IJCRT.ORG ISSN : 2320-2882

INTERNATIONAL JOURNAL OF CREATIVE RESEARCH THOUGHTS (IJCRT)

An International Open Access, Peer-reviewed, Refereed Journal

A New Approach To Solve Transportation Problem Involving Penalty - Adjusted Range Method

Shivi Tiwari[1], M. S. Chauhan[2], S.S. Shrivastava[3]

Department of Mathematics[1,2,3]

Institute for Excellence in Higher Education (IEHE), Bhopal (M.P.)

Abstract

The transportation problem focuses on minimizing distribution costs while meeting supply and demand. Conventional methods like NWCR, LCM, and VAM often yield suboptimal solutions. The Coefficient of Range improves efficiency but has limitations. This paper introduces a penalty–range hybrid approach, combining penalty sensitivity with variability. Numerical results demonstrate lower costs and improve performance compared to existing techniques.

Keywords- Transportation Problem, Coefficient of Range, Range Penalty Factor, Linear Programming, NWCR Method, LCM Method, VAM Method, COR Method, PARM Method,.

I. INTRODUCTION

The transportation problem, introduced by Hitchcock [4] and later formalized by Koopmans [5] and Dantzig [2], plays a significant role in logistics, supply chain management, and industrial operations. The main objective is to minimize transportation cost while satisfying supply and demand constraints.

Traditional methods such as NWCR, LCM, and VAM differ in their accuracy and complexity. NWCR is simple but often produces suboptimal results, LCM considers costs but may require multiple iterations, and VAM introduces penalties but is computationally intensive. To address these limitations, statistical methods have been introduced, including the Coefficient of Range (COR) approach, which reduces computational complexity while yielding better feasible solutions.

However, there is still a need for robust techniques that combine simplicity, accuracy, and computational efficiency. This paper introduces a new penalty-based range methodology that extends the Coefficient of Range approach by applying a multiplicative adjustment factor, improving allocation decisions in cost-sensitive.

II. LITERATURE REVIEW

Seethalakshmy et al. [7] proposed a method to obtain an optimum solution for real-life problems using the transportation model. Their study focused on a travel agency company to maximize profit of a travel agency and minimize transportation cost, applying a new algorithm where maximum or minimum values are marked row-wise and column-wise depending on the problem type, and allocations are made based on the greatest maximum value. This approach- ensures optimal solutions with fewer iterations.

A study by Radthy et al. [10] applies transportation models to real data from a domestic food company aiming to distribute products to four locations through three branches. The company faced high transportation and marketing costs due to reliance on staff experience without a scientific approach. To minimize cost, three methods were used minimum cell cost method, Vogel's approximation method, and a new technique for initial basic solution after which the stepping-stone test was applied to obtain the optimum solution.

A paper by Sharma et al. [8] addresses the transportation problem, a special class of linear programming used to allocate commodities from multiple sources to different destinations while minimizing shipping costs. The study proposes an alternative to the North-West Corner method by applying a statistical tool called the Coefficient of Range. Numerical examples are provided to validate and justify the effectiveness of the proposed method.

A study by Zabiba et al. [10] proposes a new technique, named NOOR1, for solving transportation problems formulated as linear programming problems. The method is designed to generate an initial solution that is either optimal or very close to optimal in most cases. It is simple, effective, and applicable to both balanced and unbalanced transportation problems with a minimization objective function.

III. PROPOSED METHOD: THE PENALTY – ADJUSTED RANGE METHOD (PARM)

The Penalty – Adjusted Range Method (PARM) is proposed as an advanced approach to generate efficient initial feasible solutions for transportation problems by integrating the concepts of penalty-based allocation and statistical variation. The procedure begins with the computation of penalties for each row and column, defined as the difference between the two least cost entries. Subsequently, the Coefficient of Range is calculated for every row and column, capturing the relative variation in transportation costs. To incorporate both penalty sensitivity and range variation, an adjustment factor is introduced, expressed as:

$$\text{Adjusted Score} = \text{Penalty} \times \left(\frac{R_k}{R_{max}}\right)$$

where R_k denotes the Coefficient of Range of the current row or column, and R_{max} is the maximum Coefficient of Range observed across all rows and columns. The row or column attaining the highest adjusted score is then prioritized, and allocation is made to the minimum cost cell within it. Following this procedure, the transportation table is updated, and the process iteratively continues until the demand and supply constraints are fully satisfied.

By synthesizing the penalty consideration employed in Vogel's Approximation Method (VAM) with the statistical robustness of the Coefficient of Range technique, the Penalty–Adjusted Range Method achieves a balanced allocation strategy. This integration enhances sensitivity to cost differences while simultaneously accounting for data variability, ultimately yielding more cost-efficient and reliable initial solutions.

III. ALGORITHM FOR THE PENALTY – ADJUSTED RANGE METHOD (PARM)

To enhance the Coefficient of Range method, we propose a Penalty - Adjusted Range Method as under:

1. Calculate the penalty for each row/column (difference between the two lowest costs).

2. Calculate the Coefficient of Range for current each row/column. (difference between maximum value and minimum value)

3. Calculate Range Penalty Adjustment Factor for each row / column using the formula.

$$\text{Adjusted Score} = \text{Penalty} \times \left(\frac{R_k}{R_{max}}\right)$$

where R_k is the Coefficient of Range of the current row/column, and R_{max} is the maximum Coefficient of Range among all rows/columns.

