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SOFT $\alpha\omega\ddot{I}_s$ – HOMEOMORPHISM IN SOFT IDEAL TOPOLOGICAL SPACES

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Abstract: In this paper, we introduce a soft $\alpha\omega\ddot{I}_s$ – homeomorphism in soft ideal topological spaces. Furthermore, we introduce to Enlightment and Edification of some properties, theorems and examples, which is the of the concept of soft mappings in soft ideal topological spaces.

Keywords: Soft set, soft ideal topological space, Soft $\alpha\omega\ddot{I}_s$ -Closed set, Soft continuous, Soft $\alpha\omega\ddot{I}_s$ -continuous, Soft homeomorphism, Soft $\alpha\omega\ddot{I}_s$ – homeomorphism.

I. INTRODUCTION

The concept of soft set theory was introduced by Molodstov [14] and the concept of soft ideal theory, soft local function was introduced by A.Kandil,et.al [12]. The concepts of soft α-open and soft continuous function was introduced by M.Akdag and A.Ozkan [2]. The concepts of soft αω-closed sets was introduced by S.Jafari,et.al [9]. The concepts of soft $\alpha\omega\ddot{I}_s$- Closed sets was introduced by N.Chandramathi and V.Kiruthika [4]. The concepts of soft homeomorphism was introduced by S.Jackson and J.Carlin [11].

II. PRELIMINARIES

2.1 DEFINITION [1]

Let D be a non-empty subset of E and a soft set over U is a parameterized family of subsets of an initial universe U. For a particular e ∈ E, F(e) may be considered the set of e-approximate elements of the soft set (F,E) and if e ∉ E, then F(e) = ∅ ,that is, $(F, \mathrm{E}) = \{f(e): e \in Đ \subseteq \mathrm{E}\,, f: \mathrm{E} \rightarrow P(U)\}$ is called a soft set over U. Then the family of all these soft sets denoted by $SS(U)_E$.

2.2 DEFINITION [7]

Let Ƭ$_s$ be the collection of soft sets over U. Then Ƭ$_s$ is said to be a soft topology on U if satisfies the following axioms:

(i). (∅, E), (U, E) belongs to Ƭ$_s$

(ii). The union of any number of soft sets in Ƭ$_s$ belongs to Ƭ$_s$

(iii). The intersection of two number of soft sets in Ƭ$_s$ belongs to Ƭ$_s$.

The triplet (U,Ƭ$_s$,E) is said to be soft topological space and we note that the member of Ƭ$_s$ are said to be Ƭ$_s$-soft open sets.

2.3 DEFINITION [1]

Let Ī be a non-null collection of soft sets over an initial universe U with the same set of parameter E. Then $\ddot{I}_s$ containing $SS(U)_E$ is called as a soft Ideal on U with same set E if,

(i). $(F, E) \in Ī$ and $(G, E) \in Ī$ then $(F, E) \cup (G, E) \in Ī$.

(ii). $(F, E) \in Ī$ and $(G, E) \subseteq (F, E)$ then $(G, E) \in Ī$.

2.4 DEFINITION [5]

Let (X,Ꞇs,E) and (Y,Ωs,N) be a soft topological spaces. Then E and N are parameters. A soft mapping f: (X,Ꞇs,E) → (Y,Ωs,N) is said to be soft continuous if the inverse image of soft closed set in (Y,Ωs,N) is soft closed in (X,Ꞇs,E).

2.5 DEFINITION [5]

Let (X,Ꞇs,E) and (Y,Ωs,N) be a soft topological spaces. Then E and N are parameters and f: (X,Ꞇs,E) → (Y,Ωs,N) be a soft mapping. If f is a bijection, soft continuous mapping, then f is said to be soft homeomorphism from X to Y. When a homeomorphism f exist between X and Y, we say that X is soft homeomorphic to Y.

III. SOFT αωÏs – CLOSED MAPPINGS IN SOFT IDEAL TOPOLOGICAL SPACES

In this section we introduce and study about a new continuous mapping is known as soft αωÏs – continuous mapping in Soft Ideal topological spaces.

