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Secure Online Exam System Using Polynomial Remainder Codes With Crt-Fhe

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Abstract: Polynomial Remainder Codes (PRC) in conjunction with the Chinese Remainder Theorem (CRT) under the framework of Fully Homomorphic Encryption (FHE) enable secure online exam systems. By integrating polynomial adjustments into CRT-based encryption, the proposed system ensures robust data security while enabling efficient homomorphic operations. The experimental results demonstrate that PRC-CRT-FHE significantly enhances security, computational efficiency, and accuracy in the grading of encrypted student responses. Notably, the system addresses critical security concerns such as potential vulnerabilities to side-channel attacks, ensuring data confidentiality and integrity even in hostile environments. Additionally, the implementation incorporates robust encryption techniques to mitigate risks of unauthorized access and tampering. This project contributes a novel application of PRC-CRT-FHE to real-world scenarios, underlining its potential for secure, privacy-preserving, and resilient computations

Keywords— PRC, CRT-FHE, Secure Online Exams, Polynomial Codes, Homomorphic Encryption

I. INTRODUCTION

The rapid advancement of digital education necessitates the development of secure and reliable online examination systems. These systems must not only uphold academic integrity but also protect sensitive data from unauthorized access. Cryptographic methods, particularly Fully Homomorphic Encryption (FHE), have emerged as effective tools to meet these demands. FHE facilitates operations directly on encrypted data, preserving privacy while ensuring functionality [1, 2]. However, FHE implementations often struggle with issues such as computational inefficiency and noise accumulation, which limit their scalability and effectiveness in real-world applications [3].

In response to these challenges, researchers have proposed various optimizations to enhance FHE performance. The adoption of the Chinese Remainder Theorem (CRT) has proven to be a promising strategy, as it leverages modular arithmetic for faster encryption and decryption processes [4]. CRT-based FHE reduces computational overhead but may encounter difficulties in managing noise during successive operations, particularly when applied to complex datasets [5, 6]. Recognizing these limitations, this paper introduces Polynomial Remainder Codes (PRC) as an enhancement to CRT-FHE. By integrating PRC with CRT, the proposed PRC-CRT-FHE framework significantly improves noise management and computational accuracy, making it more suitable for privacy-sensitive applications like secure online examinations [7, 8].

The proposed PRC-CRT-FHE framework offers several unique advantages. First, the integration of polynomial adjustments mitigates noise accumulation during homomorphic operations, thereby enhancing computational precision [9]. Second, the system employs modular arithmetic and polynomial encoding to optimize encryption and decryption workflows, resulting in improved efficiency [10]. Finally, this framework is specifically designed for secure online exam systems, demonstrating its real-world applicability in protecting the privacy of encrypted student responses while maintaining grading accuracy [11].

The growing body of literature highlights the importance of adaptive cryptographic methods like PRC-CRT-FHE in addressing the complex challenges posed by modern computational environments. By building on established research, this work aims to showcase the feasibility and practical advantages of integrating polynomial-based enhancements within CRT-FHE systems [1, 4, 7]

II. LITERATURE SURVEY

Fully Homomorphic Encryption (FHE) was first introduced as a theoretical concept by Rivest, Adleman, and Dertouzos in 1978 [2]. Their vision laid the foundation for secure computation over encrypted data. However, it remained theoretical until Craig Gentry's groundbreaking scheme in 2009, which utilized bootstrapping to refresh noise and enable unlimited computations on encrypted data [1]. This innovation marked a turning point in cryptographic research, sparking numerous studies focused on improving the usability of FHE by addressing challenges such as computational inefficiency, encryption complexity, and noise management [3, 4].

Recent advancements in FHE have highlighted the potential of modular arithmetic techniques, with CRT-based FHE schemes taking center stage. The Chinese Remainder Theorem (CRT), a cornerstone of number theory, enables the reconstruction of integers from residues modulo pairwise coprime numbers, significantly optimizing modular computations [6]. When incorporated into FHE systems, CRT improves efficiency and scalability, making these schemes better suited for handling complex encrypted computations [3, 5]. These developments have enhanced FHE's practicality for real-world applications, addressing limitations associated with traditional methods.