4. Select the row / column with the highest Adjusted Score.

5. Allocate supply / demand to the minimum cost cell in that row/column.

6. Update the table and repeat until completion.

This method balances penalty sensitivity (from VAM) with statistical variation (from Coefficient of Range), leading to more cost efficient initial solutions.

Problem 1- Petrol Refineries Problem

India Oil Corporation operates three refineries (S_1, S_2, S_3) that refine crude oil into petrol. The petrol has to be transported to four distribution depots (D_1, D_2, D_3, D_4) which further supply it to retail petrol pumps in the region.

The transportation cost (in ₹ thousand per tanker) from each refinery to each depot is shown below:

Refinery → Depot	D_1	D_2	D_3	D_4	Supply
S_1	19	30	50	10	7
S_2	70	30	40	60	9
S_3	40	8	70	20	18
Demand	5	8	7	14	29

Objective of the Problem (Real-Life Context):

The company must prepare a petrol distribution plan such that:

1. All four depots receive their required number of tankers.

2. No refinery supplies more than its refining capacity.

3. The overall transportation cost (in lakhs of rupees) is minimized.

Thus, the problem is to determine how many petrol tankers should be transported from each refinery to each depot in order to meet demand at minimum cost.

Mathematically, the decision variables (units shipped from supply to demand) x_{ij} : i = 1, ---3,

j = 1, --- 4 are to be determined so that the objective function:

$$Z = \sum_{i=1}^{3} \sum_{j=1}^{4} c_{ij}\, x_{ij}$$

is minimized, subject to the supply and demand balance constraints.

Here c_{ij} are per unit cost of transportation from S_i to D_j.

(1) Solution of the problem by NWCR Method-

Refinery → Depot	D_1	D_2	D_3	D_4	Supply
S_1	19(5)	30(2)	50(0)	10(0)	7
S_2	70(0)	30(6)	40(3)	60(0)	9
S_3	40(0)	8(0)	70(4)	20(14)	18
Demand	5	8	7	14	29

Total Cost by NWCR Method = 19 × 5 + 30 × 2 + 30 × 6 + 40 × 3 + 70 × 4 + 20 × 14 = 1015

(2) Solution of the problem by LCM Method-

Refinery → Depot	D_1	D_2	D_3	D_4	Supply
S_1	19(0)	30(0)	50(0)	10(7)	7
S_2	70(2)	30(0)	40(7)	60(0)	9
S_3	40(3)	8(0)	70(8)	20(7)	18
Demand	5	8	7	14	29

Total Cost by LCM Method = 10 × 7 + 70 × 2 + 40 × 7 + 40 × 3 + 70 × 8 + 20 × 7 = 814

(3) Solution of the problem by VAM Method-

Refinery → Depot	D_1	D_2	D_3	D_4	Supply
S_1	19(5)	30(0)	50(0)	10(2)	7
S_2	70(0)	30(0)	40(7)	60(2)	9
S_3	40(0)	8(8)	70(0)	20(10)	18
Demand	5	8	7	14	29

Total Cost by VAM Method = 19 × 5 + 20 × 2 + 40 × 7 + 60 × 2 + 8 × 8 + 20 × 10 = 779

(4) Solution of the problem by COR Method-

Refinery → Depot	D_1	D_2	D_3	D_4	Supply
S_1	19(0)	30(0)	50(0)	10(7)	7
S_2	70(2)	30(0)	40(7)	60(0)	9
S_3	40(3)	8(8)	70(0)	20(7)	18
Demand	5	8	7	14	29

Total Cost by COR Method = 10 × 7 + 70 × 2 + 40 × 7 + 40 × 3 + 8 × 8 + 20 × 7 = 814

(5) Solution of the problem by PARM Method-

Iteration 1-

Rows -
Row S_1 (costs: 19, 30, 50, 10)

Two smallest cost = 10 and 19 → Penalty = 19 − 10 = 9.

Min = 10, Max = 50 → CR = 50 − 10 = 40.
Row S_2 (70, 30, 40, 60)
Two smallest cost = 30 and 40 → Penalty = 40 − 30 = 10.
Min = 30, Max = 70 → CR = 70 − 30 = 40.
Row S_3 (40, 8, 70, 20)
Two smallest cost = 8 and 20 → Penalty = 20 − 8 = 12.
Min = 8, Max = 70 → CR = 70 − 8 = 62.
Columns-

Column D_1 (19, 70, 40)

Two smallest cost = 19 and 40 → Penalty = 40 − 19 = 21.
Min = 19, Max = 70 → CR = 70 − 19 = 51.

Column D_2 (30, 30, 8)

Two smallest cost = 8 and 30 → Penalty = 30 − 8 = 22.

Min = 8, Max = 30 → CR = 30 − 8 = 22.

Column D_3 (50, 40, 70)

Two smallest cost = 40 and 50 → Penalty = 50 − 40 = 10.

Min = 40, Max = 70 → CR = 70 − 40 = 30.

Column D_4 (10, 60, 20)

Two smallest cost = 10 and 20 → Penalty = 20 − 10 = 10.

Min = 10, Max = 60 → CR = 60 − 10 = 50.

From the CRs above: {40,40,62,51,22,30,50} → R_{max} = 62 (from row S_3}

Adjusted Score (AS) = Penalty × (CR/R_{max}). Since R_{max} = 62, CR/R_{max} will be CR/62.

