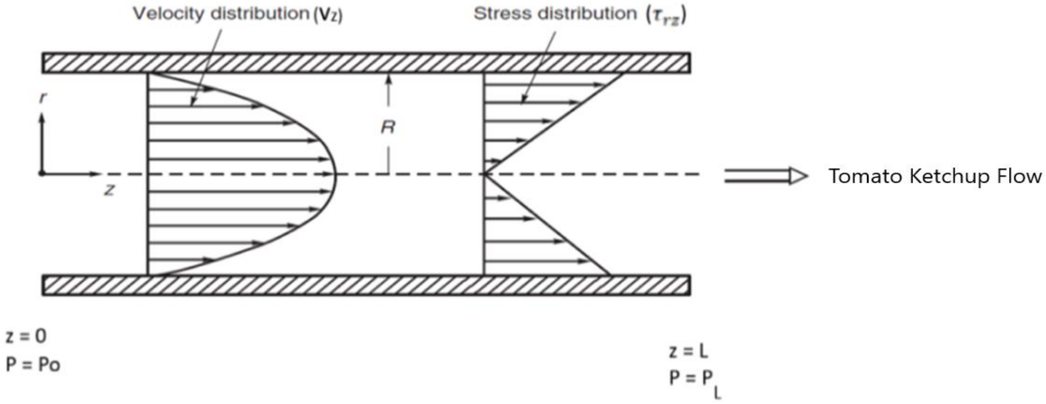
A comprehensive assessment of the latest research on rheology of products made from fruit and vegetables has been presented by Diamante and Umemoto (2015). Abdulagatov et al. (2008) investigated how temperature, pressure, and concentration influenced fruit juice viscosity. The rheologic properties of concentrated mandarin juice at lower temperatures were investigated by Falguera et al. (2010). Vandresen et al. (2009) illustrated the temperature effects on carrot juices. The impact of temperature and concentration on the orange juice concentrate flow was demonstrated by Vitali et al. (1984). Mahmoud (2011) investigated the influence of slip factor on the non-Newtonian flow past a moving porous surface with heat generation. The critical examination of fluids' rheological characteristics of clarified passion fruit was carried out by Ibarz et al. (1996). The master-curve model considering the rheological characteristics of concentrates of pummelo juice was established by Chin et al. (2009). Koocheki et al. (2009) investigated tomato ketchup flow behaviour considering experimentally obtained rheological flow parameters. For industrial applications, Goula and Adamopoulos (2011) established the rheomertical models of kiwifruit juice. Augusto et al. (2012) conducted research on the properties of fluid rheology for both steady and unsteady states of tomato juice models. Abdullah, N. et al. (2018) examined the rheological behaviour of guava, pomelo and soursop juice concentrates. Hazarika, B. et al. (2020) discussed in details about unexplored underutilized fruit crops of Assam. Massa et al. (2010), examined rheological characterization of peach purees using Power-Law Fluid model. Zhang et. al. (2019) examined the heat transmission of a non-Newtonian power law fluid through pipes with various cross sections.

This paper aims to investigate mathematically the rheological behaviour of tomato ketchup through a circular pipe using power-law fluid model and to find the impact of involved flow parameters on the velocity, average velocity, volumetric flow rate and friction factor of tomato ketchup.

2. MATHEMATICAL FORMULATION

The tomato ketchup flow through a circular pipe due to pressure difference is considered. The shear-thinning nature of flow of tomato ketchup under certain prescribed conditions is investigated employing power-law fluid model. The pipe under consideration is of length 10 m, diameter 80 mm, radius 0.04 m and the fluid motion is initiated taking pressure difference 1 kPA. To obtain the governing equations of fluid motion, the following assumptions are considered.

i. The flow is incompressible, laminar and steady.
ii. Gravity is negligible and iso-thermal condition.
iii. The flow is fully developed and symmetry in θ-direction.
iv. The flow direction is in z-direction.



