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Flow Of Non-Newtonian Hyperbolic Tangent Fluid Through Porous Medium In A Planar Channel With Peristalsis With Slip And Hall Effects

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Abstract— In this paper, flow of Non-Newtonian Hyperbolic tangent fluid through a porous medium in a planar channel with peristalsis the effect of Slip and Hall effects under the assumption of long wavelength is investigated. A Closed form solutions are obtained for axial velocity and pressure gradient by employing perturbation technique. The effects of various emerging parameters on the pressure gradient, time averaged volume flow rate and frictional force are discussed with the aid of graphs.

Keywords: Hall, Non-Newtonian Hyperbolic tangent fluid, Hartmann number, long wavelength, peristaltic pumping, Darcy number, porous medium, slip condition, Reynolds number, Weissenberg number, power-law index, pressure gradient, perturbation

I. INTRODUCTION

Extensive study of peristalsis has been carried out for a Non-Newtonian with a periodic train of sinusoidal peristaltic waves. The inertia – free peristaltic transport with long wavelength analysis was given by Shapiro et al. (1969). The early developments on the mathematical modelling and experimental fluid mechanics of peristaltic flow were given in a comprehensive review by Jafrin and Shapiro (1971). However, the rheological properties of the fluids can affect these characteristics significantly. Moreover, most of the physiological fluids are known to be non-Newtonian. It is well known that some fluids which are encountered in chemical applications do not adhere to the classical Newtonian viscosity prescription and are accordingly known as non-Newtonian fluids. One especial class of fluids which are of considerable practical importance is that in which the viscosity depends on the shear stress or on the flow rate. The viscosity of most non-Newtonian fluids, such as polymers, is usually a nonlinear decreasing function of the generalized shear rate. This is known as shear-thinning behaviour. Such fluid is a hyperbolic tangent fluid (Ai and Vafai, 2005). Nadeem and Akram (2009) have first investigated the peristaltic flow of a hyperbolic tangent fluid in an asymmetric channel. Nadeem and Akbar (2011) have analyzed the peristaltic transport of a Tangent hyperbolic fluid in an endoscope numerically. Akbar et al.

(2012) have discussed the peristaltic flow of a hyperbolic tangent fluid in an inclined asymmetric channel with slip and heat transfer Based on Experimental controls, it was shown that the controlled application of low intensity and frequency pulsing magnetic fields could modify cell and tissue behavior. Biochemistry has taught us that cells are formed of positive or negative charged molecules. This is why these magnetic fields applied to living organisms may induce deep modifications in molecule orientation and in their interaction. An impulse magnetic field in the combined therapy of patients with stone fragments in the upper urinary tract was experimentally studied by Li et al. (1994). It was found that impulse magnetic field (IMF) activates impulse activity of ureteral smooth muscles in 100% of cases. Elshahed and Haroun (2005) have investigated the peristaltic flow of a Johnson-Segalman fluid in a planar channel under the effect of a magnetic field. Hayat and Ali (2006) have investigated the peristaltic motion of a MHD third grade fluid in a tube. Hayat et al. (2007) have first investigated the Hall effects on the peristaltic flow of a Maxwell fluid trough a porous medium in channel. Magnetohydrodynamic peristaltic flow of a hyperbolic tangent fluid in a vertical asymmetric channel with heat transfer was studied by Nadeem and Akram (2011). Prasanth Reddy and Subba Reddy (2012) have analyzed the peristaltic pumping of third grade fluid in an asymmetric channel under the effect of magnetic fluid. Effect of hall and ion slip on peristaltic blood flow of Eyring Powell fluid in a non-uniform porous channel was studied by Bhatti et al. (2016). Subba Narasimhudu and Subba Reddy (2017) have studied the Hall effects on the peristaltic flow of a **Non-Newtonian** Hyperbolic tangent fluid in a channel. Shalini and Rajasekhar (2019) have investigated the effect of hall on peristaltic flow of a Newtonian fluid through a porous medium in a two-dimensional channel.

Moreover, flow through a porous medium has been studied by a number of researchers employing Darcy's law Scheidegger (1974). Some studies about this point have been given by Varshney (1979) and Raptis and Perdikis (1983). The first study of peristaltic flow through a porous medium is presented by Elsehawey et al. (1999). Elsehawey et al. (2000) investigated the peristaltic motion of a generalized Newtonian fluid through a porous medium. Hayat et al. (2007) have first investigated the Hall effects on the peristaltic flow of a Maxwell fluid trough a porous medium in channel. Peristaltic motion of a carreau fluid through a porous medium in a channel under the effect of a magnetic field was studied by Sudhakar Reddy et al. (2009). Subba Reddy and Prasnath Reddy (2010) have investigated the effect of variable viscosity on peristaltic flow of a Jeffrey fluid through a porous medium in a planar channel. Eldabe (2015) have studied the Hall Effect on peristaltic flow of third order fluid in a porous medium with heat and mass transfer.

Motivated by these, the effect of slip and Hall on the peristaltic pumping of a hyperbolic tangent fluid in a planar channel under the assumption of long wavelength is investigated. The expressions for the velocity and axial pressure gradient are obtained by employing perturbation technique. The effects of Weissenberg number, power-law index, Darcy number, Hall parameter, Hartmann number and amplitude ratio on the axial pressure gradient, time-averaged volume flow rate and the friction force at the wall are analyzed with the help of graphs.

II. MATHEMATICAL FORMULATION

We consider the peristaltic motion of a non-Newtonian hyperbolic tangent fluid through a porous medium in a two-dimensional channel of width $2a$ under the effect of magnetic field. The flow is generated by sinusoidal wave trains propagating with constant speed c along the channel walls. A uniform magnetic field B_0 is applied in the transverse direction to the flow. The magnetic Reynolds number is considered small and so induces magnetic field neglected. Fig. 1 represents the physical model of the channel.