I show exact fraction then decimal (rounded to 6 decimal places):

Rows-

AS for row $S_1 = 9 \times \frac{40}{62} = \frac{180}{31} \approx 5.806452$

AS for row $S_2 = 10 \times \frac{40}{62} = \frac{200}{31} \approx 6.451613$

AS for row $S_3 = 12 \times \frac{62}{62} = \frac{12}{1} \approx 12.000000$

Columns-

AS for column $D_1 = 21 \times \frac{51}{62} = \frac{1071}{62} \approx 17.274194$

AS for column $D_2 = 22 \times \frac{22}{62} = \frac{484}{62} \approx 7.806452$

AS for column $D_3 = 10 \times \frac{30}{62} = \frac{300}{62} \approx 4.838710$

AS for column $D_4 = 10 \times \frac{50}{62} = \frac{500}{62} \approx 8.064516$

Thus Highest Adjusted Score (≈ 17.274194) which is adjusted score for D_1.

Now we choose minimum-cost cell in D_1 which is S_1 with minimum value 19.

Allocate min {(S_1_supply = 7, D_1_demand = 5) = 5} → $x_{11} = 5$.

Update: S_1 supply 7 → 2; D_1 demand 5 → 0 (D_1 finished.)

Refinery → Depot	D_1	D_2	D_3	D_4	Supply	Penalty
S_1	5 19	30	50	10	~~7~~ 0	5.806452
S_2	70	30	40	60	9	6.451613
S_3	40	8	70	20	18	12.000000
Demand	~~5~~ 0	8	7	14		
Penalty	17.274194	7.806452	4.838710	8.064516		

Iteration 2-

Active rows: S_1 (2 left), S_2 (9), S_3 (18)

Active cols: D_2 (8), D_3 (7), D_4 (14)

Compute:

S_1 (costs 30,50,10): two smallest cost 10 & 30 → Penalty = 20; CR = 50 – 10 = 40;

S_2 (30,40,60): two smallest cost 30 & 40 → Penalty = 10; CR = 60 – 30 = 30;

S_3 (8,70,20): two smallest cost 8 & 20 → Penalty = 12; CR = 70 – 8 = 62;

D_2 (30,30,8): two smallest cost 8 & 30 → Penalty = 22; CR = 30 – 8 = 22;

D_3 (50,40,70): two smallest cost 40 & 50 → Penalty = 10; CR = 70 – 40 = 30;

D_4 (10,60,20): two smallest cost 10 & 20 → Penalty = 10; CR = 60 – 10 = 50;

R_{max} = 62 (from row S_3} We compute: AS = Penalty × (CR / R_{max}). Since R_{max} = 62, CR/R_{max} will be CR/62.

I show exact fraction then decimal (rounded to 6 decimal places):

Rows-

AS for row $S_1 = 20 \times \frac{40}{62} = \frac{800}{62} \approx 12.903226$

AS for row $S_2 = 10 \times \frac{30}{62} = \frac{300}{62} \approx 4.838710$

AS for row $S_3 = 12 \times \frac{62}{62} = \frac{12}{1} \approx 12.000000$

Columns-

AS for column $D_2 = 22 \times \frac{22}{62} = \frac{484}{62} \approx 7.806452$

AS for column $D_3 = 10 \times \frac{30}{62} = \frac{300}{62} \approx 4.838710$

AS for column $D_4 = 10 \times \frac{50}{62} = \frac{500}{62} \approx 8.064516$

Thus Highest Adjusted Score (≈ 12.903226) which is adjusted score for S_1.

Now we choose minimum-cost cell in S_1 which is D_4 with minimum value 10.

Allocate min {(S_1_supply = 2, D_4_demand = 14)} = 2 → x_{14} = 2.

Update: S_1 supply 2 → 0 (S_1 finished); D_4 demand 14 → 12.

Refinery → Depot	D_2	D_3	D_4	Supply	Penalty
S_1	30	50	[2] 10	~~2~~ 0	12.903226
S_2	30	40	60	9	4.838710
S_3	8	70	20	18	12.000000
Demand	8	7	~~14~~ 12		
Penalty	7.806452	4.838710	8.064516		

Iteration 3-

Active rows: S_2 (9), S_3 (18)

Active cols: D_2 (8), D_3 (7), D_4 (12)

Compute:

S_2 (30,40,60): two smallest cost 30 & 40 → Penalty = 10; CR = 60 – 30 = 30;

S_3 (8,70,20): two smallest cost 8 & 20 → Penalty = 12; CR = 70 – 8 = 62;

D_2 (30,8): two smallest cost 8 & 30 → Penalty = 22; CR = 30 – 8 = 22;

D_3 (40,70): two smallest cost 40 & 70 → Penalty = 30; CR = 70 – 40 = 30;

D_4 (60,20): two smallest cost 20 & 60 → Penalty = 40; CR = 60 – 20 = 40;

R_{max} = 62 (from row S_3} We compute: AS = Penalty × (CR / R_{max}). Since R_{max} = 62, CR/R_{max} will be CR/62.

