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ON (1,2)gr-COMPACTNESS IN BITOPOLOGICAL SPACES

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Abstract

In this paper, we introduce the so-called (1,2)gr-compactness in bitopological spaces. In particular, several inferred properties of the (1,2)gr-compact spaces and their connections with bitopological spaces are studied using (1,2)gr-closed sets.

Index terms: (1,2)gr-closed sets, (1,2)gr-continuous maps, (1,2)gr-irresolute, (1,2)gr-compactness.

I. INTRODUCTION

Bitopological spaces is introduced by Kelly in 1963, provide a framework for studying topological properties using two topologies. Compactness is a fundamental concept in topology, and its study has been extended to bitopological spaces. This paper focuses on (1,2)gr-compactness, a compactness notion in bitopological spaces, and explores its properties and characterizations using (1,2)gr-closed sets.

The notions of compactness is useful and fundamental notions of not only general topology but also of other advanced branches of mathematics. Many researchers have analyzed the basic properties of compactness. The notions of compactness resulted in motivating mathematicians to generalize these notions further. Bhattacharya S. introduced and studied the properties of (1,2)gr-closed sets in topological spaces. The aim of this paper is to study (1,2)gr-compactness using (1,2)gr-closed set and also discuss some of their properties.

II. PRELIMINARIES

A bitopological space consists of a set X with two topologies, say τ_1 and τ_2 defined on it. These two topologies allow us to study how different topological structures interact on the same set.

Throughout this paper (X,τ_1,τ_2), (Y,σ_1,σ_2) (or simply X and Y) represent bitopological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset A of (X,τ_1,τ_2), $\mathrm{Cl}_1(A)$ denotes the closure of the set A with respect to the topology τ_1, and $\mathrm{Int}_2(A)$ denotes the interior of the set A with respect to the topology τ_2.

2.1 DEFINITION

Let (X,τ_1,τ_2) be a bitopological space. Then a subset A of (X,τ_1,τ_2) is called regular closed set if $A = \mathrm{Cl}_1(\mathrm{Int}_2(A))$.

2.2 DEFINITION

Let (X,τ_1,τ_2) be a bitopological space. Then a subset A of (X,τ_1,τ_2) is called (1,2)gr-closed set if $A = \mathrm{rCl}_1(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X,τ_1,τ_2).

The complement of the above mentioned (1,2)gr-closed set is (1,2)gr-open set.

2.3 DEFINITION

A function f : $(X,\tau_1,\tau_2) \to (Y,\sigma_1,\sigma_2)$ is called

(i) (1,2)gr-continuous if the inverse image of every closed set in (Y,σ_1,σ_2) is (1,2)gr-closed in (X,τ_1,τ_2).

(ii) (1,2)gr-irresolute if the inverse image of every (1,2)gr-closed set in (Y,σ_1,σ_2) is (1,2)gr-closed in (X,τ_1,τ_2).

III. (1,2)GR-COMPACTNESS

3.1 DEFINITION

A collection $\{G_i : i \in I\}$ of (1,2)gr-open sets in a bitopological space (X,τ_1,τ_2) is called a (1,2)gr-open cover of a subset A of (X,τ_1,τ_2) if $A \subset \cup\{G_i : i \in I\}$ holds.

3.2 DEFINITION

A bitopological space (X,τ_1,τ_2) is (1,2)gr-compact if every (1,2)gr-open cover of (X,τ_1,τ_2) has a finite subcover.

3.3 DEFINITION

A subset A of a bitopological space (X,τ_1,τ_2) is said to be (1,2)gr-compact relative to (X,τ_1,τ_2) if, for every collection $\{G_i : i \in I\}$ of (1,2)gr-open subsets of (X,τ_1,τ_2) such that $A \subset \cup\{G_i : i \in I\}$ there exists a finite subset I_0 of I such that $A \subseteq \cup\{G_i : i \in I\}$.