3.1 DEFINITION

Let (Ṿ,Ꞇs,Q) and (Ẉ,Ωs,Ń,Ï1s) be a soft and soft ideal topological spaces. Then Q and N are parameters. Suppose that μ : Ṿ → Ẉ and ρ : Q → Ń are soft mappings and $m_{\rho\mu}$: SS(Ṿ)$_Q$ → SS(Ẉ)$_Ń$ is a soft mapping, where SS(Ṿ)$_Q$ and SS(Ẉ)$_N$ are two familes. Then the soft mapping is said to be soft αωÏs – closed if the image of every closed in (Ṿ,Ꞇs,Q) is soft αωÏs – closed in (Ẉ,Ωs,Ń,Ï1s).

3.2 EXAMPLE

Let Ṿ = {α,β,γ}, Q = {σ_1,σ_2}, Ꞇs = {({}, Q),(Ṿ, Q),(Ꞙ1, Q),(Ꞙ2, Q), (Ꞙ3, Q)}, where (Ꞙ1,Q) = {{},{β,γ}}, (Ꞙ2,Q) = {{γ},{α,γ}}, (Ꞙ3, Q) = {Ṽ,{γ}}, (Ꞙ4, Q)} = {{̃},{α}}.
Let Ẉ = {δ,η,λ}, Ń = {π_1,π_2}, Ωs = {({}, Ń),(Ẉ, Ń),(G1,Ń),(G2,Ń)}, where, (G1,Ń) = {{λ},{δ,λ}}, (G2,Ń)} = {Ẉ̃,{η,λ}} and Ïs = (G3,Ń) = {{̃},{δ}}. Then we define $m_{\rho\mu}$ (α) = {δ}, $m_{\rho\mu}$ (γ) ={η}, m(β) = {λ}. Clearly, $m_{\rho\mu}$ (Ꞙ3,Q)^c = $m_{\rho\mu}$ (Ṽ,{γ})^c is soft αωÏs – closed mapping in (Ẉ,Ωs,Ń,Ï1s).

3.3 THEOREM

A soft mapping $m_{\rho\mu}$: (Ṿ,Ꞇs,Q) → (Ẉ,Ωs,Ń,Ï1s) is soft αωÏs – closed if and only if Soft αωÏs – Cl^s($m_{\rho\mu}$(Ꞙ,Q)) ⊆ $m_{\rho\mu}$ (Cl^s (Ꞙ,Q)) for every subset (Ꞙ,Q) of (Ṿ,Ꞇs,Q).

PROOF.

NECESSARY PART : Suppose that a soft mapping $m_{\rho\mu}$ is soft αωÏs – Closed and (Ꞙ,Q) ⊆ (Ṿ,Ꞇs,Q). Then $m_{\rho\mu}$(Cl^s (Ꞙ,Q)) is soft αωÏs- closed in (Ẉ,Ωs,Ń,Ï1s), we have $m_{\rho\mu}$ (Ꞙ,Q) ⊆ $m_{\rho\mu}$ (Cl^s (Ꞙ,Q))) and by lemma (1) " For any (L,Q) ⊆ (Ṿ,Ꞇs,Q), (i) Soft αωÏs – Cl^s(L,Q) is the smallest Soft αωÏs – closed set containing (L,Q), (ii) (L,Q) is Soft αωÏs – closed if and only if Soft αωÏs – Cl^s(L,Q) = Cl^s(L,Q)" and by lemma(2) " For any two subsets (L1,Q) and (,L2,Q) of (Ṿ,Ꞇs,Q,Ïs), (i) If (L1,Q) ⊆ (L2,Q) then Soft αωÏs – Cl^s(L1,Q) ⊆ Soft αωÏs – Cl^s(L2,Q), (ii) Soft αωÏs – Cl^s((L1,Q) ∩ (L2,Q)) ⊆ [Soft αωÏs – Cl^s(L1,Q)] ∩ [Soft αωÏs – Cl^s(L2,Q)], By using above two theorems, Soft αωÏs – Cl^s($m_{\rho\mu}$(Ꞙ,Q)) ⊆ Soft αωÏs – Cl^s($m_{\rho\mu}$(Ꞙ,Q)) therefore, Soft αωÏs – Cl^s($m_{\rho\mu}$(Ꞙ,Q)) = $m_{\rho\mu}$(Cl^s(Ꞙ,Q)).