I show exact fraction then decimal (rounded to 6 decimal places):

Rows-

AS for row $S_2 = 10 \times \frac{30}{62} = \frac{300}{62} \approx 4.838710$

AS for row $S_3 = 12 \times \frac{62}{62} = \frac{12}{1} \approx 12.000000$

Columns-

AS for column $D_2 = 22 \times \frac{22}{62} = \frac{484}{62} \approx 7.806452$

AS for column $D_3 = 30 \times \frac{30}{62} = \frac{900}{62} \approx 14.516129$

AS for column $D_4 = 40 \times \frac{40}{62} = \frac{1600}{62} \approx 25.806452$

Thus Highest Adjusted Score (≈ 25.806452) which is adjusted score for D_4.

Now we choose minimum-cost cell in D_4 which is S_3 with minimum value 60.

Allocate min {(S_3_supply = 18, D_4_demand = 12)} = 12 → x_{34} = 12.

Update: S_3 supply 18 → 6; D_4 demand 12 → 0. (D_4 finished)

Refinery → Depot	D_2	D_3	D_4	Supply	Penalty
S_2	30	40	60	9	4.838710
S_3	8	70	12 20	~~18~~ 6	12.000000
Demand	8	7	~~12~~ 0		
Penalty	7.806452	14.516129	25.806452		

Iteration 4 -

Supplies: S_2 = 9, S_3 = 6

Demands: D_2 = 8, D_3 = 7

Compute Penalty and CR:

S_2(30, 40): two entries 30 & 40 → Penalty = 10. CR = 40 − 30 = 10.

$S_3(8,70)$: two entries 8 & 70 → Penalty = 62. CR = 70 − 8 = 62.

D_2 (30, 8): two entries 8 & 30 → Penalty = 22. CR = 30 − 8 = 22.

D_3 (40, 70): two entries 40 & 70 → Penalty = 30. CR = 70 − 40 = 30.

R_{max} = 62 (from row S_3} We compute: AS = Penalty × (CR / R_{max}). Since R_{max} = 62, CR / R_{max} will be CR / 62.

I show exact fraction then decimal (rounded to 6 decimal places):

Rows-

AS for row $S_2 = 10 \times \frac{10}{62} = \frac{100}{62} \approx 1.612903$

AS for row $S_3 = 62 \times \frac{62}{62} = \frac{62}{1} \approx 62.000000$

Columns-

AS for column $D_2 = 22 \times \frac{22}{62} = \frac{484}{62} \approx 7.806452$

AS for column $D_3 = 30 \times \frac{30}{62} = \frac{900}{62} \approx 14.516129$

Thus Highest Adjusted Score (≈ 62.000000) which is adjusted score for S_3.

Now we choose minimum-cost cell in S_3 which is D_2 with minimum value 8.

Allocate min {(S_3_supply = 6, D_2_demand = 8)} = 6 → $x_{32} = 6$

Update: S_3 supply 6 → 0 (S_3 finished); D_2 demand 8 → 2.

Refinery → Depot	D_2	D_3	Supply	Penalty
S_2	30	40	9	1.612903
S_3	12 8	70	~~6~~ 0	62.000000
Demand	~~8~~ 2	7		
Penalty	7.806452	14.516129		

Iteration 5-

Supplies: S_2 = 9

Demands: D_2 = 2, D_3 = 7

Compute Penalty and CR:

Rows-

$S_2(30, 40)$: two entries 30 & 40 → Penalty = 10. CR = 40 − 30 = 10.

Columns-

D_2 (Only S_1 = 30): → Penalty = 30 − 30 = 0, CR = 30 − 30 = 0.

D_3 (Only $S_2 = 30$): $\rightarrow$ Penalty $= 30 - 30 = 0$, CR $= 30 - 30 = 0$.

$R_{max} = 10$ (from row S_2} We compute: AS = Penalty × (CR / R_{max}). Since $R_{max} = 10$, CR/R_{max} will be CR/10.

I show exact fraction then decimal (rounded to 6 decimal places):

Rows-

AS for row $S_2 = 10 \times \frac{10}{10} = \frac{100}{10} \approx 10.000000$

Columns-

AS for column $D_2 = 0 \times \frac{0}{62} = 0$

AS for column $D_3 = 0 \times \frac{0}{62} = 0$

Thus Highest Adjusted Score (≈ 10.000000) which is adjusted score for S_2.

Now we choose minimum - cost cell in S_3 which is D_2 with minimum value 30.

Allocate min $\{(S_2\_supply = 9, D_2\_demand = 2)\} = 2 \rightarrow x_{22} = 2$

Update: S_2 supply $9 \rightarrow 7$; D_2 demand $2 \rightarrow 0$. (D_2 = finished)

Refinery → Depot	D_2	D_3	Supply	Penalty
S_2	[2] 30	40	~~9~~ 7	10.000000
Demand	~~2~~ 0	7		
Penalty	0	0		

Iteration 6-

Supplies: $S_2 = 7$

Demands: $D_3 = 7$

Only one feasible cell: $S_2 \rightarrow D_3$. Allocate $x_{23} = 7$ (S_2, D_3 finished)

Refinery → Depot	D_3	Demand
S_2	[7] 40	~~7~~ 0
Demand	~~7~~ 0	

Final allotment-

Refinery → Depot	D_1	D_2	D_3	D_4	Supply

S_1	[5] 19	30	50	[2] 10	~~7~~ ~~2~~ 0
S_2	70	[2] 30	[7] 40	60	~~9~~ ~~7~~ 0
S_3	40	[6] 8	70	[12] 20	~~18~~ ~~6~~ 0
Demand	~~5~~ 0	~~8~~ ~~2~~ 0	~~7~~ 0	~~14~~ ~~12~~ 0	34

Total Cost by PARM Method = $19 \times 5 + 10 \times 2 + 30 \times 2 + 40 \times 7 + 8 \times 6 + 20 \times 12$

$= 743$

Consider the transportation problem in Table.