Clearly, continuity equation in cylindrical coordinate system is satisfied due to above assumptions. The momentum equation along the coordinate axes are obtained as follows:

$$\text{r-component: } \frac{\partial p}{\partial r} = 0 \text{ i.e. p is not a function of r} \tag{2.1}$$

$$\theta\text{- component: } \frac{\partial p}{\partial \theta} = 0 \text{ i.e. p is not a function of } \theta \qquad (2.2)$$

Clearly, p is a function of z only

$$\text{z-component: } \frac{\partial p}{\partial z} = \frac{1}{r}\frac{\partial}{\partial r}(r\tau_{rz}) \qquad (2.3)$$

3. METHOD OF SOLUTION

Since LHS is a function of z and RHS is a function of r only so the equation above takes the form

$$\frac{dp}{dz} = \frac{1}{r}\frac{d}{dr}(r\tau_{rz}) \qquad (3.1)$$

Again, since p is a function of z and RHS is independent of z so LHS of (3.1) is a constant and thus using boundary conditions at z = 0: p = P_0 and z = L: p = P_L, we get

$$\frac{dp}{dz} = \left(\frac{P_L - P_0}{L}\right) \qquad (3.2)$$

Since RHS of (2.4) is function of r only and LHS is independent of r, thus

$$\frac{d}{dr}(r\tau_{rz}) = r\frac{dp}{dz} \qquad (3.3)$$

Integrating (4.3), shear stress τ_{rz} is obtained as follows

$$\tau_{rz} = \left(\frac{P_L - P_0}{2L}\right) r \qquad (3.4)$$

Putting r = R, shear stress at the wall is obtained as

$$\tau_\omega = \left(\frac{P_L - P_0}{2L}\right) R \qquad (3.5)$$

The shear stress for Power-Law fluid is

$$\tau_{rz} = k\left(-\frac{dv_z}{dr}\right)^n \qquad (3.6)$$

Solving using the boundary condition $r = R$, we get

$$v_z = \left(\frac{\tau_\omega}{k}\right)^{\frac{1}{n}} \left(\frac{nR}{n+1}\right) \left\{1 - \left(\frac{r}{R}\right)^{\frac{n+1}{n}}\right\} \qquad (3.7)$$

Volumetric flow rate in case of Power-Law fluid is

$$Q = \left(\frac{\pi R^3 n}{3n+1}\right) \left[\left(\frac{P_L - P_0}{2Lk}\right) R\right]^{\frac{1}{n}} \qquad (3.8)$$

Using above result average velocity is obtained as

$$v_{avg} = \left(\frac{nR}{3n+1}\right) \left[\left(\frac{P_L - P_0}{2Lk}\right) R\right]^{\frac{1}{n}} \qquad (3.9)$$

Using (3.9) in (3.7), we get

$$v_z = v_{avg}\left(\frac{3n+1}{n+1}\right) \left\{1 - \left(\frac{r}{R}\right)^{\frac{n+1}{n}}\right\} \qquad (3.10)$$

Friction factor for Power-Law fluid is

$$f = \frac{16}{Re_{PL}} \qquad (3.11)$$

where, $Re_{PL} = \frac{\rho (v_{avg})^{2-n} D^n}{8^{n-1} k \left(\frac{3n+1}{4n}\right)^n}$, symbols have their usual meaning.

4. RESULTS AND DISCUSSION

The graphical representation of velocity distribution for various values of consistency coefficient and flow index of tomato ketchup as shown in Table 4.1 are plotted to examine the flow behaviour in Fig. 4.1- 4.4. As the ketchup moves along the axis of the circular pipe, backward flow velocity is observed above the axis of the pipe whereas below the axis it moves forward and touches the lower surface. The flow development in the circular pipe is diminishing with the reducing values of consistency coefficients (k) and flow index (n). The variation of consistency coefficients (k) and flow index parameters (n) do not affect the flow pattern significantly.

The velocity at different radial distance from the axis is plotted in Fig. 4.2. The values of velocity are well accord with the pipe flow characteristic of fluid. As the ketchup enters the pipe, initially it moves forward along the axis but gradually backward fluid motion is noticed above the pipe's axis but it moves forward below the axis and meet the bottom surface of the pipe for different values of consistency coefficients (k) and flow index (n). Further, it shows that the velocity of the tomato ketchup is maximum when it is processed at 35^0 C.

The profiles of volumetric rate of flow and average velocity are shown in Figs. 4.5 and 4.6. The volumetric flow rate and average velocity is maximum when the ketchup is processed at moderate temperature of 35^0 C. Temperature plays a significant role on the volumetric flow rate and average velocity as the maximum volumetric flow rate and average velocity is obtained at 35^0 C. The values of skin friction factor for different values of consistency coefficients (k) and flow index (n) are depicted in Table 4.2. The friction factor value is minimum at 35^0 C and maximum at 25^0 C. The derived values of average velocity and volumetric flow rate are in good agreement with the outcome.