3.4 DEFINITION

A subset B of a topological space X is said to be (1,2)gr-compact if B is (1,2)gr-compact as a subspace of X.

3.5 DEFINITION

In bitopological spaces, a surjective function is a continuous function f : $(X,\tau_1,\tau_2) \to (Y,\sigma_1,\sigma_2)$ between two bitopological spaces (X,τ_1,τ_2) and (Y,σ_1,σ_2) such that f is surjective (onto): For every $y \in Y$, there exists $x \in X$ such that f$(x) = y$.

In other words, a surjective function in bitopological spaces is a continuous function that maps every point in the domain X to a point in the codomain Y, and every open set in Y has an open preimage in X with respect to both topologies.

3.6 THEOREM

If A be an open subset and a generalized regular closed subset of (X,τ_1,τ_2) then it is a regular closed subset of (X,τ_1,τ_2).

Proof

Let if possible, A is a generalized regular closed subset and an open subset of (X,τ_1,τ_2). Therefore $A = \mathrm{Int}_2(A)$ (an open subset of (X,τ_1,τ_2)). Hence from definition $\mathrm{rCl}_1 \subseteq \mathrm{Int}_2(A) = A$. But we know that A $\subseteq \mathrm{rCl}_1$. So, $A = \mathrm{rCl}_1$ i.e., A is a regular closed subset of (X,τ_1,τ_2).

3.7 THEOREM

If f : $(X,\tau_1,\tau_2) \to (Y,\sigma_1,\sigma_2)$ and g : $(Y,\sigma_1,\sigma_2) \to (Z,\gamma_1,\gamma_2)$ are functions, then if f : $(X,\tau_1,\tau_2) \to (Y,\sigma_1,\sigma_2)$ and are both (1,2)gr-irresolute functions, then g ∘ f: $(X,\tau_1,\tau_2) \to (Z,\gamma_1,\gamma_2)$.

Proof

Let U be a generalized regular open set in (Z,γ_1,γ_2), since g is (1,2)gr-irresolute ,then $g^{-1}(U)$ is (1,2)gr-open in (Y,σ_1,σ_2), since f is (1,2)gr-irresolute ,then f^{-1} $(g^{-1}(U))$ is (1,2)gr-open in (X,τ_1,τ_2). Since $(g \circ f)^{-1}(U)= f^{-1}(g^{-1}(U))$,then $(g \circ f)^{-1}(U)$ is a (1,2)gr-open set in (X,τ_1,τ_2). Thus g ∘ f is (1,2)gr-irresolute.

3.8 THEOREM

If f : $(X,\tau_1,\tau_2) \to (Y,\sigma_1,\sigma_2)$ and g : $(Y,\sigma_1,\sigma_2) \to (Z,\gamma_1,\gamma_2)$ are functions, then if f : $(X,\tau_1,\tau_2) \to (Y,\sigma_1,\sigma_2)$ is (1,2)gr-irresolute and g : $(Y,\sigma_1,\sigma_2) \to (Z,\gamma_1,\gamma_2)$ is (1,2)gr-continuous ,then g ∘ f: $(X,\tau_1,\tau_2) \to (Z,\gamma_1,\gamma_2)$ is (1,2)gr-continuous .

Proof

Let F be a closed set in (Z,γ_1,γ_2), since g is (1,2)gr-continuous ,then $g^{-1}(F)$ is (1,2)gr-closed in (Y,σ_1,σ_2) , since f is (1,2)gr-irresolute ,then $f^{-1}(g^{-1}(F))$ is (1,2)gr-closed in (X,τ_1,τ_2). Since $(g \circ f)^{-1}(F)= f^{-1}(g^{-1}(F))$,then $(g \circ f)^{-1}(U)$ is a (1,2)gr-closed set in (X,τ_1,τ_2). Thus g ∘ f is (1,2)gr-continuous.

3.9 THEOREM

Every (1,2)gr-closed subset of a (1,2)gr-compact space (X,τ_1,τ_2) is (1,2)gr-compact relative to (X,τ_1,τ_2).