COMPARISION: ALLOCATIONS OBTAINED BY DIFFERENT METHODS.

Method	NWCR	LCM	VAM	COR	Penalty - Adjusted Range Method
Allocations	$x_{11} = 5$, $x_{12} = 2$, $x_{22} = 6$, $x_{23} = 3$, $x_{33} = 4$, $x_{34} = 14$	$x_{14} = 7$, $x_{21} = 2$, $x_{23} = 7$, $x_{31} = 3$, $x_{32} = 8$, $x_{34} = 7$	$x_{11} = 5$, $x_{14} = 2$, $x_{23} = 7$, $x_{24} = 2$, $x_{32} = 8$, $x_{34} = 10$	$x_{14} = 7$, $x_{21} = 2$, $x_{23} = 7$, $x_{31} = 3$, $x_{32} = 8$, $x_{34} = 7$	$x_{11} = 5$, $x_{14} = 2$, $x_{22} = 2$, $x_{23} = 7$, $x_{32} = 6$, $x_{34} = 12$
Total cost	1015	814	779	814	743

Thus, our proposed method The Penalty – Adjusted Range Method (PARM) gives minimum transportation cost (743) compared to all other methods.

Problem 2- Food Distribution Problem

A food distribution agency needs to transport packets of food grains from its three godowns located in different cities (S_1, S_2, S_3) to four relief camps (D_1, D_2, D_3, D_4) set up for flood-

5 packets.

The transportation cost (in ₹ per packet) from each godown to each relief camp is given below:

Godown → Relief Camp	D_1	D_2	D_3	D_4	Supply
S_1	2	3	11	7	6
S_2	1	0	6	1	1
S_3	5	8	15	9	10
Demand	7	5	3	2	17

Objective of the Problem (Real-Life Context):

The food distribution agency wants to prepare a transportation schedule such that:

1. Each relief camp receives exactly the required number of food packets.

2. The stock available at each godown is not exceeded.

3. The total transportation cost (in ₹) of delivering the packets is as low as possible.

Thus, the task is to determine how many packets should be transported from each godown to each relief camp so that the minimum cost of transportation is achieved.

Mathematically, the decision variables (units shipped from supply to demand) are to be determined so that the objective function:

$$Z = \sum_{i=1}^{3} \sum_{j=1}^{4} c_{ij}\, x_{ij}$$

is minimized, subject to the supply and demand balance constraints.

Here c_{ij} are per unit cost of transportation from S_i to D_j.

(1) Solution of the problem by NWCR Method--

Godown → Relief Camp	D_1	D_2	D_3	D_4	Supply
S_1	2(6)	3(0)	11(0)	7(0)	6
S_2	1(1)	0 (0)	6 (0)	1 (0)	1
S_3	5 (0)	8(5)	15(3)	9(2)	10
Demand	7	5	3	2	17

Total Cost by NWCR Method = 2 × 6 + 1× 1+ 8 × 5 + 15 × 3 + 9 × 2 = 116

(2) Solution of the problem by LCM Method-

Godown → Relief Camp	D_1	D_2	D_3	D_4	Supply
S_1	2(6)	3(0)	11(0)	7(0)	6
S_2	1(0)	0 (1)	6 (0)	1 (0)	1
S_3	5 (1)	8(4)	15(3)	9(2)	10
Demand	7	5	3	2	17

Total Cost by LCM Method = 2 × 6 + 0 × 1 + 5 × 1 + 8 ×4 +15 × 3 + 9 × 2 = 112

(3) Solution of the problem by VAM Method-

Godown → Relief Camp	D_1	D_2	D_3	D_4	Supply
S_1	2(6)	3(0)	11(0)	7(0)	6
S_2	1(0)	0 (0)	6 (0)	1 (1)	1
S_3	5 (6)	8(0)	15(6)	9(1)	10
Demand	7	5	3	2	17

Total Cost by VAM Method = 2 × 6 + 1 × 1+ 5 × 6 +15 × 6 + 9 × 1 = 102

(4) Solution of the problem by COR Method-

Godown → Relief Camp	D_1	D_2	D_3	D_4	Supply
S_1	2(6)	3(0)	11(0)	7(0)	6
S_2	1(0)	0 (0)	6 (0)	1 (1)	1
S_3	5 (1)	8(5)	15(3)	9(1)	10
Demand	7	5	3	2	17

Total Cost by COR Method = 2 × 6 + 1 × 1 + 5 × 1 + 8 × 5 + 15 × 3 + 9 ×1 = 112

(5) Solution of the problem by PARM Method-

Costs and supplies/demands:

S_1: [2, 3, 11, 7], supply = 6
S_2: [1, 0, 6, 1], supply = 1
S_3: [5, 8, 15, 9], supply = 10
Demands: D_1 = 7, D_2 = 5, D_3 = 3, D_4 = 2 (total supply = total demand = 17).

Iteration 1-

(initial rows & columns)
Compute Penalty = (difference between two smallest costs) and CR = (max − min) for every full row/column.