Table 4.1: Rheological parameters of tomato ketchup [1]

Product	Temp., °C	k, Pa.s^n	n
Tomato Ketchup	25	19.34	0.228
	35	11.38	0.225
	45	9.53	0.219
	55	8.42	0.213

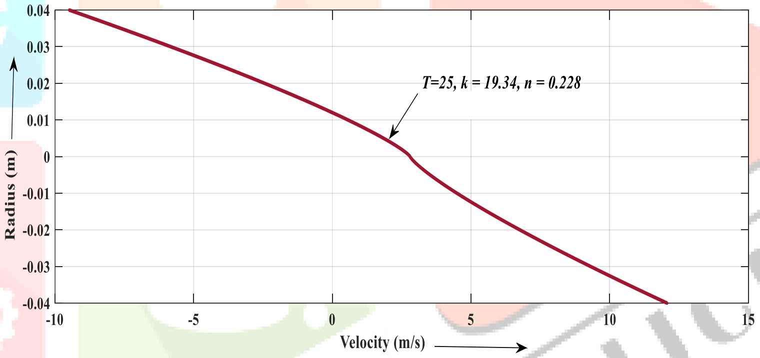


Fig. 4.1 Velocity profile k = 19.34 and n = 0.228

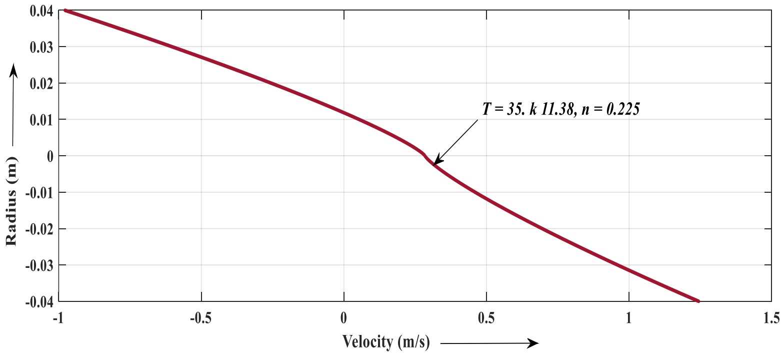


Fig. 4.2 Velocity profile for k = 11.38 and n = 0.225

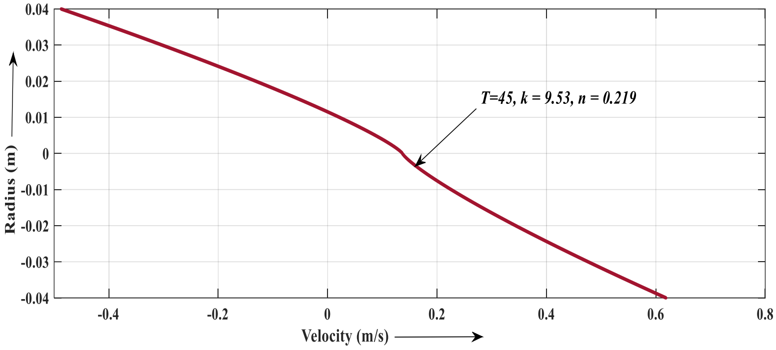


Fig. 4.3 Velocity profile for k = 9.35 and n = 0.219

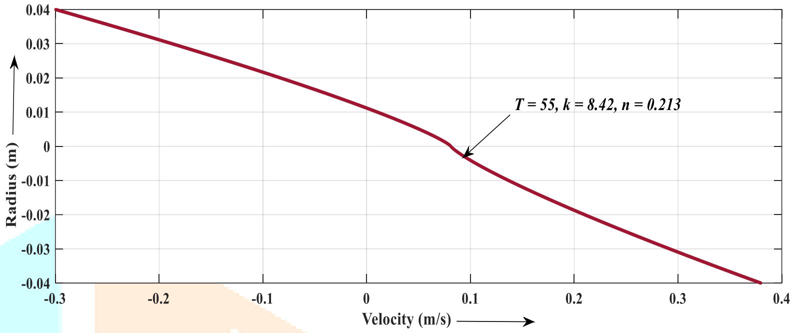


Fig. 4.4 Velocity profile for k = 8.42 and n = 0.213

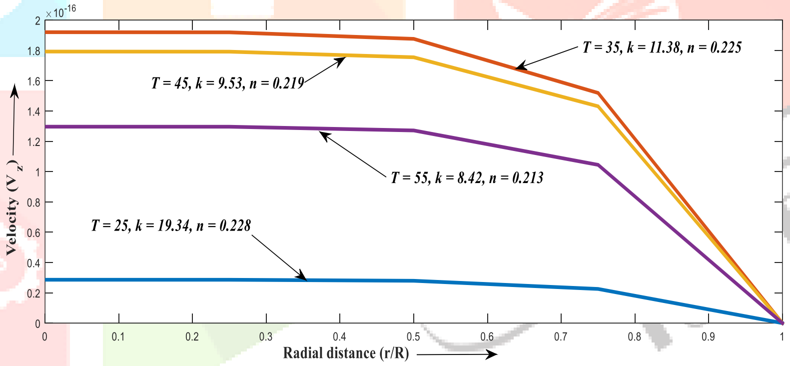


Fig. 4.5 Velocity profile at different radial distance

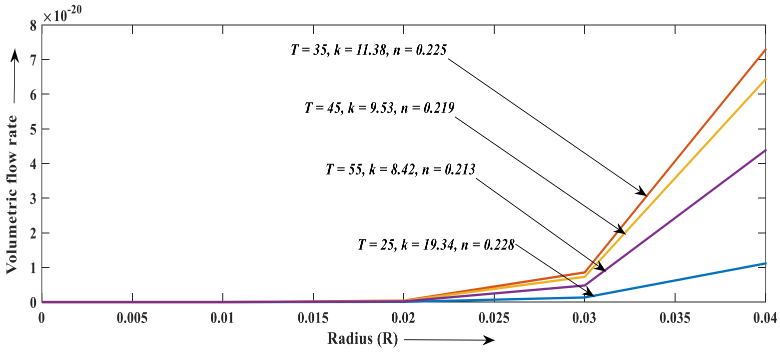


Fig. 4.6 Volumetric flow rate profile for variation of rheological parameters

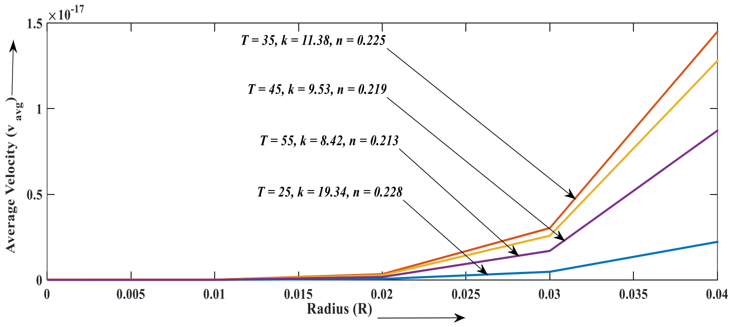


Fig. 4.7 Average velocity profile for variation of rheological parameters

Table 4.2 Calculation of friction factor

Temp., ^{o}C	k, Pa.s^{n}	n	f
25	19.34	0.228	2.3330e+36
35	11.38	0.225	6.2180e+34
45	9.53	0.219	1.0437e+35
55	8.42	0.213	2.9815e+35

5. CONCLUSIONS

The following conclusion can be drawn from this study:

- The flow of tomato ketchup diminishes gradually with the reducing values of flow index and consistency coefficients.
- Backward flow pattern is observed above the pipe's axis but it moves forward below the axis and contacts the bottom surface.
- The deformation rate of tomato ketchup is high when it is processed at moderate temperature.
- Volumetric flow rate and average velocity of tomato ketchup is maximum at moderate temperature.
- Friction factor value is maximum at low temperature but it is minimum at moderate temperature which validate the obtained results.

6. SCOPE FOR FUTURE WORK

- This work can be extended further due to its diversified applications in food and engineering sciences.
- Different fruit and vegetable products of similar kind can also be studied with the help of same fluid model.
- As per requirement of food industry, different flow geometry can also be considered.
- Flow simulation can be useful in visualising the flow pattern for various rheological parameter values.

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