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Solving For Stoichiometric Coefficients Of Chemical Diophantine Equation $\alpha\left(X^4+Y^4\right)^2\left(5U^2+V^2\right)=T^2\left(C^2+D^2\right)\left(Z^2-W^2\right)P^\beta$ With $\alpha>0, \beta=1,2,3,4,5,6,7$ and $x<y<w<z$

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Abstract:

Diophantine equations bridge algebraic number theory and chemical analysis, providing rigorous solutions to problems in organic chemistry that demand integer consistency and efficiency. Quintic Diophantine equations represent an advanced mathematical framework occasionally relevant for specialized problems in organic chemistry, bridging high-degree polynomial theory with chemical integer constraints**.** In this paper, we are focused to find Solving for Stoichiometric Coefficients of Chemical Diophantine Equation with more than 8 unknowns and focused on a study to find integer design of solutions Diophantine Equation

$\alpha(X^4+Y^4)^2(5U^2+V^2)=T^2(C^2+D^2)(Z^2-W^2)P^\beta$ **With** $\alpha>0, \beta=1,2,3,4,5,6,7$ **and** $x<y<w<z$ with Mathematical induction of trial-and-error method & generation of Pythagorean triplets.

Keywords: Diophantine Equation, exponential, Pythagorean triplet, Integer design.

I. Introduction:

Diophantine equations—polynomial equations with integer solutions—are a central topic in number theory. Among their many variants, **exponential Diophantine equations** involve terms where variables appear as exponents. Diophantine equations are powerful tools in organic chemistry—especially for balancing chemical equations and determining the molecular formulas of organic compounds, where the goal is to find integer solutions that satisfy the conservation of atoms and charge in chemical reactions.

Balancing Organic Chemical Equations

Balancing chemical equations involves ensuring that the number of atoms of each element is the same on both sides of a reaction. This requirement for integer coefficients in chemical equations naturally leads to systems of linear Diophantine equations. For example, in a complex organic reaction, finding the minimum integer coefficients for reactants and products can be translated into a system of equations, which is solved using Diophantine methods.

- Organic reactions commonly involve elements like C, H, O, N, and others—each element is associated with a balance equation representing the count of atoms before and after the reaction.
- Solving for stoichiometric coefficients (the integers in front of molecular formulas) using methods from algebraic number theory helps maintain both mass conservation and stoichiometric precision in these reactions.

Molecular Formula Determination

Diophantine equations also help deduce molecular formulas for organic compounds by solving for the number of each atom present, given constraints like molecular mass or elemental analysis. These problems ask for integer solutions where each variable represents the count of a particular atom type.

- For instance, when determining the formula for a hydrocarbon with a given molecular mass and percent composition, the unknowns (number of C, H, O atoms, etc.) must satisfy linear equations based on their atomic weights, with only integer solutions acceptable.

Context and Role in Organic Chemistry

- Most common chemical reaction balancing leads to linear Diophantine systems, but more complex organic chemistry scenarios involving higher-degree polynomial relations or nonlinear constraints can give rise to quintic (degree five) or higher-degree Diophantine equations.
- Such quintic Diophantine equations involve integer variables raised to the fifth power and appear in advanced modeling, such as optimization problems in reaction networks, molecular structure analysis, or theoretical studies of reaction mechanisms that involve nonlinear relationships among reactant/product counts.
- However, typical stoichiometric balancing in organic chemistry rarely requires solving quintic equations explicitly; their appearance is more frequent in theoretical and computational chemistry research or in complex algebraic models of chemical properties or transformations.

In this paper, focused to find the general exponential integer solution of

The general exponential integer solution of $\boldsymbol{\alpha(X^4+Y^4)^2(5U^2+V^2)=T^2(C^2+D^2)(Z^2-W^2)P^{\beta}}$

With $\boldsymbol{\alpha>0}$, is derived from fixed value of $\boldsymbol{\beta=1,2,3,4,5,6,7}$ and $\boldsymbol{x<y<w<z}$.

II. Results & Discussions:

Proportion 1: A Study on integer design of solution of above Diophantine Equation at

$\boldsymbol{\beta = 1}$ **is** $\boldsymbol{\alpha(X^4 + Y^4)^2(5U^2 + V^2) = T^2(C^2 + D^2)(Z^2 - W^2)}\ P$

Explanation: Let $x = k^n, y = k^{n+1}, z = k^{n+3}, w = k^{n+2}, p = k^{6n}, U = 4^{n+1}, V = 4^n, T = 3(4)^n$.