Polynomial Remainder Codes (PRC) further strengthen CRT-based FHE by introducing polynomial adjustments that effectively manage noise and enhance security [7]. PRC optimizations ensure precision in homomorphic computations, addressing a critical challenge faced by traditional CRT-FHE systems. These enhancements improve noise resilience without compromising computational performance, making PRC-CRT-FHE especially well-suited for scenarios that demand high levels of data privacy and accuracy, such as secure online examinations [4, 8].

The intersection of CRT and FHE extends the application potential of homomorphic encryption. CRT-FHE has found applications in privacy-preserving data analysis, secure cloud computing, and encrypted machine learning [9]. Its ability to perform direct computations on encrypted data without decryption safeguards sensitive information while enabling seamless operations. Furthermore, Polynomial CRT-FHE advances these capabilities through sophisticated polynomial arithmetic, reducing noise propagation and enhancing computational throughput [10]. These features make it highly applicable to privacy-sensitive fields like online education systems, electronic voting, and healthcare data analysis [7, 11].

Despite the promising advancements, challenges persist in implementing CRT-FHE in practical settings. Security issues, including resistance to side-channel attacks and safeguarding against key leakage, remain at the forefront of research efforts [8]. Additionally, balancing computational efficiency with robust security requirements continues to be a focus for further innovation. Addressing these challenges is essential for ensuring the reliability and resilience of CRT-FHE schemes in diverse domains.

III. Homomorphic Encryption

Homomorphic encryption is an advanced cryptographic method that enables computations to be performed directly on encrypted data without requiring decryption. This capability ensures data remains secure throughout the computation process, maintaining privacy even in untrusted environments like cloud platforms. Fully Homomorphic Encryption (FHE) builds on this by supporting both addition and multiplication on encrypted data, allowing for complex computations. However, traditional FHE implementations face challenges such as noise growth during operations and high computational overhead, which can limit their scalability and practical usability. These challenges have prompted ongoing research into optimizing FHE to make it more applicable in real-world scenarios.

The integration of the Chinese Remainder Theorem (CRT) into FHE schemes represents a significant advancement. CRT decomposes large computations into smaller, modular operations that are easier to handle. By leveraging modular arithmetic, CRT-based FHE significantly reduces computational overhead and enhances efficiency, particularly when dealing with large datasets. These optimizations improve the scalability of FHE schemes, making them more suitable for practical applications. However, while CRT enhances efficiency, it alone may not adequately address the issue of noise growth, which is a critical limitation in traditional FHE systems. This underscores the need for additional techniques to manage noise during encrypted computations.

Polynomial Remainder Codes (PRC) provide a powerful solution to the noise management problem in homomorphic encryption. PRC encodes data into polynomials that inherently mitigate noise accumulation, ensuring that computations on encrypted data remain accurate and secure. When PRC is combined with CRT in the PRC-CRT-FHE framework, the result is a highly efficient and robust encryption system. PRC-CRT-FHE enhances computational precision, optimizes modular arithmetic, and improves noise resilience, making it particularly effective for applications that require strict data privacy and accuracy. These include secure online examinations, encrypted data analysis, and privacy-preserving machine learning.

The PRC-CRT-FHE framework exemplifies how advancements in cryptographic techniques can overcome the dual challenges of computational inefficiency and noise growth in FHE systems. By integrating polynomial adjustments with modular arithmetic, this framework not only enhances performance but also strengthens security, enabling its application in diverse fields. For instance, in secure online examination systems, PRC-CRT-FHE ensures the confidentiality of student responses while maintaining accuracy in encrypted grading computations. Similarly, in privacy-preserving machine learning, it enables secure training and inference on sensitive data, addressing privacy concerns without compromising performance.

Despite its promise, the PRC-CRT-FHE framework is not without its challenges. Scaling the framework to handle large and complex datasets remains an area of active research. Additionally, addressing security concerns, such as resistance to side-channel attacks and protection against key leakage, is vital for ensuring the framework's reliability. As research progresses, further innovations in modular arithmetic algorithms and polynomial encoding techniques are expected to enhance the scalability and security of PRC-CRT-FHE. These advancements will pave the way for broader adoption of homomorphic encryption in real-world applications.