The wall deformation is given by

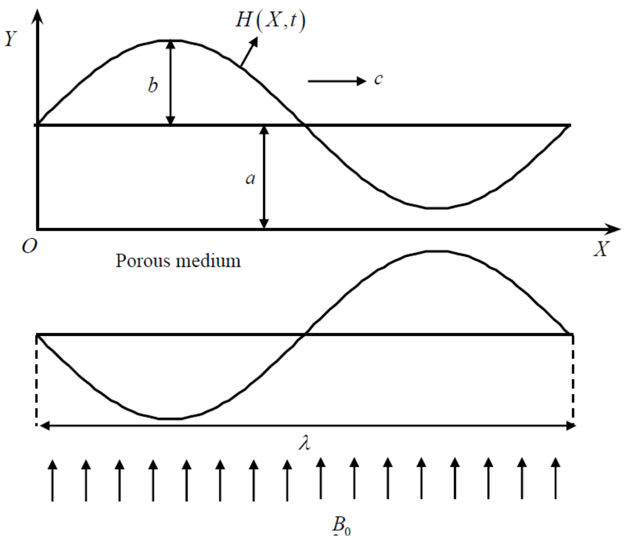
$$Y = \pm H(X,t) = \pm a \pm b\cos\frac{2\pi}{\lambda}(X - ct), \tag{2.1}$$

where b is the amplitude of the wave, λ - the wave length and X and Y - the rectangular co-ordinates with X measured along the axis of the channel and Y perpendicular to X. Let (U,V) be the velocity components in fixed frame of reference (X,Y).

The flow is unsteady in the laboratory frame (X,Y). However, in a co-ordinate system moving with the propagation velocity c (wave frame (x, y)), the boundary shape is stationary. The transformation from fixed frame to wave frame is given by

$$x = X - ct, y = Y, u = U - c, v = V \tag{2.2}$$

where (u,v) and (U,V) are velocity components in the wave and laboratory frames respectively.



The constitutive equation for a **Non-Newtonian** Hyperbolic Tangent fluid is

$$\tau = -\left[\eta_\infty + (\eta_0 + \eta_\infty)\tanh\left(\Gamma\dot{\gamma}\right)^n\right]\dot{\gamma} \tag{2.3}$$

where τ is the extra stress tensor, η_∞ is the infinite shear rate viscosity, η_o is the zero shear rate viscosity, Γ is the time constant, n is the power-law index and $\dot{\gamma}$ is defined as

$$\dot{\gamma} = \sqrt{\frac{1}{2}\sum_i \sum_j \dot{\gamma}_{ij}\dot{\gamma}_{ji}} = \sqrt{\frac{1}{2}\pi} \tag{2.4}$$

where π is the second invariant stress tensor. We consider in the constitutive equation (2.3) the case for which $\eta_\infty = 0$ and $\Gamma\dot{\gamma} < 1$, so the Eq. (2.3) can be written as

$$\tau = -\eta_0 \left(\Gamma\dot{\gamma}\right)^n \dot{\gamma} = -\eta_0 \left(1+\Gamma\dot{\gamma}-1\right)^n \dot{\gamma} = -\eta_0 \left(1+n\left[\Gamma\dot{\gamma}-1\right]\right)\dot{\gamma} \tag{2.5}$$

The above model reduces to Newtonian for $\Gamma = 0$ and $n = 0$.

The equations governing the flow in the wave frame of reference are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{2.6}$$

$$\rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = -\frac{\partial p}{\partial x} - \frac{\partial \tau_{xx}}{\partial x} - \frac{\partial \tau_{yx}}{\partial y} + \frac{\sigma B_0^2}{1+m^2}\left(mv - (u+c)\right) - \frac{\eta_0}{k}(u+c) \tag{2.7}$$

$$\rho\left(u\frac{\partial v}{\partial x} + v\frac{\partial u}{\partial y}\right) = -\frac{\partial p}{\partial y} - \frac{\partial \tau_{xy}}{\partial x} - \frac{\partial \tau_{yy}}{\partial y} - \frac{\sigma B_0^2}{1+m^2}\left(m(u+c) + v\right) - \frac{\eta_0}{k}v \tag{2.8}$$

where ρ is the density, k is the permeability of the porous medium, σ is the electrical conductivity, B_0 is the magnetic field strength and m is the Hall parameter.

The corresponding dimensional boundary conditions are

$$u + \beta\tau_{xy} = -c \quad \text{at} \quad y = H \quad \text{(slip condition)} \tag{2.9}$$

$$\frac{\partial u}{\partial y} = 0 \quad \text{at } y = 0 \quad \text{(symmetry condition)} \tag{2.10}$$

here β is the slip parameter.

Introducing the non-dimensional variables defined by

$$\bar{x} = \frac{x}{\lambda},\ \bar{y} = \frac{y}{a},\ \bar{u} = \frac{u}{c},\ \bar{v} = \frac{v}{c\delta},\ \delta = \frac{a}{\lambda},$$
$$\bar{p} = \frac{pa_0^2}{\eta_0 c\lambda}, \phi = \frac{b}{a}$$

$$h=\frac{H}{a},\ \bar{t}=\frac{ct}{\lambda},\ \bar{\tau}_{xx}=\frac{\lambda}{\eta_0 c}\tau_{xx},\ \bar{\tau}_{xy}=\frac{a}{\eta_0 c}\tau_{xy},\ \bar{\tau}_{yy}=\frac{\lambda}{\eta_0 c}\tau_{yy},$$

$$\mathrm{Re}=\frac{\rho ac}{\eta_0},\ We=\frac{\Gamma c}{a},\ \bar{\dot{\gamma}}=\frac{\dot{\gamma}a}{c},\ \bar{q}=\frac{q}{ac} \tag{2.11}$$

into the Equations (2.6) - (2.8), reduce to (after dropping the bars)

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{2.12}$$

$$\mathrm{Re}\,\delta\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=-\frac{\partial p}{\partial x}-\delta^2\frac{\partial \tau_{xx}}{\partial x}-\frac{\partial \tau_{xy}}{\partial y}+\frac{M^2}{1+m^2}\left(m\delta v-(u+1)\right)-\frac{1}{Da}(u+1) \tag{2.13}$$

$$\mathrm{Re}\,\delta^3\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)=-\frac{\partial p}{\partial y}-\delta^2\frac{\partial \tau_{xy}}{\partial y}-\delta\frac{\partial \tau_{yy}}{\partial y}-\frac{\delta M^2}{1+m^2}\left(m(u+1)+\delta v\right)-\frac{\delta^2}{Da}v \tag{2.14}$$

where $Da=\frac{k}{a^2}$ is the Darcy number,

$$\tau_{xx}=-2\left[1+n\left(We\dot{\gamma}-1\right)\right]\frac{\partial u}{\partial x}\quad \tau_{xy}=-\left[1+n\left(We\dot{\gamma}-1\right)\right]\left(\frac{\partial u}{\partial y}+\delta^2\frac{\partial v}{\partial x}\right)$$

$$\tau_{yy}=-2\delta\left[1+n\left(We\dot{\gamma}-1\right)\right]\frac{\partial v}{\partial y}\quad \dot{\gamma}=\left[2\delta^2\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial u}{\partial y}+\delta^2\frac{\partial v}{\partial x}\right)^2+2\delta^2\left(\frac{\partial v}{\partial y}\right)^2\right]^{\frac{1}{2}}$$ and

$M=aB_0\sqrt{\frac{\sigma}{\eta_0}}$ is the Hartmann number.