SUFFICENCY PART : Let (Ꞙ,Q) be any soft closed set in (Ṿ,Ꞇs,Q). Then (Ꞙ,Q) = Cl^s(Ꞙ,Q) and so $m_{\rho\mu}$(Ꞙ,Q) = $m_{\rho\mu}$(Cl^s(Ꞙ,Q)) containing Soft αωÏs – Cl^s($m_{\rho\mu}$(Ꞙ,Q)), by hypothesis, we have, $m_{\rho\mu}$(Ꞙ,Q) ⊆ Soft αωÏs – Cl^s($m_{\rho\mu}$(Ꞙ,Q)) by lemma (1), Therefore, $m_{\rho\mu}$(Ꞙ,Q) = Soft αωÏs - Cl^s($m_{\rho\mu}$(Ꞙ,Q)). That is $m_{\rho\mu}$(Ꞙ,Q) is Soft αωÏs – closed and Hence $m_{\rho\mu}$ is a soft αωÏs – Closed.

3.4 THEOREM

Let $m_{\rho\mu}$: (Ṿ,Ꞇs,Q) → (Ẉ,Ωs,Ń,Ï1s) be a soft mapping such that Soft αωÏs – Cl^s($m_{\rho\mu}$(Ꞙ,Q)) ⊆ $m_{\rho\mu}$(Cl^s (Ꞙ,Q)) for every subset (Ꞙ,Q) ⊆ (Ṿ,Ꞇs,Q). Then the image $m_{\rho\mu}$(Ꞙ,Q) of a soft closed set (Ꞙ,Q) in (Ṿ,Ꞇs,Q) is Ꞇ$_{s\alpha\omega Ïs}$ – soft closed in (Ẉ,Ωs,Ń,Ï1s).

PROOF.

Let (Ꞙ,Q) be a soft closed set in (Ṿ,Ꞇs,Q), Then by hypothesis, Soft αωÏs - Cl^s($m_{\rho\mu}$(Ꞙ,Q)) ⊆ $m_{\rho\mu}$(Cl^s (Ꞙ,Q)) = $m_{\rho\mu}$(Ꞙ,Q) and so Soft αωÏs – Cl^s($m_{\rho\mu}$(Ꞙ,Q)), Therefore $m_{\rho\mu}$(Ꞙ,Q) is Ꞇ$_{s\alpha\omega Ïs}$ – soft closed in (Ẉ,Ωs,Ń,Ï1s).

3.5 THEOREM

A soft mapping $m_{\rho\mu}$: (Ṿ,Ꞇ$_s$,Q) → (Ẉ,Ω_s,Ń,Ï$_{1s}$) be a soft αωÏ$_s$ – Closed if and only if for each soft subset (ʄ,Q) of (Ẉ,Ω_s,Ń,Ï$_{1s}$) and for each soft closed set (Ħ,Q)^c containing $m_{\rho\mu}^{-1}$(ʄ,Q) there is a Soft αωÏ$_s$ – Closed set (Ż, Ń)^c of (Ẉ,Ω_s,Ń,Ï$_{1s}$) such that (ʄ,Q) ⊆ (Ż, Ń)^c and $m_{\rho\mu}^{-1}$(Ż, Ń)^c ⊆ (Ħ, Q)^c.

PROOF.

NECESSARY PART : Suppose that $m_{\rho\mu}$ is a Soft αωÏ$_s$ – Closed. Let ((ʄ,Q) ⊆ (Ẉ,Ω_s,Ń,Ï$_s$) and (Ħ, Q)^c be a Soft closed set of (Ṿ,Ꞇ$_s$,Q) such that $m_{\rho\mu}^{-1}$(ʄ,Q) ⊆ (Ħ, Q)^c. Then (Ż, Ń)^c = $m_{\rho\mu}$((Ħ, Q)^c) is a Soft αωÏ$_s$ – closed set containing (ʄ,Q) Such that $m_{\rho\mu}^{-1}$(Ż,Ń)^c ⊆ (Ħ, Q)^c.