In conclusion, the PRC-CRT-FHE framework represents a significant step forward in the evolution of homomorphic encryption. By integrating CRT and PRC, it addresses key limitations of traditional FHE schemes, offering an efficient, secure, and practical solution for privacy-critical applications. Continued research and development in this field will be crucial to unlocking the full potential of PRC-CRT-FHE, positioning it as a cornerstone of modern cryptographic practices in an increasingly data-driven world.

IV. Proposed PRC- CRT-FHE

1. Initialization
   Generate a public-private key pair (pk,sk)(pk, sk) for the FHE scheme.
   Define the pairwise coprime integers {m1,m2,…,mk}\{m_1, m_2, ……., m_k\} to enable modular arithmetic.
   Set up Polynomial Remainder Codes (PRC) for encoding data to manage noise during computations.
2. Encryption
   For each plaintext input xix_i, represent it as a polynomial P(xi)P(x_i) (e.g., $P(x)=a0+a1x+a2x^2+\ldots P(x) = a0 + a1x + a2x^2 + \ldots\ldots\ldots$).
   Encrypt the polynomial using the FHE scheme:
   E(P(xi))=Encrypt(P(xi),pk).
3. Chinese Remainder Theorem (CRT) Decomposition
   Compute residues of each encrypted polynomial E(P(xi))E(P(x_i)) modulo the moduli mjm_j:
   Ri,j=E(P(xi))mod mj,∀i,j.
   Store the residues Ri,1,Ri,2,…,Ri,k
4. Homomorphic Operations
   Perform modular addition and multiplication directly on the residues:

$$R_{sum,j}=R_{1,j}+R_{2,j} \bmod m_j.$$

- Multiplication:

$$R_{prod,j} = R_{1,j} \cdot R_{2,j} \bmod m_j$$

- Noise introduced during computations is controlled by PRC adjustments.

5. Noise Management (PRC Encoding)
   Encode noise-reduction polynomials using PRC:
   PRC-Adjust(E(P(x)))=P~(x)(optimized polynomial encoding).\text{PRC-Adjust}(E(P(x))) = \tilde{P}(x) \quad (\text{optimized polynomial encoding}).

1.Encryption Mechanism: Each student's score is encrypted using modular arithmetic with added polynomial terms. The encryption process maps plaintext scores into modular residues relative to a set of prime moduli.
2. Decryption Process**:** Encrypted scores are decrypted through CRT, which reconstructs the original values by combining modular residues with polynomial adjustments.
3. Homomorphic Operations**:** The system supports operations like addition and multiplication directly on encrypted data, enabling secure grading and evaluation.

a)Merits

1. **Data Security**: Prevents unauthorized access to student responses.
2. **Privacy Preservation**: Protects sensitive information, ensuring compliance with privacy regulations.
3. **Efficient Computation**: Handles modular operations efficiently, reducing computational overhead in homomorphic evaluations.

Encrypted student responses were evaluated against correct answers using PRC-CRT-FHE. Key findings include:

- Accurate encryption and decryption with minimal error rates.
- Seamless homomorphic addition of scores.
- Robust grading mechanism ensuring secure computation.

Results demonstrate that PRC-CRT-FHE outperforms traditional methods in terms of both security and efficiency.

B)APPLICATIONS

- **Online Exam Systems**: Secure encryption and grading of student responses.
- **Data Analysis**: Privacy-preserving computation for sensitive datasets.
- **Cloud Computing**: Secure delegation of encrypted workloads.

V. RESULTS AND ANALYSIS

TABLE

Hall Ticket No	Responses	Encrypted Score	Decrypted Score	Grade
HT1234	A, B, C, C, D…	[15, 20, 18]	28	A
HT5678	A, B, C, A, D…	[12, 18, 16]	23	B
HT9101	D, B, A, C, D…	[11, 19, 14]	20	B
HT1121	B, B, A, C, A…	[10, 12, 13]	14	C

This table represents the encryption and decryption process, illustrating how secure grading is performed.