Under lubrication approach, neglecting the terms of order δ and Re, the Eqs. (2.13) and (2.14) become

$$\frac{\partial p}{\partial x}=\frac{\partial}{\partial y}\left\{\left[1+n\left(We\frac{\partial u}{\partial y}-1\right)\right]\frac{\partial u}{\partial y}\right\}-\left(\frac{M^2}{1+m^2}+\frac{1}{Da}\right)(u+1) \tag{2.15}$$

$$\frac{\partial p}{\partial y}=0 \tag{2.16}$$

From Eq. (2.15) and (2.16), we get

$$\frac{dp}{dx}=(1-n)\frac{\partial^2 u}{\partial y^2}+nWe\frac{\partial}{\partial y}\left[\left(\frac{\partial u}{\partial y}\right)^2\right]-\left(\frac{M^2}{1+m^2}+\frac{1}{Da}\right)(u+1) \tag{2.17}$$

The corresponding non-dimensional boundary conditions in the wave frame are given by

$$u+\beta\left[1+n\left(We\frac{\partial u}{\partial y}-1\right)\right]\frac{\partial u}{\partial y}=-1 \quad \text{at}$$

$$y=h=1+\phi\cos 2\pi x \qquad (2.18)$$

$$\frac{\partial u}{\partial y}=0 \qquad \text{at} \qquad y=0 \qquad (2.19)$$

The volume flow rate q in a wave frame of reference is given by

$$q=\int_0^h u dy\,. \qquad (2.20)$$

The instantaneous flow $Q(X,t)$ in the laboratory frame is

$$Q(X,t)=\int_0^h U dY=\int_0^h (u+1)dy=q+h \qquad (2.21)$$

The time averaged volume flow rate $\overline{Q}$ over one period $T\left(=\frac{\lambda}{c}\right)$ of the peristaltic wave is given by

$$\overline{Q}=\frac{1}{T}\int_0^T Q dt=q+1 \qquad (2.22)$$

III. SOLUTION

Since Eq. (2.17) is a non-linear differential equation, it is not possible to obtain closed form solution. Therefore we employ regular perturbation to find the solution.

For perturbation solution, we expand $u, \frac{dp}{dx}$ and q as follows

$$u=u_0+We\,u_1+O\left(We^2\right) \qquad (3.1)$$

$$\frac{dp}{dx}=\frac{dp_0}{dx}+We\frac{dp_1}{dx}+O\left(We^2\right) \qquad (3.2)$$

$$q=q_0+We\,q_1+O\left(We^2\right) \qquad (3.3)$$

Substituting these equations into the Eqs. (2.17) - (2.19), we obtain

3.1. System of order We^0

$$\frac{dp_0}{dx}=(1-n)\frac{\partial^2 u_0}{\partial y^2}-\left(\frac{M^2}{1+m^2}+\frac{1}{Da}\right)(u_0+1) \qquad (3.4)$$

and the respective boundary conditions are

$$u_0+\beta(1-n)\frac{\partial u_0}{\partial y}=-1 \quad \text{at } y=h \tag{3.5}$$

$$\frac{\partial u_0}{\partial y}=0 \quad \text{at} \quad y=0 \tag{3.6}$$

3.2. System of order We^1

$$\frac{dp_1}{dx}=(1-n)\frac{\partial^2 u_1}{\partial y^2}+\frac{\partial}{\partial y}\left[\left(\frac{\partial u_o}{\partial y}\right)^2\right]-\left(\frac{M^2}{1+m^2}+\frac{1}{Da}\right)u_1 \tag{3.7}$$

and the respective boundary conditions are

$$u_1+\beta(1-n)\frac{\partial u_1}{\partial y}+\beta n\left(\frac{\partial u_0}{\partial y}\right)^2=0 \quad \text{at } y=h \tag{3.8}$$

$$\frac{\partial u_1}{\partial y}=0 \quad \text{at} \quad y=0 \tag{3.9}$$

3.3 Solution for system of order We^0

Solving Eq. (3.4) using the boundary conditions (3.5) and (3.6), we obtain

$$u_0=\frac{1}{\alpha^2(1-n)}\frac{dp_0}{dx}\left[\frac{\cosh\alpha y}{\cosh\alpha h+\alpha\beta(1-n)\sinh\alpha h}-1\right]-1 \tag{3.10}$$

where $\alpha=\sqrt{\frac{1}{1-n}\left(\frac{M^2}{1+m^2}+\frac{1}{Da}\right)}$.

The volume flow rate q_0 is given by

$$q_0=\frac{1}{\alpha^3(1-n)}\frac{dp_0}{dx}\left[\frac{\sinh\alpha h-\alpha h\left(\cosh\alpha h+\alpha\beta(1-n)\sinh\alpha h\right)}{\cosh\alpha h+\alpha\beta(1-n)\sinh\alpha h}\right]-h \tag{3.11}$$

From Eq. (3.11), we have

$$\frac{dp_0}{dx}=\frac{(q_0+h)\alpha^3(1-n)\left(\cosh\alpha h+\alpha\beta(1-n)\sinh\alpha h\right)}{\left[\sinh\alpha h-\alpha h\left(\cosh\alpha h+\alpha\beta(1-n)\sinh\alpha h\right)\right]} \tag{3.12}$$