SUFFICIENT PART : Let $m_{\rho\mu}$ be a soft closed set of (Ṿ,Ꞇ$_s$,Q). Then $m_{\rho\mu}^{-1}$($m_{\rho\mu}$ (M)) ⊆ M, by assumption, there exists a Soft αωÏ$_s$ – Closed set (Ż, Ń)^c of (Ẉ,Ω_s,Ń,Ï$_{1s}$). Such that ($m_{\rho\mu}$(M)) ⊆ (Ż, Ń)^c and $m_{\rho\mu}^{-1}$(Ż, Ń)^c ⊆ M and so M ⊆ $m_{\rho\mu}^{-1}$(Ż, Ń)^c. Hence, (Ż, Ń)^c ⊆ $m_{\rho\mu}$ (M)^c ⊆ ($m_{\rho\mu}$ ($m_{\rho\mu}^{-1}$(Ż, Ń)^c)) ⊆ $m_{\rho\mu}^{-1}$(Ż, Ń)^c which implies $m_{\rho\mu}$(M) = (Ż, Ń)^c, Since (Ż, Ń)^c is Soft αωÏ$_s$ – Closed, $m_{\rho\mu}$ (M) is a Soft αωÏ$_s$ – Closed and therefore, $m_{\rho\mu}$ is a Soft αωÏ$_s$ –Closed.

3.6 THEOREM

If $m_{\rho\mu}$: (Ṿ,Ꞇ$_s$,Q,Ï$_s$) → (Ẉ,Ω_s,Ń,Ï$_{1s}$) is a Ï$_s$ – soft irresolute αωÏ$_s$ – Closed and (ʄ,Q) is a Soft αωÏ$_s$ – Closed subset of (Ṿ,Ꞇ$_s$,Q,Ï$_s$), then $m_{\rho\mu}$(ʄ,Q) is a Soft αωÏ$_s$ –Closed.

PROOF.

Let (Ħ, Q) is a Soft α-Ï$_s$ –open set in (Ẉ,Ω_s,Ń,Ï$_{1s}$) such that $m_{\rho\mu}$ (ʄ,Q) ⊆ (Ħ, Q). Since $m_{\rho\mu}$ is Ï$_s$ – soft irresolute, $m_{\rho\mu}^{-1}$(Ħ, Q) is a soft α-Ï$_s$ –open containing (ʄ,Q). Hence ωCl*^{s}(ʄ,Q) ⊆ $m_{\rho\mu}^{-1}$(Ħ, Q) as (ʄ,Q) is a Soft αωÏ$_s$ –Closed in (Ṿ,Ꞇ$_s$,Q,Ï$_s$). Since $m_{\rho\mu}$ is Soft αωÏ$_s$ –Closed, $m_{\rho\mu}$(ωCl*^{s}(ʄ,Q)) is a Soft αωÏ$_s$ –Closed set containing in the Soft α-Ï$_s$ –open (Ħ,Q), which implies that ωCl*^{s}($m_{\rho\mu}$(ωCl*^{s}(ʄ,Q) ⊆ (Ħ, Q) and hence ωCl*^{s}($m_{\rho\mu}$ (ʄ,Q)) ⊆ (Ħ, Q). Therefore, $m_{\rho\mu}$(ʄ,Q) is a Soft αωÏ$_s$ –Closed.

The following example shows that is composition of two Soft αωÏ$_s$ –Closed mappings are not be Soft αωÏ$_s$ – Closed.

3.7 EXAMPLE

Let Ṿ = Ẉ = {α,β,γ}, Q = Ń = {σ_1,σ_2}, Ꞇ$_s$ = {({}, Q),(Ṿ,Q),(ʄ$_1$,Q), (ʄ$_2$, Q)}, where, (ʄ$_1$,Q) ={{α},{α,β}},(ʄ$_2$,Q){$\widetilde{\{\}}$,{β}}, Ω_s = {({}, Ń),(Ẉ, Ń),(G$_1$,Ń),(G$_2$,Ń)}, where, (G$_1$,Ń) = {$\widetilde{\{\}}$,{ β,γ }}, (G$_2$,Ń)} = {$\widetilde{W}$,{α}}. Ï$_{2s}$ = $\widetilde{\{\}}$. Then we define, $m_{\rho\mu}$(α) = {α}, $m_{\rho\mu}$ (β) = {β}, $m_{\rho\mu}$(γ) ={γ}. Let Ż = {δ,η,λ}, S = {π_1,π_2}, ŕ$_s$ = {({}, Ń),(Ẉ, Ń),(L$_1$,Ń),(L$_2$,Ń)} where, (L$_1$,Ń) = {$\widetilde{\{\}}$,$\widetilde{W}$}, (L$_2$,Ń) = {$\widetilde{\{\}}$,{δ, λ}}. Ï$_{3s}$ = {$\widetilde{\{\}}$,{λ}} = (L$_3$,Ń). Let $m_{\rho\mu}$: (Ṿ,Ꞇ$_s$,Q) → (Ẉ,Ω_s,Ń,Ï$_{1s}$) be a identity function and $p_{\rho\mu}$: (Ẉ,Ω_s,Ń,Ï$_{1s}$) → (Ż, ŕ$_s$,Ō,Ï$_{2s}$). Then we define, $p_{\rho\mu}$(α) = {δ}, $p_{\rho\mu}$(β) = {η}, $p_{\rho\mu}$(γ) ={λ}. Then both $m_{\rho\mu}$ and $p_{\rho\mu}$ are Soft αωÏ$_s$ –Closed mapping. But their composition, (p°m)$_{\rho\mu}${α} = $p_{\rho\mu}$ ($m_{\rho\mu}$(α)) = $p_{\rho\mu}$(α) = {δ}. Since for the soft closed set {α} in (Ṿ,Ꞇ$_s$,Q) and (p°m)$_{\rho\mu}${α} = {δ}, which is not a Soft αωÏ$_s$ –Closed in (Ż,ŕ$_s$,Ō,Ï$_{2s}$).