Proof

Let A be (1,2)gr-closed subset of (1,2)gr-compact space X. Then A^C is (1,2)gr-open in X. Let $M = \{G_\alpha : \alpha \in I\}$ be a cover of A by (1,2)gr-open sets in X. Then, $M^* = M \cup A^C$ is a (1,2)gr-open cover of X. Since X is (1,2)gr-compact, M^* is reducible to a finite subcover of X, say $X=G_{\alpha_1} \cup G_{\alpha_2} \cup ... \cup G_{\alpha_m} \cup A^C$, $G_{\alpha_k} \in M$. But, A and A^C are disjoint hence $A \subset G_{\alpha_1} \cup G_{\alpha_2} \cup ... \cup G_{\alpha_m}$, $G_{\alpha_k} \in M$, which implies that any (1,2)gr-open cover M of A contains a finite subcover. Therefore, A is (1,2)gr-compact relative to X. Thus, every (1,2)gr-closed subset of (1,2)gr-compact space X is (1,2)gr-compact.

3.10 THEOREM

Every (1,2)gr-compact space is compact.

Proof

Let (X,τ_1,τ_2) be a (1,2)gr-compact space. Let $\{A_i : i \in I\}$ be an open cover of X. Then $\{A_i : i \in I\}$ is a (1,2)gr-open cover of (X,τ_1,τ_2) as every open set is (1,2)gr-open set. Since (X,τ_1,τ_2) is (1,2)gr-compact, the (1,2)gr-open cover $\{A_i : i \in I\}$ of X has a finite subcover, say $\{A_i : i=1,2...n\}$ for X. Hence X is compact.

3.11 THEOREM

Let f : $(X,\tau_1,\tau_2) \to (Y,\sigma_1,\sigma_2)$ be surjective, (1,2)gr-continuous function. If (X,τ_1,τ_2) is (1,2)gr-compact, then (Y,σ_1,σ_2) is compact.

Proof

Let $\{A_i : i \in I\}$ be an open cover of (Y,σ_1,σ_2) . Since f is (1,2)gr-continuous function, then $\{f^{-1}.\ (A_i) : i \in I\}$ is (1,2)gr-open cover of (X, τ_1, τ_2) has a finite subcover, say$\{A_i : i=1,2...n\}$. Therefore, $X=\bigcup_{i=1}^{n} f^{-1}(A_i)$ which implies $f(X)= \bigcup_{i=1}^{n}(A_i)$. Since f is surjective, $Y= \bigcup_{i=1}^{n} f(A_i)$ Thus, $\{A_1, A_2, ..., A_n\}$ is a finite subcover of $\{A_i : i \in I\}$ for (Y,σ_1,σ_2) . Hence (Y,σ_1,σ_2) is compact.

3.1.12 THEOREM

If a map f : $(X,\tau_1,\tau_2) \to (Y,\sigma_1,\sigma_2)$ is (1,2)gr-irresolute and a subset B of (X,τ_1,τ_2) is (1,2)gr-compact relative to (Y,σ_1,σ_2)then the image f(B) is (1,2)gr-compact relative to (Y,σ_1,σ_2).

Proof

Let $\{A_\alpha : \alpha \in I\}$ be any collection of (1,2)gr-open subsets of (Y,σ_1,σ_2)such that f $(B) \subset \cup\{A_\alpha : \alpha \in I\}$. Then, $B \subset \{f^{-1}(A_\alpha)$: $\alpha \in$ I} holds. From the hypothesis, B is (1,2)gr-compact relative to (X,τ_1,τ_2). Then, there exists a finite subset I_0 of I such that $B \subset \{f^{-1}(A_\alpha)$: $\alpha \in I_0\}$. Therefore, we have f $(B) \subset \cup\{A_\alpha$: $\alpha \in I_0$ }, which shows that f(B) is (1,2)gr-compact relative to (Y,σ_1,σ_2).

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