3.4 Solution for system of order We^1

Substituting Eq. (3.10) in the Eq. (3.7) and solving the Eq. (3.7), using the boundary conditions (3.8) and (3.9), we obtain

$$u_1 = \frac{1}{\alpha^2(1-n)}\frac{dp_1}{dx}\left[\frac{\cosh\alpha y}{(\cosh\alpha h+\alpha\beta(1-n)\sinh\alpha h)}-1\right] + \frac{n}{3}\frac{\left(\frac{dp_0}{dx}\right)^2}{c_1}g(y) \tag{3.13}$$

Where $c_1 = \left[\alpha(1-n)(\cosh\alpha h+\alpha\beta(1-n)\sinh\alpha h)\right]^3$

$$g(y)=\left\{\begin{array}{l}\left[\begin{array}{l}(\sinh 2\alpha h-2\sinh\alpha h)\cosh\alpha y+\\(2\sinh\alpha y-\sinh 2\alpha y)\cosh\alpha h\end{array}\right]\\+\alpha\beta(1-n)\left[\begin{array}{l}2(\cosh 2\alpha h-\cosh\alpha h)\cosh\alpha y\\-\sinh^2\alpha h\cosh\alpha y\\+\sinh\alpha h(2\sinh\alpha y-\sinh 2\alpha y)\end{array}\right]\end{array}\right\}.$$

The volume flow rate q_1 is given by

$$q_1 = \frac{1}{\alpha^3(1-n)}\frac{dp_1}{dx}\left[\frac{\sinh\alpha h-\alpha h(\cosh\alpha h+\alpha\beta(1-n)\sinh\alpha h)}{(\cosh\alpha h+\alpha\beta(1-n)\sinh\alpha h)}\right]+ c_2\left(\frac{dp_0}{dx}\right)^2 \tag{3.14}$$

where $c_2 = n\left(\frac{\begin{pmatrix}4-3\cosh\alpha h+2\sinh 2\alpha h\sinh\alpha h-\\\cosh 2\alpha h\cosh\alpha h\end{pmatrix}+c_3}{6\alpha^4(1-n)^3(\cosh\alpha h+\alpha\beta(1-n)\sinh\alpha h)^3}\right)$ and

$$c_3 = \alpha\beta(1-n)\begin{pmatrix}3\sinh\alpha h(\cosh 2\alpha h-1)\\-2\sinh^3\alpha h\end{pmatrix}.$$

From Eq. (3.14) and (3.12), we have
$$\frac{dp_1}{dx} = \frac{q_1N^3(1-n)(\cosh\alpha h+\alpha\beta(1-n)\sinh\alpha h)}{\left[\sinh\alpha h-\alpha h(\cosh\alpha h+\alpha\beta(1-n)\sinh\alpha h)\right]}-c_4\left(\frac{dp_0}{dx}\right)^2 \tag{3.15}$$

where $c_4 = n\left(\frac{\begin{array}{c}4-3\cosh\alpha h+2\sinh 2\alpha h\sinh\alpha h-\\\cosh 2\alpha h\cosh\alpha h+c_3\end{array}}{\begin{array}{c}6\alpha(1-n)^2(\cosh\alpha h+\alpha\beta(1-n)\sinh\alpha h)^2\\(\sinh\alpha h-\alpha h(\cosh\alpha h+\alpha\beta(1-n)\sinh\alpha h))\end{array}}\right).$

Substituting Equations (3.12) and (3.15) into the Eq. (3.2) and using the relation $\frac{dp_0}{dx}=\frac{dp}{dx}-We\frac{dp_1}{dx}$ and neglecting terms greater than $O(We)$, we get

$$\frac{dp}{dx}=\frac{(q+h)\alpha^3(1-n)(\cosh\alpha h+\alpha\beta(1-n)\sinh\alpha h)}{[\sinh\alpha h-\alpha h(\cosh\alpha h+\alpha\beta(1-n)\sinh\alpha h)]}- We c_4(q+h)^2 c_5 \tag{3.16}$$

Where $c_5=\left(\frac{\alpha^3(1-n)\begin{pmatrix}\cosh\alpha h+\\ \alpha\beta(1-n)\sinh\alpha h\end{pmatrix}}{\left[\sinh\alpha h-\alpha h\begin{pmatrix}\cosh\alpha h+\\ \alpha\beta(1-n)\sinh\alpha h\end{pmatrix}\right]}\right)^2$

The dimensionless pressure rise per one wavelength in the wave frame is defined as

$$\Delta p=\int_0^1\frac{dp}{dx}dx \tag{3.17}$$

Note that, as $Da\to\infty$ and $\beta\to 0$ our results coincide with the results of Subba Narasimhudu and Subba reddy (2017).

IV. DISCUSSIONS OF THE RESULTS

Fig. 2 shows the variation of the axial pressure gradient $\frac{dp}{dx}$ with We for $n=0.5$, $m=0.2$, $\beta=0.1$, $M=1$, $Da=0.1$, $\phi=0.6$ and $\overline{Q}=-1$. It is observed that, the axial pressure gradient $\frac{dp}{dx}$ increases with increasing Wiessenberg number We.

The variation of the axial pressure gradient $\frac{dp}{dx}$ with n for $We=0.01$, $m=0.2$, $M=1$, $\beta=0.1$, $Da=0.1$, $\phi=0.6$ and $\overline{Q}=-1$ is depicted in Fig. 3. It is found that, the axial pressure gradient $\frac{dp}{dx}$ decreases with an increase in power-law index n.

Fig. 4 illustrates the variation of the axial pressure gradient $\frac{dp}{dx}$ with β for $n=0.5$, $We=0.01$, $M=1$, $m=0.2$, $Da=0.1$, $\phi=0.6$ and $\overline{Q}=-1$. It is noted that, the axial pressure gradient $\frac{dp}{dx}$ decreases with increasing slip parameter β.

The variation of the axial pressure gradient $\frac{dp}{dx}$ with Da for $n=0.5$, $We=0.01$, $M=1$, $\beta=0.1$, $m=0.2$, $\phi=0.6$ and $\overline{Q}=-1$ shown in Fig. 5. It is noted that, the axial pressure gradient $\frac{dp}{dx}$ decreases with increasing Darcy number Da.

Fig. 6 depicts the variation of the axial pressure gradient $\frac{dp}{dx}$ with m for $n=0.5$, $We=0.01$, $M=1$, $\beta=0.1$, $Da=0.1$, $\phi=0.6$ and $\overline{Q}=-1$. It is noted that, the axial pressure gradient $\frac{dp}{dx}$ decreases with increasing Hall parameter m.