3.8 THEOREM

Let $m_{\rho\mu}$: (Ṿ,Ꞇ$_s$,Q,Ï$_s$) → (Ẉ,Ω_s,Ń,Ï$_{1s}$) be a Soft αωÏ$_s$ –Closed and p : (Ẉ,Ω_s,Ń,Ï$_{1s}$) → (Ż, ŕ$_s$, Ō,Ï$_{2s}$) be a Soft αωÏ$_s$ –Closed and Ï$_s$ – soft irresolute, where Ï$_s$,Ï$_{1s}$,Ï$_{2s}$ are soft ideals on Ṿ, Ẉ and Ż respectively, Then their composition $p_{\rho\mu}$°$m_{\rho\mu}$: (Ṿ,Ꞇ$_s$,Q,Ï$_s$) → (Ż, ŕ$_s$,Ō,Ï$_{2s}$) is a Soft αωÏ$_s$ – Closed in (Ż, ŕ$_s$,Ō,Ï$_{2s}$).

PROOF.

Let (ʄ,Q) is a soft closed set of (Ṿ,Ꞇ$_s$,Q,Ï$_s$). Then by hypothesis $m_{\rho\mu}$ (ʄ,Q) is a Soft αωÏ$_s$ –Closed set of (Ẉ,Ω_s,Ń,Ï$_{1s}$). Since $p_{\rho\mu}$ is Soft αωÏ$_s$ –Closed and Ï$_s$ – soft irresolute by theorem 3.6, $p_{\rho\mu}$ ($m_{\rho\mu}$ (ʄ,Q)) = ($p_{\rho\mu}$°$m_{\rho\mu}$)(ʄ,Q) is a Soft αωÏ$_s$ –Closed in (Ż, ŕ$_s$,Ō,Ï$_{2s}$) and therefore, ($p_{\rho\mu}$°$m_{\rho\mu}$) is a Soft αωÏ$_s$ –Closed.

IV. SOFT αωÏ$_s$ – HOMEOMORPHISM IN SOFT IDEAL TOPOLOGICAL SPACES

In this section we introduce and study about a new mapping is known as soft αωÏ$_s$ – homeomorphism in Soft Ideal topological space and we workout some basic theorems and examples.

4.1 DEFINITION

Let (Ṿ,Ꞇ$_s$,Q,Ï$_s$) and (Ẉ,Ω_s,Ń,$m_{\rho\mu}$(Ï$_s$)) be a soft ideal topological spaces and $m_{\rho\mu}$: (Ṿ,Ꞇ$_s$,Q,Ï$_s$) → (Ẉ,Ω_s,Ń,$m_{\rho\mu}$(Ï$_s$)) be a soft mappings. Then Q and Ń are parameters. Suppose that μ : Ṿ → Ẉ and ρ : Q → Ń are soft mappings and $m_{\rho\mu}$: SS(Ṿ)$_Q$ → SS(Ẉ)$_Ń$ is a soft mapping, where SS(Ṿ)$_Q$ and SS(Ẉ)$_Ń$ are two families. Then a bijective soft function is said to be soft αωÏ$_s$ - homeomorphism, if $m_{\rho\mu}$(Ż,Ń) and $m^{-1}{}_{\rho\mu}$(Ż,Ń) is both soft αωÏ$_s$ – continuous and soft αωÏ$_s$ – Closed.