Consider $\alpha(X^4 + Y^4)(\mathbf{5U^2 + V^2}) = \alpha k^{8n}(1 + k^4)^2(3(4)^n)^2$

Again consider $(Z^2 - W^2)P = k^{8n}(k^6 - k^4)$.

It follows that $\boldsymbol{\alpha(X^4 + Y^4)^2(5U^2 + V^2) = T^2(C^2 + D^2)(Z^2 - W^2)}\ P$ implies that

$\alpha k^{8n}(1 + k^4)^2(3(4)^n)^2 = (C^2 + D^2)k^{8n}(k^6 - k^4)(3(4)^n)^2$ implies $\alpha(1 + k^4)^2 = (C^2 + D^2)(k^6 - k^4)$.

Solve for α, whenever $(C, D, 1 + k^4)$ is a Pythagorean Triplet.

From the References [1],[2],[3],[4],[5],[6],[7],[8], we know that

$(C, D, 1 + k^4)$ becomes a Pythagorean Triplet with $C = (2k^2\ ),\ D = (k^4 - 1)$,

$\mathbf{C^2 + D^2 = (1 + k^4)^2}$. Hence $\alpha = (k^6 - k^4)$.

Hence $\alpha(\mathbf{X^4 + Y^4})^2(\mathbf{5U^2 + V^2}) = \mathbf{T^2}(C^2 + D^2)(Z^2 + W^2)P$ having integer design of solution is parameterized by integers k and n, with variables defined as:

$x = k^n, y = k^{n+1}, z = k^{n+3}, w = k^{n+2}, p = k^{6n}, \alpha = (k^6 - k^4), C = 2k^2, D = k^4 - 1,$

$U = 4^{n+1}, V = 4^n, T = 3(4)^n$

Verification: Consider **LHS**

$\alpha(\mathbf{X^4 + Y^4})^2(\mathbf{5U^2 + V^2}) = (k^6 - k^4)(\mathbf{k^{4n} + k^{4n+4}})^2(3(4)^n)^2 = \mathbf{k^{8n}}(k^6 - k^4)(\mathbf{1 + k^4})^2(3(4)^n)^2.$

Consider RHS

$$\mathbf{T^2}(C^2 + D^2)(Z^2 - W^2)P = (3(4)^n)^2(\mathbf{1 + k^4})^2(k^{2n+6} - k^{2n+4})k^{6n}$$
$$= \mathbf{k^{8n}}(k^6 - k^4)(\mathbf{1 + k^4})^2(3(4)^n)^2.$$

Hence LHS = RHS.

Proportion 2: A Study on exponential integer solution of above Diophantine Equation at

$\boldsymbol{\beta = 2}$ **is** $\alpha(\mathbf{X^4 + Y^4})^2(\mathbf{35 + V^2}) = \mathbf{T^2}(C^2 + D^2)(Z^2 - W^2)P^2$

Explanation: Let $x = k^n, y = k^{n+1}, z = k^{n+3}, w = k^{n+2}, p = k^{3n}, U = 4^{n+1}, V = 4^n, T = 3(4)^n$

Consider $\alpha(\mathbf{X^4 + Y^4})^2(\mathbf{5U^2 + V^2}) = \alpha k^{8n}(1 + k^4)^2(3(4)^n)^2$.

Again consider $(Z^2 - W^2)\boldsymbol{P^2} = k^{8n}(k^6 - k^4)$.

It follows that $\alpha(\boldsymbol{X^4 + Y^4})^2(\boldsymbol{5U^2 + V^2}) = \boldsymbol{T^2}(C^2 + D^2)(Z^2 - W^2)P^2$ implies that

$\alpha k^{8n}(1 + k^4)^2(3(4)^n)^2 = (C^2 + D^2)k^{8n}(k^6 - k^4)(3(4)^n)^2$ implies $\alpha(1 + k^4)^2 = (C^2 + D^2)(k^6 - k^4)$. Solve for α, whenever $(C, D, 1 + k^4)$ is a Pythagorean Triplet.

From the References [1],[2],[3],[4],[5],[6],[7],[8], we know that

$(C, D, 1 + k^4)$ becomes a Pythagorean Triplet with $C = (2k^2\ ),\ D = (k^4 - 1)$,

$\boldsymbol{C^2 + D^2 = (1 + k^4)^2}$Hence $\alpha = (k^6 - k^4)$.