The variation of the axial pressure gradient $\frac{dp}{dx}$ with M for $n=0.5$, $m=0.2$, $We=0.01$, $\beta=0.1$, $Da=0.1$, $\phi=0.6$ and $\overline{Q}=-1$ is depicted in Fig. 7. It is observed that, on increasing Hartmann number M increases the axial pressure gradient $\frac{dp}{dx}$.

Fig. 8 shows the variation of the axial pressure gradient $\frac{dp}{dx}$ with ϕ for $n=0.5$, $m=0.2$, $\beta=0.1$, $M=1$, $We=0.01$, $Da=0.1$ and $\overline{Q}=-1$. It is found that, the axial pressure gradient $\frac{dp}{dx}$ increases with increasing amplitude ratio ϕ.

The variation of the pressure rise Δp with $\overline{Q}$ for different values of We with $n=0.5$, $m=0.2$, $\beta=0.1$, $M=1$, $Da=0.1$ and $\phi=0.6$ is shown in Fig. 9. It is noted that, the time-averaged volume flow rate $\overline{Q}$ increases with increasing Wiessenberg number We in pumping $(\Delta p>0)$, free-pumping $(\Delta p=0)$ and co-pumping $(\Delta p<0)$ regions.

Fig. 10 illustrates the variation of the pressure rise Δp with $\overline{Q}$ for different values of n with $We=0.01$, $m=0.2$, $\beta=0.1$, $Da=0.1$, $M=1$ and $\phi=0.6$. It is noted that, the time-averaged flow rate $\overline{Q}$ decreases with increasing n in both the pumping and free pumping regions, while it increases with increasing n in the co-pumping region.

The variation of the pressure rise Δp with $\overline{Q}$ for different values of β with $n=0.5$, $We=0.01$, $m=0.2$, $Da=0.1$, $M=1$ and $\phi=0.6$ is presented in Fig. 11. It is found that, the time-averaged flow rate $\overline{Q}$ decreases with increasing β in both the pumping and the free pumping regions, while it increases with increasing β in the co-pumping region.

Fig. 12 depicts the variation of the pressure rise Δp with $\overline{Q}$ for different values of Da with $n=0.5$, $We=0.01$, $\beta=0.1$, $m=0.2$, $M=1$ and $\phi=0.6$ is presented in Fig. 11. It is found that, the time-averaged flow rate $\overline{Q}$ decreases with increasing Da in the pumping region, while it increases with increasing Da in both the free pumping and the co-pumping regions.

The variation of the pressure rise Δp with $\overline{Q}$ for different values of m with $n=0.5$, $We=0.01$, $\beta=0.1$, $Da=0.1$, $M=1$ and $\phi=0.6$ is illustrated in Fig. 13. It is observed that, the time-averaged flow rate $\overline{Q}$

decreases with increasing m in the pumping region, while it increases with increasing m in both the free pumping and co-pumping regions.

Fig. 14 depicts the variation of the pressure rise Δp with $\overline{Q}$ for different values of M with $n = 0.5$, $\beta = 0.1$, $m = 0.2$, $Da = 0.1$, $We = 0.01$ and $\phi = 0.6$. It is observed that, the time-averaged flow rate $\overline{Q}$ increases with increasing M in the pumping region, while it decreases with increasing M in both the free-pumping and co-pumping regions.

The variation of the pressure rise Δp with $\overline{Q}$ for different values of ϕ with $n = 0.5$, $m = 0.2$, $\beta = 0.1$, $Da = 0.1$, $M = 1$ and $We = 0.01$ is depicted in Fig. 15. It is found that, the time-averaged flow rate $\overline{Q}$ increases with increasing ϕ in both the pumping and free pumping regions, while it decreases with increasing n in the co-pumping region for chosen $\Delta p(<0)$.

V. CONCLUSIONS

In this chapter, we studied the effects of slip and Hall on the peristaltic flow of a **Non-Newtonian** Hyperbolic tangent fluid through a porous medium in a planar channel under the assumption of long wavelength. The expressions for the velocity and axial pressure gradient are obtained by employing perturbation technique. It is found that, the axial pressure gradient and time-averaged flow rate in the pumping region increases with increasing the Weissenberg number We, the Hartmann number M and the amplitude ratio ϕ, while they decreases with increasing power-law index n, slip parameter β, Darcy number Da and Hall parameter m.

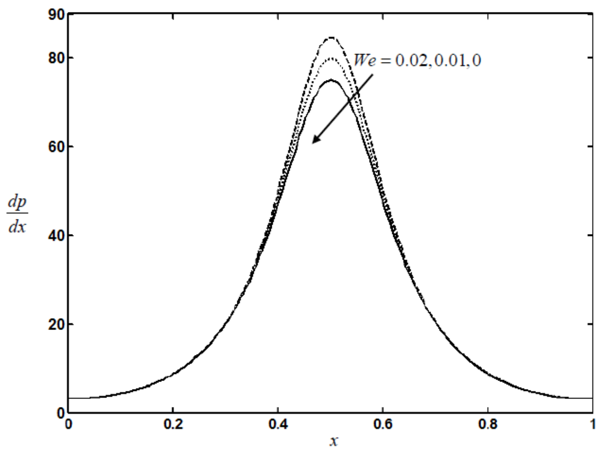


Fig. 2 The variation of the axial pressure gradient $\frac{dp}{dx}$ with We for $n = 0.5$, $m = 0.2$, $\beta = 0.1$, $M = 1$, $Da = 0.1$, $\phi = 0.6$ and $\overline{Q} = -1$.

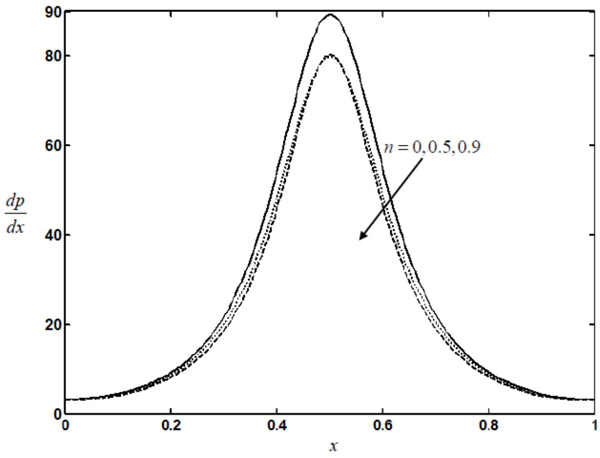


Fig. 3 The variation of the axial pressure gradient $\frac{dp}{dx}$ with n for $We=0.01$, $m=0.2$, $M=1$, $\beta=0.1$, $Da=0.1$, $\phi=0.6$ and $\overline{Q}=-1$.