Rows-

S_1: two smallest cost {2,3} → Penalty = 1. min = 2, max = 11 → CR = 11 − 2 = 9.

S_2: two smallest cost {0,1} → Penalty = 1. min = 0, max = 6 → CR = 6 − 0 = 6.

S_3: two smallest cost {5,8} → Penalty = 3. min = 5, max = 15 → CR = 15 − 5 = 10.

Columns-

D_1 (2,1,5): two smallest cost {1,2} → Penalty = 1.
min =1, max = 5 → CR = 5 − 1 = 4.

D_2 (3,0,8): two smallest cost {0,3} → Penalty = 3.

min = 0, max = 8 → CR = 8 − 0 = 8.

D_3 (11,6,15): two smallest cost {6,11} → Penalty = 5.
min = 6, max = 15 → CR = 15 − 6 = 9.

D_4 (7,1,9): two smallest {1,7} → Penalty = 6. min = 1, max = 9 → CR = 9 − 1 = 8.

R_{max} = 10 (from S_3). Compute Adjusted Score (AS) = Penalty × (CR / R_{max}).
AS values (rounded when shown):
Rows: S_1 = 1 * (9/10) = 0.9; S_2 = 1 * (6/10) = 0.6; S_3 = 3 * (10/10) = 3.0
Cols: D_1 = 1 * (4/10) = 0.4; D_2 = 3 * (8/10) = 2.4; D_3 = 5 * (9/10) = 4.5
D_4 = 6 * (8/10) = 4.8
Highest AS = D_4 (≈ 4.8) → choose column D_4.
Minimum-cost cell in column D_4 is S_2 → D_4 (cost = 1).
Allocate min (S_2_supply = 1, D_4_demand = 2) = 1 → x_{24} = 1.
Update: S_2 supply 1 → 0 (S_2 finished); D_4 demand 2 → 1.

Godown → Relief Camp	D_1	D_2	D_3	D_4	Supply	Penalty
S_1	2	3	11	7	6	0.9
S_2	1	0	6	[1] 1	~~1~~ 0	0.6
S_3	5	8	15	9	10	3
Demand	7	5	3	~~2~~ 1		
Penalty	0.4	2.4	4.5	4.8		

Iteration 2-

(S_2 finished)
Active rows: S_1 (6), S_3 (10). Active columns: D_1 (7), D_2 (5), D_3 (3), D_4 (1).
Compute Penalty & CR on active cells:
Rows-
S_1 (2,3,11,7): Penalty = 1, CR = 11 − 2 = 9.
S3 (5,8,15,9): Penalty = 3, CR = 15 − 5 = 10.
Columns (only S1 & S3 entries):
D_1: {2,5} → Penalty = 3, CR = 5 − 2 = 3.

D_2: {3,8} → Penalty = 5, CR = 8 − 3 = 5.
D_3: {11,15} → Penalty = 4, CR = 15 − 11 = 4.
D_4: {7,9} → Penalty = 2, CR = 9 − 7 = 2.
R_{max} still = 10 (S_3). AS = Penalty × (CR / 10):
Rows: S_1 = 0.9; S_3 = 3.0
Cols: D_1 = 3 * (3/10) = 0.9; D_2 = 5 * (5/10) = 2.5; D_3 = 4 * (4/10) = 1.6
D_4 = 2 * (2/10) = 0.4
Highest AS = S_3 (3.0) → choose row S_3.
Minimum-cost cell in S_3 (active cols) is S_3 → D_1 (cost = 5).
Allocate min {(S_3_supply = 10, D_1_demand = 7)} = 7 → x_{31} = 7.
Update: S_3 supply 10 → 3; D_1 demand 7 → 0 (D_1 finished).

Godown → Relief Camp	D_1	D_2	D_3	D_4	Supply	Penalty
S_1	2	3	11	7	6	0.9
S_3	[7] 5	8	15	9	~~10~~ 3	3
Demand	~~7~~ 0	5	3	1		
Penalty	0.9	2.5	1.6	0.4		

Iteration 3-

Active rows: S_1 (6), S_3 (3). Active cols: D_2 (5), D_3 (3), D_4 (1).
Compute Penalty & CR based on active cells:
Rows-
S_1 (D_2, D_3, D_4) = [3,11,7] → two smallest cost {3,7} → Penalty = 4. CR = 11 − 3 = 8.
S_3 (8,15,9) → two smallest cost {8,9} → Penalty = 1. CR = 15 − 8 = 7.
Cols - D_2: {3,8} → Penalty = 5, CR = 8 − 3 = 5.
D_3: {11,15} → Penalty = 4, CR = 15 − 11 = 4.
D_4: {7,9} → Penalty = 2, CR = 9 − 7 = 2.
R_{max} = 8 (from S_1). AS = Penalty × (CR / 8)
Rows: S_1 = 4 * (8/8) = 4.0; S_3 = 1 * (7/8) = 0.875
Cols: D_2 = 5 * (5/8) = 3.125; D_3 = 4 * (4/8) = 2.0; D_4 = 2 * (2/8) = 0.5
Highest AS = S_1 (4.0) → choose row S_1.
Minimum-cost cell in S_1 among active columns is S_1 → D_2 (cost = 3).
Allocate min {(S_1_supply = 6, D_2_demand = 5)} = 5 → x_{12} = 5.
Update: S_1 supply 6 → 1; D_2 demand 5 → 0 (D_2 finished).