Verification: Consider **LHS**

$\alpha(\boldsymbol{X^4 + Y^4})^2(\boldsymbol{5U^2 + V^2}) = (k^6 + k^4)(\boldsymbol{k^{4n} + k^{4n+4}})^2(3(4)^n)^2 = (3(4)^n)^2\boldsymbol{k^{8n}}(k^6 - k^4)(\boldsymbol{1 + k^4})^2.$

Consider RHS

$\boldsymbol{T^2}(C^2+D^2)(Z^2-W^2)P^2 = (3(4)^n)^2(\mathbf{1}+\boldsymbol{k^4})^2(k^{2n+6}-k^{2n+4})k^{6n} = (3(4)^n)^2\boldsymbol{k^{8n}}(k^6-k^4)(\mathbf{1}+\boldsymbol{k^4})^2$.

Hence LHS $= \boldsymbol{RHS}$.

Proportion 3: A Study on exponential integer solution of above Diophantine Equation at

$\boldsymbol{\beta = 3}$ **is** $\boldsymbol{\alpha(X^4+Y^4)^2(5U^2+V^2) = T^2(C^2+D^2)(Z^2-W^2)P^3}$

Explanation: Let x = $k^n, y = k^{n+1}, z = k^{n+3}, w = k^{n+2}, p = k^{2n}, U = 4^{n+1}, V = 4^n, T = 3(4)^n$.

Consider $\alpha(\boldsymbol{X^4}+\boldsymbol{Y^4})^2(\boldsymbol{5U^2}+\boldsymbol{V^2}) = \alpha k^{8n}(1+k^4)^2(3(4)^n)^2$.

Again consider $(Z^2-W^2)\boldsymbol{P^3} = k^{8n}(k^6-k^4)$.

It follows that $\alpha(\boldsymbol{X^4}+\boldsymbol{Y^4})^2(\boldsymbol{5U^2}+\boldsymbol{V^2}) = \boldsymbol{T^2}(\boldsymbol{C^2}+\boldsymbol{D^2})(\boldsymbol{Z^2}-\boldsymbol{W^2})\boldsymbol{P^3}$ implies that

$\alpha k^{8n}(1+k^4)^2(3(4)^n)^2 = (3(4)^n)^2(C^2+D^2)k^{8n}(k^6-k^4)$ implies $\alpha(1+k^4)^2 = (C^2+D^2)(k^6-k^4)$. Solve for α, whenever $(C, D, 1+k^4)$ is a Pythagorean Triplet.

From the References [1],[2],[3],[4],[5],[6],[7],[8], we know that

$(C, D, 1+k^4)$ becomes a Pythagorean Triplet with $C = (2k^2)$, $D = (k^4-1)$,

$\boldsymbol{C^2}+\boldsymbol{D^2} = (\mathbf{1}+\boldsymbol{k^4})^2$Hence $\alpha = (k^6-k^4)$.

Verification: Consider **LHS**

$\alpha(\boldsymbol{X^4}+\boldsymbol{Y^4})^2(\boldsymbol{5U^2}+\boldsymbol{V^2}) = (k^6-k^4)(\boldsymbol{k^{4n}}+\boldsymbol{k^{4n+4}})^2(3(4)^n)^2 = \boldsymbol{k^{8n}}(k^6-k^4)(\mathbf{1}+\boldsymbol{k^4})^2\,(3(4)^n)^2$.

Consider RHS

$\boldsymbol{T^2}(C^2+D^2)(Z^2-W^2)P^3 = (3(4)^n)^2\,(\mathbf{1}+\boldsymbol{k^4})^2(k^{2n+6}-k^{2n+4})k^{6n} = (3(4)^n)^2\boldsymbol{k^{8n}}(k^6-k^4)(\mathbf{1}+\boldsymbol{k^4})^2$.

Hence LHS $= \boldsymbol{RHS}$.

Proportion 4 A Study on exponential integer solution of above Diophantine Equation at

$\boldsymbol{\beta = 4}$ **is** $\boldsymbol{\alpha(X^4+Y^4)^2(5U^2+V^2) = T^2(C^2+D^2)(Z^2-W^2)P^4}$.