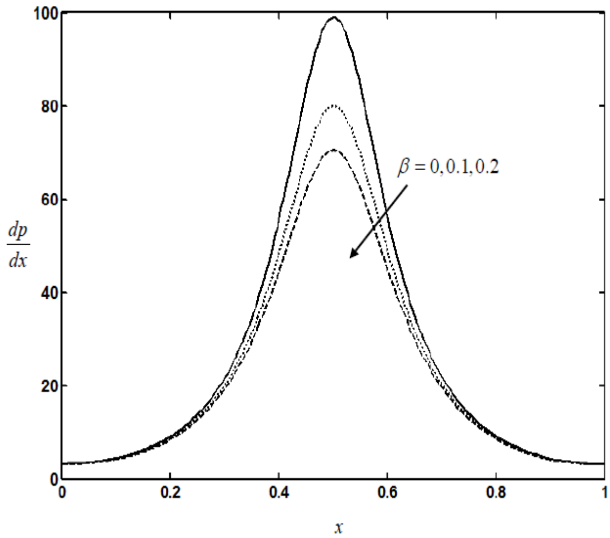


Fig. 4 The variation of the axial pressure gradient

$\frac{dp}{dx}$ with β for $n=0.5$, $We=0.01$, $M=1$, $m=0.2$, $Da=0.1$, $\phi=0.6$ and $\overline{Q}=-1$.

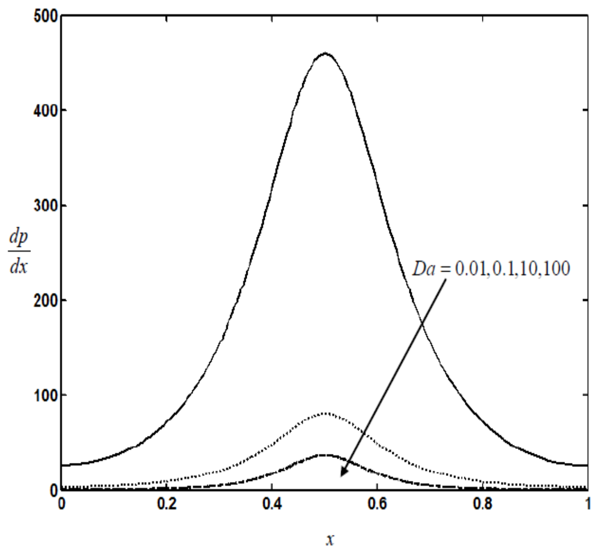


Fig. 5 The variation of the axial pressure gradient $\frac{dp}{dx}$ with Da for $n = 0.5$, $We = 0.01$, $M = 1$, $m = 0.2$, $\beta = 0.1$, $\phi = 0.6$ and $\overline{Q} = -1$.

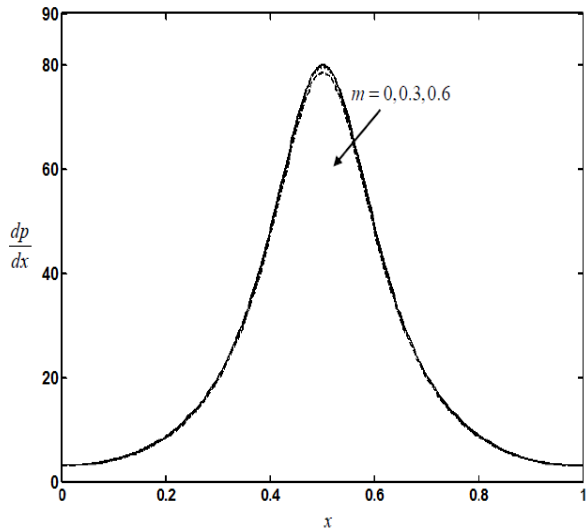


Fig. 6 The variation of the axial pressure gradient $\frac{dp}{dx}$ with m for $n = 0.5$, $We = 0.01$, $M = 1$, $\beta = 0.1$, $Da = 0.1$, $\phi = 0.6$ and $\overline{Q} = -1$.

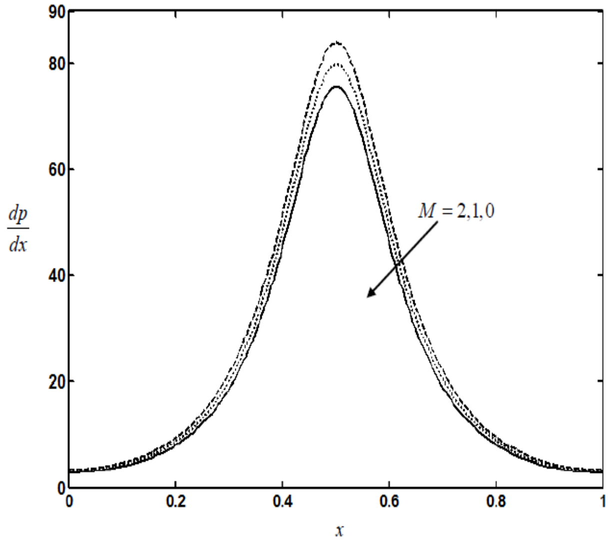


Fig. 7 The variation of the axial pressure gradient

$\frac{dp}{dx}$ with M for $n=0.5$, $m=0.2$, $We=0.01$, $Da=0.1$, $\beta=0.1$, $\phi=0.6$ and $\overline{Q}=-1$.