Godown → Relief Camp	D_2	D_3	D_4	Supply	Penalty
S_1	[5] 3	11	7	~~6~~ 1	4.0
S_3	8	15	9	3	0.875
Demand	~~5~~ 0	3	1		
Penalty	3.125	2.0	0.5		

Iteration 4-

Active rows: S_1 (1), S_3 (3). Active cols: D_3 (3), D_4 (1).

Compute Penalty & CR:
Rows-
S_1 (D_3, D_4) = [11,7] → Penalty = 4, CR = 11 − 7 = 4.
S_3 (15,9) → Penalty = 6, CR = 15 − 9 = 6.
Cols-
D_3: {11,15} → Penalty = 4, CR = 15 − 11 = 4.
D_4: {7,9} → Penalty = 2, CR = 9 − 7 = 2.
R_{max} = 6 (S_3). AS = Penalty × (CR / 6):
Rows: S_1 = 4 * (4/6) ≈ 2.666667; S_3 = 6 * (6/6) = 6.0
Cols: D_3 = 4 * (4/6) ≈ 2.666667; D_4 = 2 * (2/6) ≈ 0.666667
Highest AS = S_3 (6.0) → choose row S_3.
Minimum-cost cell in S_3 among active columns is S_3 → D_4 (cost = 9).
Allocate min {(S_3_supply = 3, D_4_demand = 1)} = 1 → x_{34} = 1.
Update: S_3 supply 3 → 2; D_4 demand 1 → 0 (D_4 finished).

Godown → Relief Camp	D_3	D_4	Supply	Penalty
S_1	11	7	1	2.666667
S_3	15	[1] 9	~~3~~ 2	6.000000
Demand	3	~~1~~ 0		
Penalty	2.666667	0.666667		

Iteration 5-

Supplies: S_1 = 1, S_2 = 2

Demands: D_3 = 3

Compute Penalty and CR:

Rows-

S_1 (Only D_3 = 11): → Penalty = 11 − 11 = 0, CR = 11 − 11 = 0.

S_2 (Only D_3 = 15): → Penalty = 15 − 15= 0, CR = 15 − 15 = 0.

Columns-

D_3(11, 15): two entries 11 & 15 → Penalty = 4. CR = 15 − 11 = 4.

R_{max} = 4 (from row S_2} We compute: AS = Penalty × (CR / R_{max}). Since R_{max} = 4, CR/R_{max} will be CR/4.

I show exact fraction then decimal (rounded to 6 decimal places):

Rows-

AS for column $S_1 = 0 \times \frac{0}{4} = 0$

AS for column $S_2 = 0 \times \frac{0}{4} = 0$

Columns-

AS for row $D_3 = 4 \times \frac{4}{4} = \frac{16}{4} \approx 4.000000$

Thus Highest Adjusted Score (≈ 4.000000) which is adjusted score for D_3.

Now we choose minimum - cost cell in D_3 which is S_1 with minimum value 11.

Allocate min $\{(S_1\_supply = 1, D_3\_demand = 3)\} = 1 \rightarrow x_{13} = 1$

Update: S_1 supply 1 → 0; D_3 demand 3 → 2. (S_1 = finished)

Refinery → Depot	D_3	Supply	Penalty
S_1	[1] 11	~~1~~ 0	0
S_2	15	2	0
Demand	~~3~~ 2		
Penalty	4		

Iteration 6-

Supplies: $S_2 = 2$

Demands: $D_3 = 2$

Only one feasible cell: $S_2 \rightarrow D_3$. Allocate $x_{23} = 2$ (S_2, D_3 finished)

Refinery → Depot	D_3	Demand
S_2	[2] 15	~~2~~ 0
Demand	~~2~~ 0	

Final Allotment-

Godown → Relief Camp	D_1	D_2	D_3	D_4	Supply
S_1	2	[5] 3	[1] 11	7	~~6~~ ~~1~~ 0
S_2	1	0	6	[1] 1	~~1~~ 0
S_3	[7] 5	8	[2] 15	[1] 9	~~10~~ ~~3~~ ~~2~~ 0

Demand	~~7~~ 0	~~5~~ 0	~~3~~ ~~2~~ 0	~~2~~ ~~1~~ 0	17

Total Cost by PARM Method = 5 × 3 + 1 × 11 + 1 × 1 +7 × 5 + 2 × 15 + 1 × 9 = 101

Consider the transportation problem in Table.

COMPARISION: ALLOCATIONS OBTAINED BY DIFFERENT METHODS.

Method	NWCR	LCM	VAM	COR	Penalty - Adjusted Range Method
Allocations	$x_{11} = 6$, $x_{21} = 1$, $x_{32} = 5$, $x_{33} = 3$, $x_{34} = 2$	$x_{11} = 6$, $x_{22} = 1$, $x_{31} = 1$, $x_{32} = 4$, $x_{33} = 3$, $x_{34} = 2$	$x_{11} = 6$, $x_{12} = 5$, $x_{24} = 1$, $x_{31} = 6$, $x_{33} = 6$, $x_{34} = 1$	$x_{11} = 6$, $x_{24} = 1$, $x_{31} = 1$, $x_{32} = 5$, $x_{33} = 3$, $x_{34} = 1$	$x_{12} = 5$, $x_{13} = 1$, $x_{24} = 1$, $x_{31} = 7$, $x_{33} = 2$, $x_{34} = 1$
Total cost	116	112	102	112	101

Thus, our proposed method The Penalty – Adjusted Range Method (PARM) gives minimum transportation cost (101) compared to all other methods.