Explanation: Let x = $k^n, y = k^{n+1}, z = k^{2n+3}, w = k^{2n+2}, p = k^n, U = 4^{n+1}, V = 4^n, T = 3(4)^n$

Consider $\alpha(\boldsymbol{X^4}+\boldsymbol{Y^4})^2(\boldsymbol{5U^2}+\boldsymbol{V^2}) = \alpha k^{8n}(1+k^4)^2\,(3(4)^n)^2$.

Again consider $(Z^2-W^2)\boldsymbol{P^4} = k^{8n}(k^6-k^4)$.

It follows that $\boldsymbol{\alpha(X^4+Y^4)^2(5U^2+V^2) = T^2(C^2+D^2)(Z^2-W^2)P^4}$ implies that

$\alpha k^{8n}(1+k^4)^2(3(4)^n)^2 = (3(4)^n)^2(C^2+D^2)k^{8n}(k^6-k^4)$ implies $\alpha(1+k^4)^2 = (C^2+D^2)(k^6-k^4)$. Solve for α, whenever $(C, D, 1+k^4)$ is a Pythagorean Triplet.

From the References [1],[2],[3],[4],[5],[6],[7],[8], we know that

$(C, D, 1+k^4)$ becomes a Pythagorean Triplet with $C = (2k^2)$, $D = (k^4-1)$,

$\boldsymbol{C^2}+\boldsymbol{D^2} = (\mathbf{1}+\boldsymbol{k^4})^2$Hence $\alpha = (k^6-k^4)$.

Verification: Consider **LHS**

is $\boldsymbol{\alpha(X^4+Y^4)^2(5U^2+V^2)} = (k^6-k^4)(\boldsymbol{k^{4n}}+\boldsymbol{k^{4n+4}})^2(3(4)^n)^2 = \boldsymbol{k^{8n}}(k^6-k^4)(\mathbf{1}+\boldsymbol{k^4})^2\,(3(4)^n)^2$

Consider RHS

$$\boldsymbol{T^2}(\boldsymbol{C^2}+\boldsymbol{D^2})(\boldsymbol{Z^2}-\boldsymbol{W^2})\boldsymbol{P^4} = (3(4)^n)^2\,(\boldsymbol{1}+\boldsymbol{k^4})^2(k^{2n+6}-k^{2n+4})k^{6n}$$
$$= \boldsymbol{k^{8n}}(k^6-k^4)(\boldsymbol{1}+\boldsymbol{k^4})^2(3(4)^n)^2$$

Hence LHS $=$ ***RHS*.**

Proportion 5: A Study on exponential integer solution of above Diophantine Equation at

$\boldsymbol{\beta = 5}$ **is** $\boldsymbol{\alpha(X^4+Y^4)^2(5U^2+V^2) = T^2(C^2+D^2)(Z^2-W^2)P^5}$**.**

Explanation: Let x $= k^n, y = k^{n+1}, z = k^{2n+3}, w = k^{2n+2}, p = k^n, U = 4^{n+1}, V = 4^n, T = 3(4)^n$

Consider $\alpha(\boldsymbol{X^4}+\boldsymbol{Y^4})^2(\boldsymbol{5U^2}+\boldsymbol{V^2}) = \alpha k^{8n}(1+k^4)^2\,(3(4)^n)^2$.

Again consider $(Z^2-W^2)\boldsymbol{P^5} = k^{9n}(k^6-k^4)$.

It follows that $\boldsymbol{\alpha(X^4+Y^4)^2(5U^2+V^2) = T^2(C^2+D^2)(Z^2-W^2)P^5}$ implies that

$\alpha k^{8n}(1+k^4)^2(3(4)^n)^2 = (3(4)^n)^2(C^2+D^2)k^{9n}(k^6-k^4)$ implies $\alpha(1+k^4)^2 = (C^2+D^2)k^n(k^6-k^4)$. Solve for α, whenever $(C, D, 1+k^4)$ is a Pythagorean Triplet.

From the References [1],[2],[3],[4],[5],[6],[7],[8], we know that

$(C, D, 1+k^4)$ becomes a Pythagorean Triplet with $C = (2k^2)$, $D = (k^4-1)$,

$\boldsymbol{C^2}+\boldsymbol{D^2} = (\boldsymbol{1}+\boldsymbol{k^4})^2$Hence $\alpha = k^n(k^6-k^4)$.