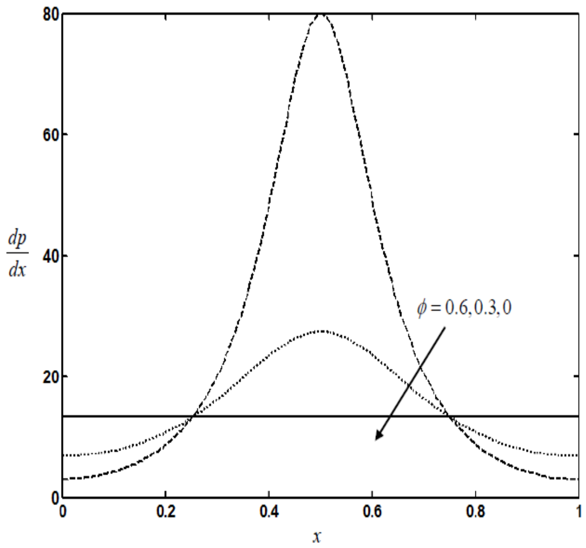


Fig. 8 The variation of the axial pressure gradient $\frac{dp}{dx}$ with ϕ for $n=0.5$, $m=0.2$, $M=1$, $\beta=0.1$, $Da=0.1$, $We=0.01$ and $\overline{Q}=-1$.

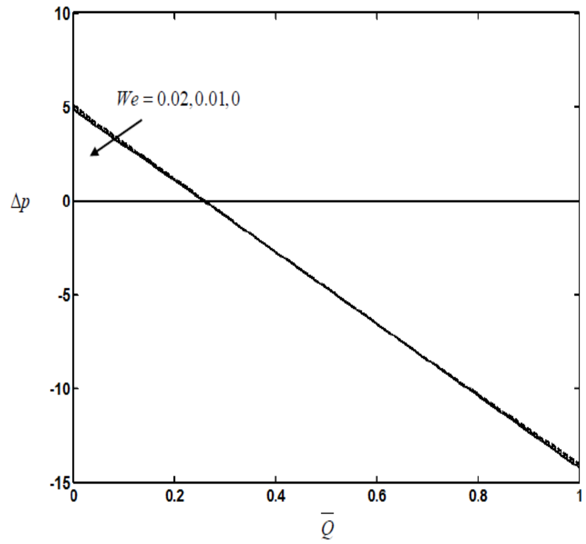


Fig. 9 The variation of the pressure rise Δp with $\overline{Q}$ for different values of We with $n = 0.5$, $\beta = 0.1$, $m = 0.2$, $Da = 0.1$, $M = 1$ and $\phi = 0.6$.

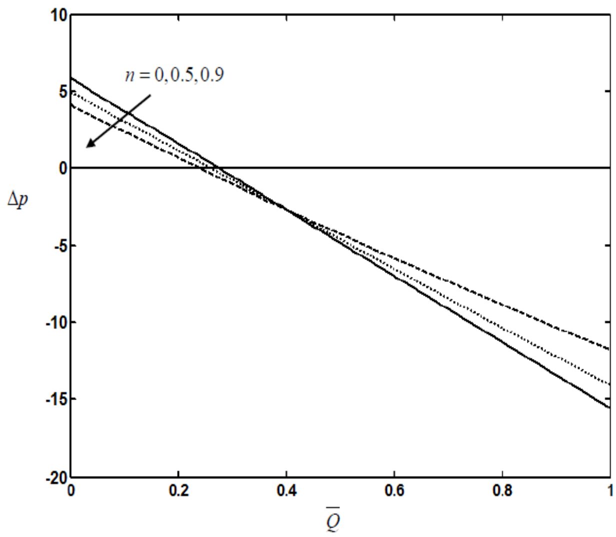


Fig. 10 The variation of the pressure rise Δp with $\overline{Q}$ for different values of n with $We = 0.01$, $\beta = 0.1$, $m = 0.2$, $Da = 0.1$, $M = 1$ and $\phi = 0.6$.

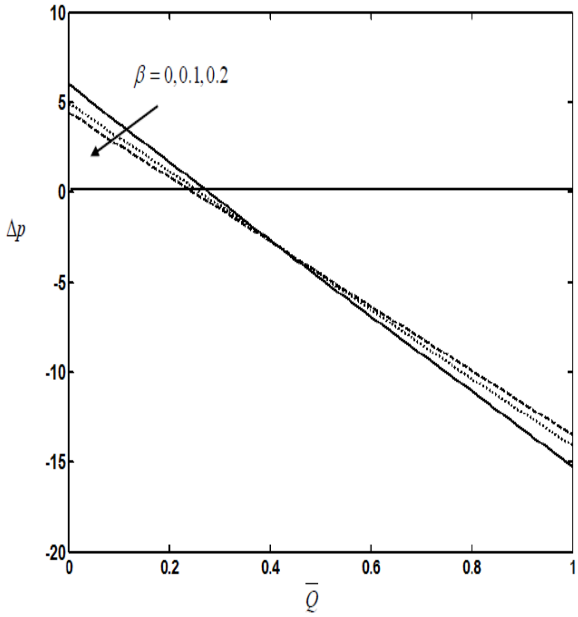


Fig. 11 The variation of the pressure rise Δp with $\overline{Q}$ for different values of β with $n = 0.5$, $m = 0.2$, $We = 0.01$, $Da = 0.1$, $M = 1$ and $\phi = 0.6$.

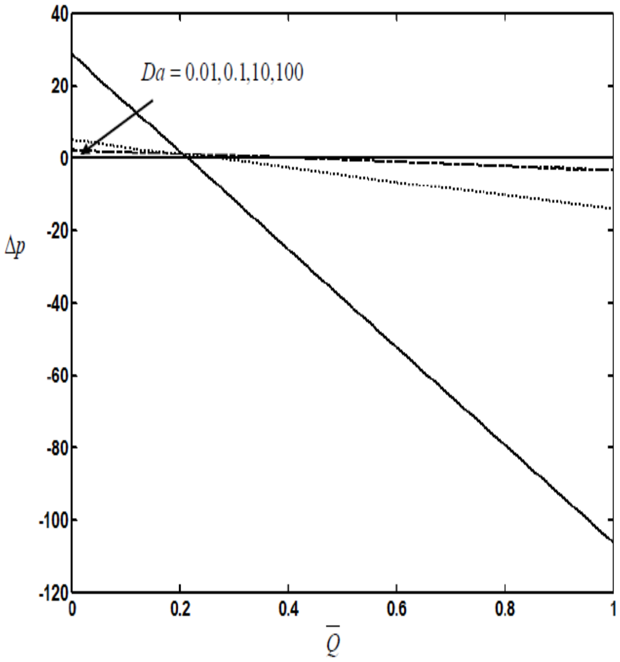


Fig. 12 The variation of the pressure rise Δp with $\overline{Q}$ for different values of Da with $n = 0.5$, $m = 0.2$, $We = 0.01$, $\beta = 0.1$, $M = 1$ and $\phi = 0.6$.