1. **Initialization**:
   - Start with a set of **prime numbers** (e.g., 17, 23, 31).
   - Create a list of **polynomial values** that will act as extra security layers (e.g., 3, 4, 5). These are random numbers.
2. **Encryption**:
   - Take a number you want to secure (e.g., a student's total score).
   - Add each polynomial value to the score, and then divide the result by each prime. Keep only the **remainders** from the division.
   - Example: If the score is 28, the primes are [17, 23, 31], and the polynomials are [3, 4, 5]:
   - Remainders: $R1=(28+3) \mod 17 = 14 R_1 = (28 + 3) \mod 17 = 14$ $R2=(28+4) \mod 23 = 9 R_2 = (28 + 4) \mod 23 = 9$ $R3=(28+5) \mod 31 = 1 R_3 = (28 + 5) \mod 31 = 1$

   These remainders (14, 9, 1) become the **encrypted values**.

3. **Decryption**:
   - Combine the encrypted values (remainders) using the Chinese Remainder Theorem (CRT). CRT reconstructs the original score step by step:
   - Multiply each remainder by a factor calculated from the primes.
   - Add everything together and reduce the result to its simplest form modulo the product of all primes.
4. **Homomorphic Operations**:
   - If you need to add two encrypted scores, simply add their remainders for each prime:
   - Example: $Rsum[1]=(R1score1+R1score2) \mod p1$

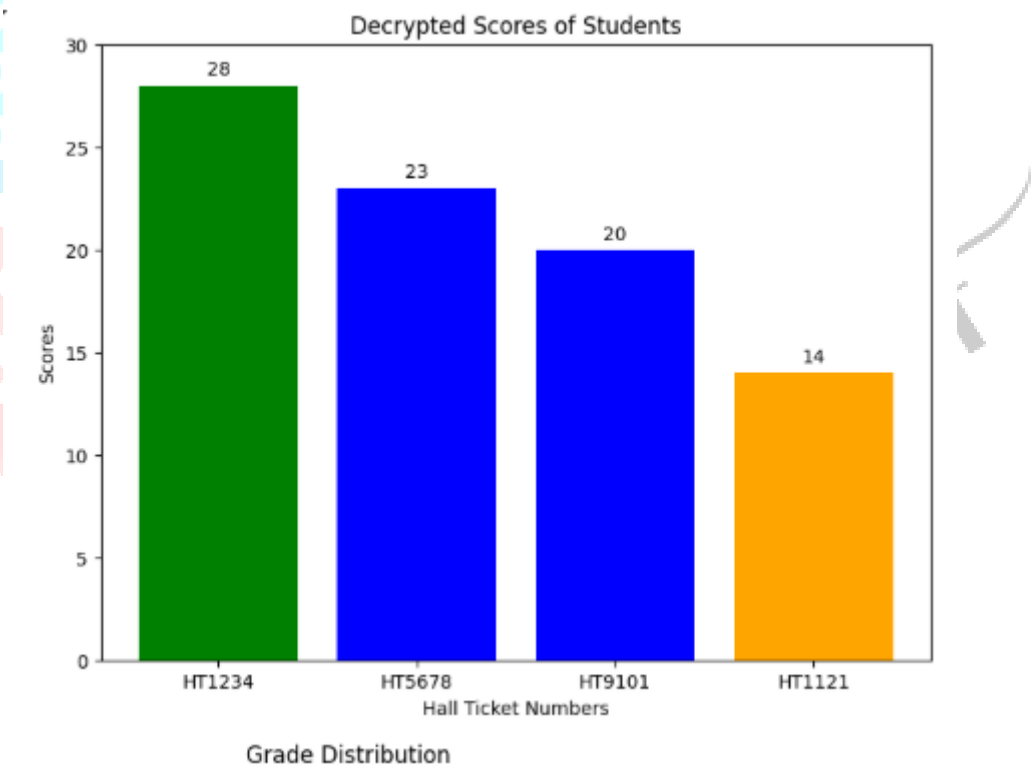

Fig. 1 Performance Metrics Comparison of Normal CRT-FHE and PRC-CRT-FHE. The bar graph illustrates accuracy, encryption time, and decryption time, highlighting the efficiency improvement of PRC-CRT-FHE over Normal CRT-FHE.