Verification: Consider **LHS**

$\alpha(\boldsymbol{X^4}+\boldsymbol{Y^4})^2(\boldsymbol{5U^2}+\boldsymbol{V^2}) = k^n \quad (k^6-k^4)(\boldsymbol{k^{4n}}+\boldsymbol{k^{4n+4}})^2(3(4)^n)^2 = (3(4)^n)^2\boldsymbol{k^{9n}}(k^6-k^4)(\boldsymbol{1}+\boldsymbol{k^4})^2$.

Consider RHS

$\boldsymbol{T^2}(C^2+D^2)(Z^2-W^2)P^5 = (3(4)^n)^2(\boldsymbol{1}+\boldsymbol{k^4})^2(k^{4n+6}-k^{4n+4})k^{5n} = (3(4)^n)^2\boldsymbol{k^{9n}}(k^6-k^4)(\boldsymbol{1}+\boldsymbol{k^4})^2$.

Hence LHS $=$ ***RHS*.**

Proportion 6: A Study on exponential integer solution of above Diophantine Equation at

$\boldsymbol{\beta = 6}$ **is** $\boldsymbol{\alpha(X^4+Y^4)^2(5U^2+V^2) = T^2(C^2+D^2)(Z^2-W^2)P^6}$**.**

Explanation: Let x $= k^n, y = k^{n+1}, z = k^{2n+3}, w = k^{2n+2}, p = k^n, U = 4^{n+1}, V = 4^n, T = 3(4)^n$

Consider $\alpha(\boldsymbol{X^4}+\boldsymbol{Y^4})^2(\boldsymbol{5U^2}+\boldsymbol{V^2}) = \alpha k^{8n}(1+k^4)^2(3(4)^n)^2$.

Again consider $(Z^2-W^2)\boldsymbol{P^6} = k^{10n}(k^6-k^4)$.

It follows that $\boldsymbol{\alpha(X^4+Y^4)^2(5U^2+V^2) = T^2(C^2+D^2)(Z^2-W^2)P^6}$ implies that

$\alpha k^{8n}(1+k^4)^2(3(4)^n)^2 = (3(4)^n)^2(C^2+D^2)k^{10n}(k^6-k^4)$ implies

$\alpha(1+k^4)^2 = (C^2+D^2)k^{2n}(k^6-k^4)$. Solve for α, whenever $(C, D, 1+k^4)$ is a Pythagorean Triplet.

From the References [1],[2],[3],[4],[5],[6],[7],[8], we know that

$(C, D, 1+k^4)$ becomes a Pythagorean Triplet with $C = (2k^2)$, $D = (k^4-1)$,

$\boldsymbol{C^2}+\boldsymbol{D^2} = (\boldsymbol{1}+\boldsymbol{k^4})^2$Hence $\alpha = k^{2n}(k^6-k^4)$.

Verification: Consider **LHS**

$\alpha(\boldsymbol{X^4}+\boldsymbol{Y^4})^2(\boldsymbol{5U^2}+\boldsymbol{V^2}) = k^{2n} \quad (k^6-k^4)(\boldsymbol{k^{4n}}+\boldsymbol{k^{4n+4}})^2(3(4)^n)^2 = (3(4)^n)^2\boldsymbol{k^{10n}}(k^6-k^4)(\boldsymbol{1}+\boldsymbol{k^4})^2$.

Consider RHS

$\boldsymbol{T^2(C^2+D^2)(Z^2-W^2)P^6} = (3(4)^n)^2(\boldsymbol{1+k^4})^2(k^{4n+6}-k^{4n+4})k^{6n} = (3(4)^n)^2\boldsymbol{k^{10n}}(k^6-k^4)(\boldsymbol{1+k^4})^2$.

Hence LHS $= \boldsymbol{RHS}$.

Proportion 7: A Study on exponential integer solution of above Diophantine Equation at

$\boldsymbol{\beta = 7}$ **is** $\boldsymbol{\alpha(X^4+Y^4)^2(5U^2+V^2) = T^2(C^2+D^2)(Z^2-W^2)P^7}$.

Explanation: Let $x = k^n, y = k^{n+1}, z = k^{2n+3}, w = k^{2n+2}, p = k^n, U = 4^{n+1}, V = 4^n, T = 3(4)^n$

Consider $\alpha(\boldsymbol{X^4+Y^4})^2(\boldsymbol{5U^2+V^2}) = \alpha k^{8n}(1+k^4)^2(3(4)^n)^2$.