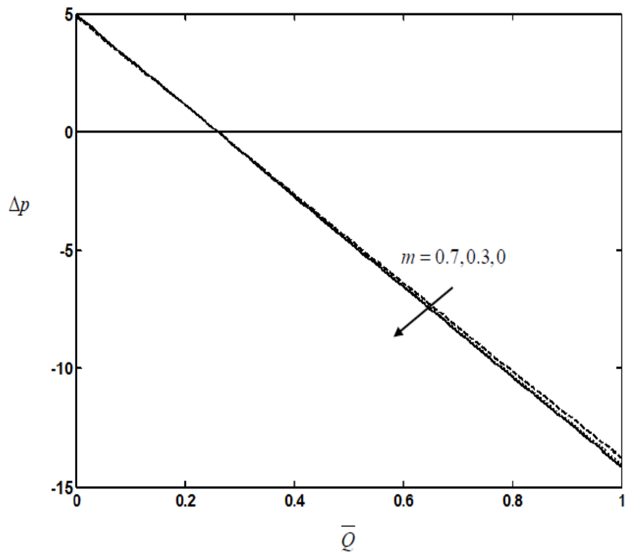


Fig. 13(i) The variation of the pressure rise Δp with $\overline{Q}$ for different values of m with $n = 0.5$, $\beta = 0.1$, $We = 0.01$, $Da = 0.1$, $M = 1$ and $\phi = 0.6$.

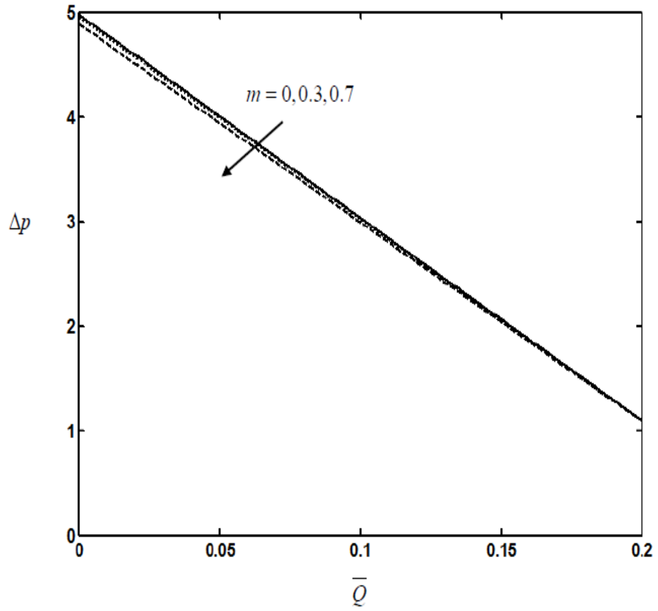


Fig. 13(ii) Enlargement of Fig. 13(i).

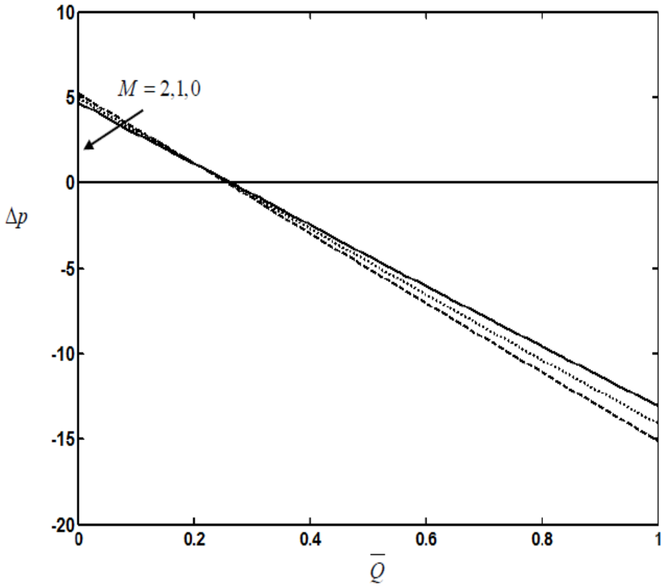


Fig. 14 The variation of the pressure rise Δp with $\overline{Q}$ for different values of M with $n = 0.5$, $\beta = 0.1$, $m = 0.2$, $Da = 0.1$, $We = 0.01$ and $\phi = 0.6$.

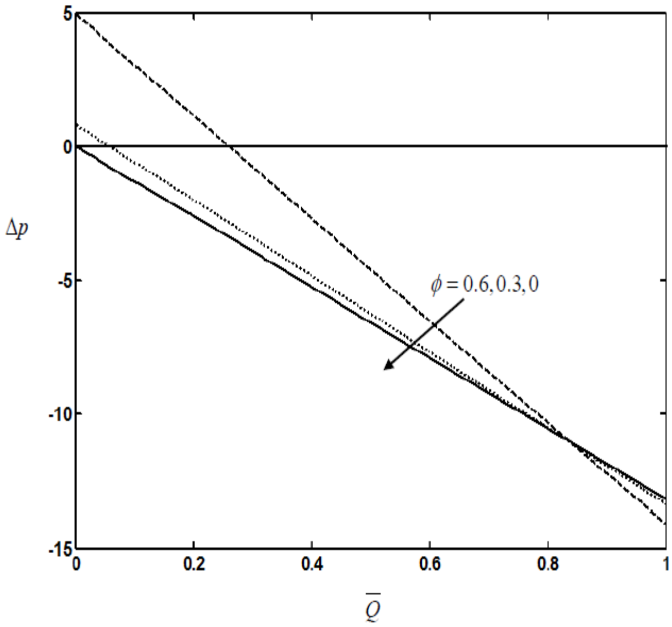


Fig. 15 The variation of the pressure rise Δp with $\overline{Q}$ for different values of ϕ with $n = 0.5$, $\beta = 0.1$, $m = 0.2$, $Da = 0.1$, $M = 1$ and $We = 0.01$.

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