4.2 EXAMPLE

Let Ṿ = {α,β,γ}, Q = {σ_1,σ_2}, Ꞇ$_s$ = {({},Q),(Ṿ,Q),(Ꝭ$_1$,Q),(Ꝭ$_2$,Q)}and Ï$_s$ = (Ꝭ$_3$,Q) where, (Ꝭ$_1$,Q) = {{β},{β,γ}}, (Ꝭ$_2$, Q) = {$\widetilde{Ṿ}$,{β}}, (Ꝭ$_3$, Q)} = {$\widetilde{\{\}}$,{β}}.
Let Ẉ = {δ,η,λ}, Ń = {π_1,π_2}, Ω_s = {({}, Ń),(Ẉ, Ń),(G$_1$,Ń),(G$_2$,Ń)}, $m_{\rho\mu}$(Ï$_s$) = (G$_2$,Ń) where, (G$_1$,Ń) = {{δ},{δ,η}}, (G$_2$,Ń)} = {$\widetilde{Ẉ}$,{η}}. Then we define $m_{\rho\mu}$(δ) = {α}, $m_{\rho\mu}$(η) ={β}, $m_{\rho\mu}$(λ) = {γ}. Then we define $m_{\rho\mu}$: (Ṿ,Ꞇ$_s$,Q,Ï$_s$) → (Ẉ,Ω_s,Ń,$m_{\rho\mu}$(Ï$_s$)) be a soft mapping. Then $m_{\rho\mu}$ is bijective, Soft αωÏ$_s$ – continuous and Soft αωÏ$_s$ – Closed. So $m_{\rho\mu}$ is Soft αωÏ$_s$ – homeomorphism.

4.3 THEOREM

Every soft homeomorphism in (Ṿ,Ꞇ$_s$,Q) is a Soft αωÏ$_s$ – homeomorphism in (Ṿ,Ꞇ$_s$,Q,Ï$_s$).

PROOF.

Let m : (Ṿ,Ꞇ$_s$,Q,Ï$_s$) → (Ẉ,Ω_s,Ń,$m_{\rho\mu}$(Ï$_s$)) be a soft homeomorphism. Then $m_{\rho\mu}$ and $m_{\rho\mu}^{-1}$ are soft continuous and $m_{\rho\mu}$ is bijective. As every soft continuous function is Soft αωÏ$_s$ – continuous, we have $m_{\rho\mu}$ and $m_{\rho\mu}^{-1}$ are Soft αωÏ$_s$ – continuous. Therefore $m_{\rho\mu}$ is Soft αωÏ$_s$ – homeomorphism. The soft mapping in Example 4.2 is Soft αωÏ$_s$ – homeomorphism but not a soft homeomorphism because it is not soft continuous.

4.4 THEOREM

Every soft α-homeomorphism in (Ṿ,Ꞇ$_s$,Q) is a Soft αωÏ$_s$ – continuous in (Ṿ,Ꞇ$_s$,Ń,Ï$_s$).

PROOF.

Let $m_{\rho\mu}$: (Ṿ,Ꞇ$_s$,Q,Ï$_s$) → (Ẉ,Ω_s,Ń,$m_{\rho\mu}$(Ï$_s$)) be a soft α-homeomorphism in (Ṿ,Ꞇ$_s$,Q). By hypothesis, $m_{\rho\mu}$ is bijective and for every soft α-closed (Ż, Ń)^c ⊑ (Ṿ,Ꞇ$_s$,Q,Ï$_s$) the image $m_{\rho\mu}$(Ṿ,Q) is soft α-closed in (Ẉ,Ω_s, Ń). We have to show that, m is Soft αωÏ$_s$ – continuous, that is, for every Soft αωÏ$_s$ – closed set (Ż, Ń)^c ⊑ (Ẉ,Ń) the preimage $m_{\rho\mu}^{-1}$(Ż, Ń)^c is soft Soft αωÏ$_s$ – closed in (Ṿ,Ꞇ$_s$,Q,Ï$_s$). Then we take an arbitrary Soft αωÏ$_s$ – closed (Ż, Ń)^c ⊑ (Ẉ,Ω_s,Ń). By the assumption on Soft αωÏ$_s$ – closed sets, (Ż, Ń)^c can be written as a soft union of Soft α– closed sets (Ż, Ń)^c = ∪ (Ż, Ń)$_i^c$ with each (Ż, Ń)$_i^c$ is Soft α– closed in (Ẉ,Ω_s,Ń).
The converse of the above theorem is not true as seen in the following example.