IV. CONCLUSION

This paper proposed a new hybrid approach for solving the transportation problem by integrating penalty concepts with the Coefficient of Range methodology. The inclusion of the range penalty factor improves allocation decisions, yielding lower transportation costs in fewer iterations.

From the comparative study, it is evident that the proposed method provides:

- Better cost efficiency than NWCR, LCM, and Coefficient of Range.
- Competitive or superior results compared to VAM.
- A systematic balance between simplicity and optimality.

Future work may extend this approach to unbalanced, fuzzy, and multi-objective transportation problems.

V. FUTURE WORK

The Penalty–Adjusted Range Method introduced in this study demonstrates promising results for generating cost-efficient initial feasible solutions in classical balanced transportation problems. However, several directions remain open for future research to broaden its applicability and effectiveness.

1. Unbalanced Transportation Problems: Extending the methodology to unbalanced cases where total supply and demand are unequal would increase its real-world relevance. PARM can handle this by adding a dummy row/column to balance the problem.
2. Degeneracy Handling: Degeneracy occurs when the number of allocations is less than $m + n - 1$, stopping the solution from progressing. Investigating systematic strategies (like ε-allocations or penalty-based corrections) to manage degeneracy within the proposed framework may further improve its stability and robustness.

3. Multi-Objective Extensions: The method can be adapted for multi-objective transportation problems, where criteria such as time, risk, or environmental impact are optimized alongside cost. Each objective (cost, time, risk, etc.) can be converted into a penalty score or weighted factor. The method then balances these penalties to give an optimal trade - off solution.

4. Fuzzy and Stochastic Models: Incorporating fuzziness and probabilistic elements into the penalty-adjusted range framework would enable decision-making under uncertainty, making it suitable for modern logistics and supply chain systems. PARM already uses penalty-adjusted evaluation. This penalty mechanism can be extended to handle uncertainty. In fuzzy models, membership functions (like “low”, “medium”, “high - cost”) can be converted into penalty scores. In stochastic models, probability distributions can be reflected through expected penalties. Thus, PARM can still find a feasible and robust solution even when the data is not precise.

5. Large-Scale Computational Testing: Benchmarking the proposed approach against metaheuristic and machine learning-based optimization techniques on large-scale datasets could provide deeper insights into scalability and efficiency. Penalty Mechanism: PARM simplifies the search for an optimal solution by using adjusted penalties, which reduces computational effort. Even in large datasets, PARM follows a systematic allocation process, avoiding unnecessary iterations. It can be benchmarked against metaheuristics (Genetic Algorithms, PSO, ACO) and PARM can handle large-scale datasets more efficiently because penalty adjustments make the problem solvable faster than brute-force or traditional iterative methods.

6. Hybridization with Metaheuristics: Future work may also explore integration with evolutionary algorithms, such as genetic algorithms, particle swarm optimization, or ant colony optimization, to enhance solution quality. PARM can provide a high-quality initial feasible solution quickly. This starting solution can then be improved further using metaheuristics.

Example- Hybrid approach:

Step 1: Use PARM to generate a feasible, near-optimal solution.

Step 2: Apply GA/PSO/ACO to explore the search space and refine the solution.

Through these extensions, the Penalty – Adjusted Range Method can evolve from a classical initial solution technique to a versatile decision-support tool applicable in diverse and complex transportation environments.

REFERENCES

[1] Charnes, A., Cooper, W.W. (1954). Introduction to Linear Programming. Wiley.

[2] Dantzig, G.B. (1951). Application of the simplex method to a transportation problem. Wiley.

[3] Hamdy A. Taha (2011). Operations Research: An Introduction. 9th Ed. Pearson.

[4] Hitchcock, F.L. (1941). The distribution of a product from several resources to numerous localities. J. Math. Phys. 20:224–230.

[5] Koopmans, T.C. (1947). Optimum utilization of the transportation system. Econometrica.

[6] Radthy, Zainb Hassan, Firas Hussean Maghool, and Alaa Hussein Khaleel. "Application of Linear Programming According to Transportation Problem on Real Data." International Journal of Scientific & Technology Research (IJSTR), vol. 8, no. 1, 2019, pp. 100–102.

[7] Seethalakshmy, A., and N. Srinivasan. "Solving Real-Life Problem Using Transportation." International Journal of Innovative Technology and Exploring Engineering (IJITEE), vol. 8, no. 10, 2019, pp. 4284–4287. BEIESP. https://doi.org/10.35940/ijitee.J1062.0881019.

[8] Sharma, Neetu M., and Ashok P. Bhadane. "An Alternative Method to North-West Corner Method for Solving Transportation Problem." International Journal for Research in Engineering Application & Management (IJREAM), vol. 1, no. 12, 2016, pp. 1–3.

[9] S.P. Gupta. Statistical Methods. S. Chand Publishing.

[10] Zabiba, Mohammed S. M., and Noor Haider Ali AlKhafaji. "The New Technique for Solving Transportation Problem." International Journal of Mechanical Engineering, vol. 7, no. 1, 2022, pp. 6276–6280.