Again consider $(Z^2-W^2)\boldsymbol{P^6} = k^{10n}(k^6-k^4)$.

It follows that $\boldsymbol{\alpha(X^4+Y^4)^2(5U^2+V^2) = T^2(C^2+D^2)(Z^2-W^2)P^7}$. implies that

$\alpha k^{8n}(1+k^4)^2(3(4)^n)^2 = (3(4)^n)^2(C^2+D^2)k^{11n}(k^6-k^4)$ implies

$\alpha(1+k^4)^2 = (C^2+D^2)k^{3n}(k^6-k^4)$. Solve for α, whenever $(C, D, 1+k^4)$ is a Pythagorean Triplet.

From the References [1],[2],[3],[4],[5],[6],[7],[8], we know that

$(C, D, 1+k^4)$ becomes a Pythagorean Triplet with $C = (2k^2)$, $D = (k^4-1)$,

$\boldsymbol{C^2+D^2} = (\boldsymbol{1+k^4})^2$. Hence $\alpha = k^{3n}(k^6-k^4)$.

Verification: Consider **LHS**

$\boldsymbol{\alpha(X^4+Y^4)^2(5U^2+V^2)} = k^{3n} \quad (k^6-k^4)(\boldsymbol{k^{4n}+k^{4n+4}})^2(3(4)^n)^2 = (3(4)^n)^2\,\boldsymbol{k^{11n}}(k^6-k^4)(\boldsymbol{1+k^4})^2$.

Consider RHS

$\boldsymbol{T^2(C^2+D^2)(Z^2-W^2)P^7} = (3(4)^n)^2(\boldsymbol{1+k^4})^2(k^{4n+6}-k^{4n+4})k^{7n} = (3(4)^n)^2\,\boldsymbol{k^{11n}}(k^6-k^4)(\boldsymbol{1+k^4})^2$.

Hence LHS $= \boldsymbol{RHS}$.

III. Main Result:

A Study on exponential integer solution of above Diophantine Equation at

$\boldsymbol{\alpha(X^4+Y^4)^2(5U^2+V^2) = T^2(C^2+D^2)(Z^2-W^2)P^\beta}$.

Explanation: Let $x = k^n, y = k^{n+1}, z = k^{2n+3}, w = k^{2n+2}, p = k^n, U = 4^{n+1}, V = 4^n, T = 3(4)^n$

Consider $\alpha(\boldsymbol{X^4+Y^4})^2(\boldsymbol{5U^2+V^2}) = \alpha k^{8n}(1+k^4)^2\,(3(4)^n)^2$.

Again consider $(Z^2-W^2)\boldsymbol{P^\beta} = k^{4n+n\beta}(k^6-k^4)$.

It follows that $\boldsymbol{\alpha(X^4+Y^4)^2(5U^2+V^2) = T^2(C^2+D^2)(Z^2-W^2)P^\beta}$.

implies that

$\alpha k^{8n}(1+k^4)^2(3(4)^n)^2 = (3(4)^n)^2(C^2+D^2)k^{4n+n\beta}(k^6-k^4)$ implies

$\alpha(1+k^4)^2 = (C^2+D^2)k^{-4n+n\beta}(k^6+k^4)$. Solve for α, whenever $(C, D, 1+k^4)$ is a Pythagorean Triplet.

From the References [1],[2],[3],[4],[5],[6],[7],[8], we know that

$(C, D, 1+k^4)$ becomes a Pythagorean Triplet with $C = (2k^2)$, $D = (k^4-1)$,

$\boldsymbol{C^2 + D^2} = (\mathbf{1} + \boldsymbol{k^4})^{\mathbf{2}}$. Hence $\alpha = k^{-4n+n\beta}(k^6 - k^4) = k^{(\beta-4)n}(k^6 - k^4)$.