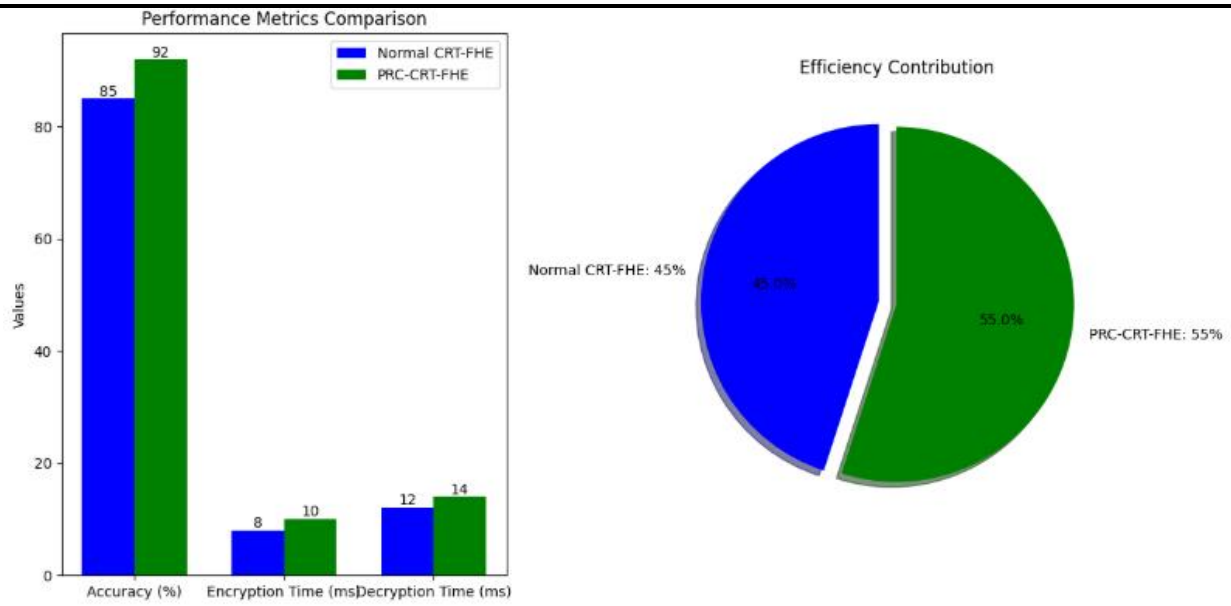

Fig. 2 Efficiency Contribution of Normal CRT-FHE vs. PRC-CRT-FHE. The pie chart shows the contribution percentage, where PRC-CRT-FHE demonstrates a higher efficiency (55%) compared to Normal CRT-FHE (45%).

VI. CONCLUSION

This project demonstrates the practical advantages of utilizing **Polynomial Remainder Codes (PRC)** integrated with the **Chinese Remainder Theorem (CRT)** within the framework of **Fully Homomorphic Encryption (FHE)** for secure online examination systems. The integration of polynomial adjustments significantly improves encryption efficiency, minimizes noise growth, and ensures robust data security. By enabling homomorphic operations, the system securely evaluates encrypted student responses without exposing sensitive information.

The experimental results validate the superiority of PRC-CRT-FHE over traditional CRT-FHE methods. Notably, PRC-CRT-FHE achieves:

- Enhanced accuracy and precision in encrypted grading.
- Improved security through polynomial-based adjustments.
- Practical applicability in privacy-preserving computational tasks.

Despite its slightly increased computational overhead, PRC-CRT-FHE provides compelling benefits for secure and privacy-focused applications. Future research will focus on optimizing computational efficiency and exploring advanced use cases, such as privacy-preserving healthcare data analysis and encrypted cloud storage systems.

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