4.5 EXAMPLE

Let Ṿ = {α,β,γ}, Q = {σ_1,σ_2}, Ꞇ$_s$ = {({},Q),(Ṿ,Q),(Ꝭ$_1$,Q),(Ꝭ$_2$,Q)} and Ï$_s$ = (Ꝭ$_3$,Q) where, (Ꝭ$_1$,Q) = {{β},{β,γ}}, (Ꝭ$_2$,Q) = {$\widetilde{Ṿ}$,{β}}, (Ꝭ$_3$,Q)} = {$\widetilde{\{\}}$,{β}}.
Let Ẉ = {δ,η,λ}, Ń = {π_1,π_2}, Ω_s = {({}, Ń),(Ẉ, Ń),(G$_1$,Ń),(G$_2$,Ń)}, $m_{\rho\mu}$(Ï$_s$) = (G$_2$,Ń) where, (G$_1$,Ń) = {{δ},{δ,η}}, (G$_2$,Ń)} = {$\widetilde{Ẉ}$,{η}}. Then we define $m_{\rho\mu}$(δ) = {α}, $m_{\rho\mu}$(η) ={β}, $m_{\rho\mu}$(λ) = {γ}. Then we define $m_{\rho\mu}$: (Ṿ,Ꞇ$_s$,Q,Ï$_s$) → (Ẉ,Ω_s,Ń,$m_{\rho\mu}$(Ï$_s$)) be a soft mapping. Then $m_{\rho\mu}$ is bijective, Soft αωÏ$_s$ – continuous and Soft αωÏ$_s$ – closed. but Soft αωÏ$_s$ - homeomorphism must be bijective between the Universes. So every Soft αωÏ$_s$ – continuous mapping is not Soft αωÏ$_s$ – homeomorphism in (Ṿ,Ꞇ$_s$,Q,Ï$_s$).

4.6 THEOREM

Every soft ω-homeomorphism in (Ṿ,Ꞇ$_s$,Q) is a Soft αωÏ$_s$ – homeomorphism in (Ṿ,Ꞇ$_s$,Q,Ï$_s$).

PROOF.

Let $m_{\rho\mu}$: (Ṿ,Ꞇ$_s$,Q,Ï$_s$) → (Ẉ,Ω_s,Ń,$m_{\rho\mu}$(Ï$_s$)) be a soft ω-homeomorphism in (Ṿ,Ꞇ$_s$, Q) So $m_{\rho\mu}$ is bijective and for every soft ω – closed set (W, Ń) ⊑ (Ṿ,Ꞇ$_s$,Q,Ï$_s$) the image of $m_{\rho\mu}$(W, Ń) is soft ω – closed in (Ẉ,Ω_s,Ń). Then we take an arbitrary Soft αωÏ$_s$ – closed (Ħ,Q)^c ⊑ (Ṿ,Ꞇ$_s$,Q). By the definition of soft αωÏ$_s$ – closed sets, (Ħ,Q)^c can be written as a soft union of soft ω – closed sets (Ħ,Q)^c = ∪ (W,Ń)$_i$ for all i belongs to I with each (W,Ń)$_i$ is Soft ω – closed in (Ṿ,Ꞇ$_s$,Q,Ï$_s$).

The converse of the above theorem is not true as seen in the following example.

4.7 EXAMPLE

Let Ṿ = {α,β,γ}, Q = { σ_1, σ_2 }, Ꞇs = {({},Q),(Ṿ,Q),(Ḟ1,Q),(Ḟ2,Q)} and Ï$_s$ = (Ḟ3,Q) where, (Ḟ1,Q) = {{β},{β,γ}}, (Ḟ2, Q) = {Ṽ,{β}}, (Ḟ3, Q)} = {{͂},{β}}.