Verification: Consider **LHS**

$\alpha(\boldsymbol{X^4} + \boldsymbol{Y^4})^2(\mathbf{5}\boldsymbol{U^2} + \boldsymbol{V^2}) = k^{(\beta-4)n}(k^6 - k^4)\ (\boldsymbol{k^{4n}} + \boldsymbol{k^{4n+4}})^2(3(4)^n)^2$

$= (3(4)^n)^2\boldsymbol{k}^{(\beta+4)n}(k^6 - k^4)(\mathbf{1} + \boldsymbol{k^4})^{\mathbf{2}}.$

Consider RHS

$\boldsymbol{T^2}(\boldsymbol{C^2} + \boldsymbol{D^2})(\boldsymbol{Z^2} - \boldsymbol{W^2})\boldsymbol{P^\beta} = (3(4)^n)^2(\mathbf{1} + \boldsymbol{k^4})^2(k^{4n+6} - k^{4n+4})k^{\beta n}$

$= (3(4)^n)^2\boldsymbol{k}^{(\beta+4)n}(k^6 - k^4)(\mathbf{1} + \boldsymbol{k^4})^{\mathbf{2}}.$

Hence LHS $= \boldsymbol{RHS}$.

IV. Conclusion:

This equation generalizes classical Diophantine problems, blending sums of fourth powers with multiplicative factorizations. While challenging, targeted parametrization and modular analysis can yield solutions. Future work may classify solutions for specific α, β or link to broader number-theoretic frameworks. The parametric framework provides infinite families of solutions by exploiting algebraic identities and modular arithmetic. Future work could explore non-parametric solutions or generalizations to higher exponents.

This paper focused on a study to find integer design of solutions Diophantine Equation

$\boldsymbol{\alpha(X^4 + Y^4)^2(5U^2 + V^2) = T^2(C^2 + D^2)(Z^2 - W^2)P^\beta}$ **With** $\boldsymbol{\alpha > 0, \gamma = 2, 3, \beta = 1, 2, 3, 4, 5, 6, 7}$ ***and***

$\boldsymbol{x < y < w < z}$ with Mathematical induction & generation of Pythagorean triplets.

for $\boldsymbol{\beta = 1}$, having integer design of solution is parameterized by positive integers k and n, with variables defined as:

$x = k^n, y = k^{n+1}, z = k^{n+3}, w = k^{n+2}, U = 4^{n+1}, V = 4^n, T = 3(4)^n,$

$p = k^{6n}, \alpha = (k^6 - k^4), C = 2k^2, D = k^4 - 1.$

for $\boldsymbol{\beta = 2}$, having integer design of solution is parameterized by integers k and n, with variables defined as:

$x = k^n, y = k^{n+1}, z = k^{n+3}, w = k^{n+2}, U = 4^{n+1}, V = 4^n, T = 3(4)^n,$

$p = k^{3n}, \alpha = (k^6 - k^4), C = 2k^2, D = k^4 - 1.$

for $\boldsymbol{\beta = 3}$ **,** having integer design of solution is parameterized by integers k and n, with variables defined as:

$x = k^n, y = k^{n+1}, z = k^{n+3}, w = k^{n+2}, U = 4^{n+1}, V = 4^n, T = 3(4)^n,$

$p = k^{2n}, \alpha = (k^6 - k^4), C = 2k^2, D = k^4 - 1.$

for $\boldsymbol{\beta = 4}$ **,** having integer design of solution is parameterized by integers k and n, with variables defined as:

$x = k^n, y = k^{n+1}, z = k^{2n+3}, w = k^{2n+2}, U = 4^{n+1}, V = 4^n, T = 3(4)^n,$

$p = k^n, \alpha = (k^6 - k^4), C = 2k^2, D = k^4 - 1.$

for $\boldsymbol{\beta = 5}$ **,** having integer design of solution is parameterized by integers k and n, with variables defined as:

$x = k^n, y = k^{n+1}, z = k^{n+3}, w = k^{n+2}, U = 4^{n+1}, V = 4^n, T = 3(4)^n,$

$p = k^{2n}, \alpha = k^n(k^6 - k^4), C = 2k^2, D = k^4 - 1.$

for $\beta = 6$, having integer design of solution is parameterized by integers k and n, with variables defined as:

$x = k^n, y = k^{n+1}, z = k^{2n+3}, w = k^{2n+2}, U = 4^{n+1}, V = 4^n, T = 3(4)^n,$

$p = k^n, \alpha = k^{2n}(k^6 - k^4), C = 2k^2, D = k^4 - 1.$

for $\beta = 7$, having integer design of solution is parameterized by integers k and n, with variables defined as:

$x = k^n, y = k^{n+1}, z = k^{2n+3}, w = k^{2n+2}, U = 4^{n+1}, V = 4^n, T = 3(4)^n,$

$p = k^n, \alpha = k^{3n}(k^6 - k^4), C = 2k^2, D = k^4 - 1.$

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