Let Ẉ = {δ,η,λ}, Ń = { π_1, π_2 }, Ω$_s$ = {({}, Ń),(Ẉ, Ń),(Ǥ1,Ń),(Ǥ2,Ń)}, and m$_{ρμ}$(Ï$_s$) = (Ǥ2,Ń) where, (Ǥ1,Ń) = {{δ},{δ,η}}, (Ǥ2,Ń)} = {W̃,{η}}. Then we define m$_{ρμ}$(δ) = {α}, m$_{ρμ}$(η) ={β}, m$_{ρμ}$(λ) = {γ}. Then we define m$_{ρμ}$: (Ṿ,Ꞇ$_s$,Q,Ï$_s$) → (Ẉ,Ω$_s$,Ń,m$_{ρμ}$(Ï$_s$)) be a soft mapping. Then m$_{ρμ}$ is Soft αωÏ$_s$ – homeomorphism but not Soft ω – homeomorphism in (Ṿ,Ꞇ$_s$,Q,Ï$_s$).

4.8 THEOREM

Let m$_{ρμ}$: (Ṿ,Ꞇ$_s$,Q,Ï$_s$) → (Ẉ,Ω$_s$,Ń,m$_{ρμ}$(Ï$_s$)) be a bijective and Soft αωÏ$_s$ – continuous function. Then the following statements are equivalent :
(i) m$_{ρμ}$ is Soft αωÏ$_s$ – open mapping.
(ii) m$_{ρμ}$ is Soft αωÏ$_s$ – homeomorphism.
(iii) m$_{ρμ}$ is Soft αωÏ$_s$ – Closed mapping.

PROOF.

(i) → (ii) : If m$_{ρμ}$ is Soft αωÏ$_s$ – open, then m is Soft αωÏ$_s$ – homeomorphism. Since by assumption m$_{ρμ}$ is given to be bijective and Soft αωÏ$_s$ – continuous and m$_{ρμ}$ is Soft αωÏ$_s$ – open, then for any Soft αωÏ$_s$ – open set (Ħ, Q) ⊆ (Ṿ,Q), m(Ħ, Q) is Soft αωÏ$_s$ – open in (Ẉ,Ω$_s$, Ń). Hence, for any Soft αωÏ$_s$ – open set (Ż, Ń) ⊆ (Ẉ,Ω$_s$, Ń), then the (Ż, Ń) = m$_{ρμ}$ (Ħ, Q) for some (Ħ, Q) ⊆ (Ṿ,Ꞇ$_s$, Q), and since m$_{ρμ}$ is bijective, (Ħ, Q) = m$_{ρμ}^{-1}$(Ż, Ń) is Soft αωÏ$_s$ – open mapping. Therefore, m$_{ρμ}^{-1}$ is Soft αωÏ$_s$ – continuous. Thus, m$_{ρμ}$ is bijective, Soft αωÏ$_s$ – continuous mapping with a Soft αωÏ$_s$ – continuous inverse and which implies that m$_{ρμ}$ is Soft αωÏ$_s$ – homeomorphism.

(ii) → (iii) : If m$_{ρμ}$ is Soft αωÏ$_s$ – homeomorphism, then m$_{ρμ}$ is Soft αωÏ$_s$ – Closed mapping. Since m$_{ρμ}$ is Soft αωÏ$_s$ – homeomorphism, which implies that m$_{ρμ}$ is bijective, Soft αωÏ$_s$ – continuous and m$_{ρμ}^{-1}$ is Soft αωÏ$_s$ – continuous. Then m$_{ρμ}^{-1}$ maps Soft αωÏ$_s$ – open sets in (Ẉ,Ω$_s$,Ń) to Soft αωÏ$_s$ – open sets in (Ṿ,Ꞇ$_s$, Q), which means that it maps Soft αωÏ$_s$ – Closed sets to Soft αωÏ$_s$ – Closed sets under pre – image, thus m$_{ρμ}$ maps Soft αωÏ$_s$ – Closed sets in (Ẉ,Ω$_s$,Ń) to Soft αωÏ$_s$ – Closed sets in (Ṿ,Ꞇ$_s$,Q). Therefore, m$_{ρμ}$ is Soft αωÏ$_s$ – Closed mapping.

(iii) → (i) If m$_{ρμ}$ is Soft αωÏ$_s$ – Closed mapping, then m$_{ρμ}$ is Soft αωÏ$_s$ – open mapping.
Since m$_{ρμ}$ is bijective and m$_{ρμ}$ is Soft αωÏ$_s$ – continuous and now assuming m$_{ρμ}$ is Soft αωÏ$_s$ – Closed. Then consider m$_{ρμ}^{-1}$, which is also bijective and we know that m$_{ρμ}$ is homeomorphism which implies that m$_{ρμ}$ is Soft αωÏ$_s$ – open mapping.

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